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Logic Puzzle #4

The total points given out in the class is T = 22+9+9 = 40. Let n be the number of papers, and x be the number of points given out total for each paper, i.e. x = p+q+r. By definition, T = n*x, so x must divide T = 40. Also, p > q > r > 0, so at minimum r = 1, q = 2, and p = 3, so 6 <=x <= 40. Thus x is 8, 10, 20, or 40. If x = 40, then n = 1, so Amie would need to receive 22, Matthew 9, and Sven 9 on the one paper, which contradicts both Matthew having the best grade on the first paper as well as q > r. If x = 20, then n = 2. Since Matthew got the best grade on the first paper, and his total score is 9, p is at most 8, for otherwise Matthew's total score would be greater than 9. But then Amie's scores are at most 7+8 < 22, a contradiction. For x = 10 and x = 8, see the table below, which delineates all possibilities for p, q, and r. Note that many times Matthew's score creates problems, as he must have the highest score on the first paper. We will label each students scores by their first initial followed by the number of the paper, eg. M2 is Matthew's second paper's score.

x	n	p	q	r	Solution?	Reason
10	4	7	2	1	no	M1 = 7, M2 = M3 = M4 >=1, with Mtotal > 9.
10	4	6	3	1	no	Since M1 = 6, Amie has at most 3 scores of 6, with A1 <= 3, meaning Atotal <= 21.
10	4	5	4	1	no	M1 = 5, and no summation of three 4's and 1's equals 4.
10	4	5	3	2	no	M1 = 5, and no summation of three 3's and 2's equals 4.
8	5	5	2	1	yes	Amie = 2+5+5+5+5. Matthew = 5+1+1+1+1. Sven = 1+2+2+2+2.
8	5	4	3	1	no	M1 = 4, and no summation of four 3's and 1's equals 5.

So, to answer the questions:

1. There were 5 papers.

2. p = 5, q = 2, r = 1.

3. Sven had the second best grade on the second paper, as he got 2's on all papers besides the first one.