Logic Puzzle Number 1: Bottles of Champagne

Solution by Yuqi Mao ’09

1 & 2. In the 16-student and the 8-student scenarios, we can employ the same method. Use a binary system of 8 digits to label all bottles in base 2 from 0 to 239. Have each of the 8 students pick a number between 1 and 8 and stick to that number, that will be the nth digit out of a bottle that they look at. If the bottle has a 1 in that digit, the student will drink a drop, if the bottle has a 0 in that digit, the student will leave it alone.

It comes to a system like this, an example with 3 students and 8 bottles:

Student	1	2	3	
Bottles	0	0	0	Bottle 0
	0	0	1	Bottle 1
	0	1	0	…
	0	1	1	…
	1	0	0	…
	1	0	1	…
	1	1	0	…
	1	1	1	Bottle 7

Using a combination of whose teeth turned blue, we can construct numbers 0 to 7, and that covers all 8 possibilities. Considering 3 students, we get 2^3 bottles, then using 8 students and the same method, we cover 2^8=256 possible bottles, which is beyond the needed 240. I hope my extrapolation is fairly easy to follow.

3. What we haven’t considered yet is the amount of time we have. We have enough time for two tests, one test shows up between 8 and 11 hours. Before we see the results of the first test, we test a second time, and that test can be arranged to show up after 11 hours from the start time and before the end of the total available time. Say one test starts at 6, and the second test starts at 10. There is not enough time to cover a third test, no matter how close we squeeze the result slots. So separating the two tests like this gives us results from the first test between 2 and 5 AM and from the second test between 6 and 9 AM.

So with two time slots, now fully using all of the information from the puzzle, we actually have a system we can model in base 3. Take, for example, if it were only me, and I had 2 time slots, I can test 3 bottles, 1 bottle at time 1, another at time 2, and

one left untested. Then, let us look at a two people scenario using base 3, we can test up to 9 bottles, numbered 0-8:

Bottle Number	Student 1	Student 2
0	0	0
1	0	t1
2	0	t2
3	t1	0
4	t1	t1
5	t1	t2
6	t2	0
7	t2	t1
8	t2	t2

0 = Student does not test this bottle

t1= Student tests at Time 1

t2= Student tests at Time 2

So we have a base 3 system that tests $3^{\text{Number of Testers}}$. Extending this solution to 5 people will yield $3^5=243$ total possible bottles. That covers 3 more bottles than the required 240. In the system where all slots are taken, each student tastes 81 bottles at t1, and 81 bottles again at t2, leaving 81 bottles alone. Of course, each bottle must be labeled and we must keep track of them, not numbering them means that Dean Urgo must tell the people at brunch that whomever gets blue teeth will get a prize.

So this is basically the same solution as in part 2, only that now we are taking full advantage of the time available to convert the solution from base 2 to base 3.

Though at first, this solution may seem a bit shaky, because what will you do if your teeth turn blue in the resulting time from test 1? But you don't actually care about anything that comes after if your teeth turn blue, the time your teeth turn blue is the only information we want. Thus, this solution is valid.

Bonus:

Since the bottle is at random, and everyone ultimately tests 2/3 of the total number of bottles. That means that any one of them has a 2/3 probability of having blue teeth by the start of the brunch.