On Quine, Ontic Commitments, and the Indispensability Argument

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Abstract:

I argue against Quine's procedure for determining ontic commitments by examining the indispensability argument. The indispensabilist relies on Quine's procedure, which involves regimenting scientific theory into first-order logic. I argue that any regimented scientific theory will suffer from various flaws, including incompleteness, which undermine Quine's procedure. The indispensabilist's main opponent, who tries to eliminate quantification over mathematical objects, also employs Quine's procedure, and I show how similar problems arise.

§1: Quine's Indispensability Argument

Contemporary philosophers often assume, either implicitly or explicitly, that disputes over what exists are best resolved by examining regimented theories. Specifically, this assumption is the Quinean allegation that we can and should represent all of our existence claims in a formal theory, cast in the canonical language of first-order logic. In this paper, I argue against Quine's procedure for determining ontic commitments by examining the indispensability argument. I expose some of the consequences of a latent reliance on this procedure.

I start by specifying Quine's procedure for determining ontic commitments:

(QP) QP.1: Our ontic commitments are those of the theory which best accounts for our empirical experience.
QP.2: The ontic commitments of our best theory are found in the existential quantifications of the theory when it is cast in first-order logic with identity.
QP.3: To determine the objects over which a theory quantifies, we look at the domain of quantification of the theory to see what objects the theory needs to come out true.

Quine's restriction of ontic commitments to the construction and modeling of a single theory used to account for empirical experience, is an element of his naturalism. QP.2 reflects Quine's preference for first-order logic as a canonical language. QP.3 leads pretty directly to his doctrine of ontological relativity.

QP is in part a reaction to logical positivism, which was notoriously hostile to metaphysics. Quine proposed a single method for determining what exists, restoring metaphysics by making it the byproduct of a disinterested theoretical construction. Quine's indispensability argument, which concludes that we are committed to mathematical objects, is a quick corollary of this procedure. Note that QI.1 summarizes QP.

(QI) QI.1: We are committed to whatever we existentially quantify over in our best theory of our empirical experience.
QI.2: We existentially quantify over mathematical objects in our best theory of our empirical experience.
QI.C: We are committed to mathematical objects.

QI works as follows: We construct our ideal physical theory, and regiment it in first-order logic. We then discover that the theory includes, in the casting of physical laws, certain functions, say, or numbers. For example, consider Coulomb's Law: $F = k\ |q_1 q_2| / r^2$. This law states that the electromagnetic force between two charged particles is proportional to the charges on the particles and, inversely, to the square of the distance between them. Here is an incomplete regimentation of Coulomb's Law, which nevertheless suffices to demonstrate its commitments, using 'Px' for 'x is a charged particle'.

(CL) $\forall x \forall y\{(Px \wedge Py) \supset (\exists f)[<q(x), q(y), d(x,y), k, F> \mid F = (k \cdot |q(x) \cdot q(y)|) / d(x,y)^2\}$

Besides the charged particles over which the universal quantifiers in front range, there is an existential quantification over a function, f. Furthermore, this function maps numbers (the Coulomb's Law constant, and measurements of charge and distance) to other numbers (measurements of force between the particles). The ideal theory under consideration includes, of course, other laws with similar mathematical elements.

In order to ensure that there are enough sets to construct these numbers and functions, and in order to round out the theory, which may be justified by considerations of simplicity, the ideal scientific theory includes set-theoretic axioms, say those of Zermelo-Fraenkel set theory, ZF. Almost all the axioms of ZF contain existential assertions (null set, pair set, union, power set, separation, infinity, the replacement axioms, foundation).

Reading existential claims seems prima facie quite straightforward. Consider the null set axiom, $(\exists x)(\forall y){\sim}(y \in x)$, which, taken at face value, requires the null set. Quine, though, urges us not to conclude that objects exist directly from the existential claims. Instead, we ascend to a metalanguage to construct a model for the theory which includes a domain of quantification in which we find values for all variables of the object theory. The elements required are those which we need as values of the variables bound by the first-order quantifiers. "To be is to be the value of a variable." (Quine (1939) p 50)

Despite the move to a model, it is clear that from CL and laws like it, using QP, we can derive a vast universe of sets. So, says the indispensabilist, CL contains or entails many mathematical existential claims. Dispensabilist responses to QI, like Field (1980), accept QI.1, and try to eliminate these types of quantifications, denying only QI.2.

Quine's defends his procedure by appealing to the myriad ways in which first-order regimentation simplifies and resolves disputes concerning commitments.[1] In individual cases, like dealing with non-existence claims, the procedure is neatly effective. But, I shall argue, applying this formal method more broadly leads to profound difficulties.

§2: Quine's Innovation and Incompleteness

The incompleteness of any regimented theory sufficiently strong to encapsulate our best science makes that theory insufficient for revealing ontic commitments. It will omit relevant information. I first sketch a bit of the history which led to Quine's innovative linking of

[1] The arguments for using first-order logic as canonical are ubiquitous in Quine's work. See especially Quine (1948), Quine (1960), and Quine (1986).

regimentation and ontic commitment to show that it is independent of the motivations of those who initially developed those formal languages.

A regimented scientific theory will consist of a set of axioms within a deductive apparatus which guides inference syntactically. Aristotle's syllogisms are the prototype for shifting the focus for inference to syntax, but he was concerned with clarity, not ontic commitments. The work on formal theories with the most historical relevance to QP started in the nineteenth century, when several problems impelled mathematicians to seek greater clarity and foundation for their work. In geometry, the questions which had been percolating about Euclid's parallel postulate reached a head around mid-century with the work of Lobachevsky and Riemann. Cantor's controversial work in set theory soon followed. While Cantor looked to foundations to defend the rigor of his work with transfinites, his set theory itself, which entailed the Burali-Forti paradox and relied on the faulty axiom of comprehension, impelled increased precision. Worries about foundational questions in mathematics had reached a tipping point, and formal systems came to be seen as essential within mathematics proper. For about fifty years, from, say, Frege's *Begriffsschrift* (1879) to Gödel's Incompleteness Theorems (1931), formal systems were explored with the hope that foundational questions would be resolved.

Mathematicians were encouraged by the clarity of formal theories, especially Peano's postulates for arithmetic (1889) and Hilbert's subsequent axiomatization of geometry (1902), if not by their fruitfulness. The key work in non-Euclidean geometry was done prior to axiomatization. Similarly, set theory was not axiomatized until 1908, when Zermelo presented the first rigorous system, well after Cantor's success with transfinites. (Dedekind had published a fragmentary development in 1888.)

Of course, there were existence questions on the minds of those who developed these formal systems, questions about the existence and plenitude of transfinites, for example. But the main worry was antinomy, not existence. Despite resistance due perhaps to worries about specific formulations of set theory, Cantor's achievements were compelling. Hilbert, for example, refused exile from Cantor's paradise, despite profound concerns to establish finitistic foundations for mathematics.

Though mathematical proofs had long existed, once the axiomatic theories of the late nineteenth and early twentieth centuries were developed, the notion of proof became grounded. A proof in any discipline is a sequence of statements each of which is either an axiom, or follows from axioms using prescribed rules of inference. Other notions of proof were either reducible to this kind of proof, or dismissed as unacceptably informal.

The main philosophical goal of axiomatizing mathematics was to explicate mathematical truth in terms of provability: Mathematical theorems are true just in case they are provable in a formal system with accepted axioms. In one direction, deriving truth from provability would ground mathematics with assurance that theorems are derived from accepted postulates. We would know that our theorems are clean. In the other direction, the equation would delimit clear boundaries on the possible theorems of mathematics.

Frege, hoping to return to the "Old Euclidean standards of rigor," (Frege (1953) p 1) looked to formalize all deduction. Formal languages like those of Frege were easily adaptable to include physical axioms. All of human knowledge, it could easily have been hoped, could be derived within a formal theory. Truth and provability could be aligned in all disciplines.

Russell's paradox for Frege's nascent set theory was the first sign of a problem. Frege did not abandon axiomatics, though one might see the paradox as a reductio on the sufficiency of axiomatizations of set theory to capture our notion of set.

After Gödel's incompleteness theorems, hopes for identifying truth with provability for sufficiently complex formal theories were dashed. Mathematical truth turned out to be provably distinct from mathematical proof in a single formal system. The divergence of mathematical truth and proof is a remarkable philosophical achievement, and it extends beyond mathematics. In any discipline whose formalization is sufficiently strong to be of interest, we must distinguish truth from proof within a single formal system. Any formal theory which might serve as our best scientific theory is strong enough to be shown incomplete. The commitments of science thus can not be found in a formal theory. In particular, the indispensability argument, which relies on the construction of formal scientific theories, is invalid.

One might think that the inference from Gödel's incompleteness theorems to the invalidity of QI is too quick. For, QI needs only the sufficiency of proof for truth, and Gödel only showed that formal theories omit commitments, not that they generate false ones. But further problems arise from relying on formal theories to reveal ontic commitments. Even a complete theory, like the first-order theory of the reals, may not be categorical. A theory is categorical if all its models are isomorphic. Failure of categoricity entails that there will be non-standard models.

Even in mathematics proper, formal theories have limited appeal. Consider Paul Benacerraf's argument that we can not choose between various adequate set-theoretic reductions of the numbers. Jerrold Katz responds that we do have tools to select determinately the objects which appropriately model our number-theoretic axioms. Calling numbers communal property, among different fields which share interests in their diverse properties, he argues that no formal system can capture all we know about numbers.

> It is in the nature of formalization and theory construction to select those properties of the objects that have a role in the structure chosen for study. Moreover, selectiveness is essential in the formal sciences because numbers and the other objects they study are not the private property of any one discipline... The mathematician's special interest in numbers is with their arithmetic structure; the philosopher's is with their ontology and epistemology. From the standpoint of the inherent selectiveness of formalization and theory construction, the assumption of Benacerraf's argument that we know nothing about the numbers except what is in number theory seems truly bizarre. (Katz (1998) p 111)

Typical axiomatizations of number theory provide no information about the abstractness of numbers, or how we come to know about them. We can formalize the notion of circle as the locus of all points equidistant from a given point, but unless the domain is strictly larger than the rationals, we do not even really get circles. Geometric axiomatizations provide no insight into the epistemology of points, or surfaces. Set theoretic axiomatizations give us no insight into the modality of sets.

QI, indeed Quine's procedure for revealing existence claims generally, demands more of formal theories than they can deliver. The tool is not up to the task.

§3: The Regimentation of Ontic Prejudice

One reason to favor QP is because the clarity of regimented language can help us to reveal our presuppositions, and avoid making errant claims. We can regiment scientific theory without consideration of its commitments. We focus on generating a simple and elegant axiomatization. Then, we look to the regimented theory to reveal its existence claims, which are byproducts of a neutral process.

The Quinean picture I just described is misleading. When we regiment to clarify our commitments, we permit existential generalization only in cases where we desire to express commitment. A nominalist with respect to any kind of entity will cast his theory in a way which avoids commitments which a realist will make. Quine recognizes this. “The resort to canonical notation as an aid to clarifying ontic commitments is of limited polemical power... But it does help us who are agreeable to the canonical forms to judge what we care to consider there to be. We can face the question squarely as a question what to admit to the universe of values of our variables of quantification.” (Quine (1960a) p 243)

For example, consider Quine’s rejection of propositional attitudes as “creatures of darkness.” (Quine (1956) p 188) We do not construct a semantic theory, and then notice whether it quantifies over propositional attitudes. We consider the world, and our minds, and make that decision.

The picture which I called misleading is closely related to the idea that formal theories are disinterpreted. Quine (1978a) argues against the disinterpretive stance, which was held by formalists who tried to eschew metaphysical controversy by emphasizing the syntactic properties of mathematical theories. Quine rightly saw that mathematical theories are useless if taken as disinterpreted. They are about mathematical objects, and we can not pretend otherwise.

This is a fairly obvious point: translating ordinary language into regimented form can aid clarity, but the regimented language is not magically protected from errant commitments. Determining our commitments is a task prior to regimentation. We can regiment the existence of unicorns as easily as that of horses.

But, QI makes exactly the mistake against which I am cautioning. It alleges that we must admit mathematical objects into our ontology since they are required for the regimentation of science. Quine's implication that we are forced to quantify over mathematical objects is misleading. We decide to quantify over mathematical objects, by adding mathematical theorems to the object language of our best theory, not by examining the domain of quantification of the metalanguage and discovering them there.

Quine violates his own strictures against disinterpretation, by emphasizing the needs of the process of regimentation over the content of the theory. "Structure is what matters to a theory, and not the choice of its objects." (Quine (1981b) p 20)

We must disconnect theory, and its structure, from ontology. Formal theory is insufficient for metaphysics just because disinterpretation is not possible. Our ontology is a constraint on theory construction, not a result of it.

§4: Dispensabilism

My argument against QI in the previous section, that we decide whether to quantify over mathematical objects, may appear too quick to any one who spends time considering dispensabilist constructions of scientific theory, such as those surveyed by Burgess and Rosen (1997). The most significant of these is Hartry Field's reformulation of Newtonian Gravitational Theory (NGT), but Charles Chihara and Geoffrey Hellman present interesting modal projects, among other attempts. For a moment, let us grant that we need to reformulate science in a way which avoids quantification over mathematical objects in order to convince ourselves that we need not believe they exist. Still, the dispensabilist suffers from the problems which affect QP.

A popular objection made to Field's program concerns the incompleteness of Field's second-order reformulation of NGT.[2] Field replaces quantification over mathematical objects, specifically real numbers, with quantification over space-time points. In order to account for the utility of mathematics in science without committing to mathematical objects, Field needs to establish conservativeness, that the addition of mathematics to a nominalist theory licenses no additional nominalist conclusions. To emphasize that a conservative mathematical theory is supposed to be compatible with any state of the world, he characterizes conservativeness as, "Necessary truth without the truth." (Field (1982a) p 59)

Conservativeness may be either deductive, which means that mathematics does not allow new theorems to be derived, or semantic, which means that no additional theorems come out true in any model of the theory which includes mathematics. In a complete theory, deductive and semantic completeness are coextensive. In an incomplete theory, they diverge.

To argue for conservativeness, Field attempts to construct representation theorems, standard mathematical functions which establish a homomorphism between two sets, or structures. For Field's project, the relevant representation theorems map space-time points onto real numbers, showing how to translate nominalist statements into statements about their abstract counterparts, the real numbers, and back. If Field's representation theorems are available, the conservativeness of the theory of real numbers over NGT follows.

The natural number structure is representable in the domain of space-time points. This means that we can construct models of the natural numbers within the nominalist theory. Thus,

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[2] Field mentions the objection in the original Field (1980), crediting John Burgess and Yiannis Moschovakis. Saul Kripke is rumored to have discussed similar criticisms, though his remarks have never been published. Stewart Shapiro (1983) works out the criticism in detail.

the Gödel incompleteness theorems apply. There is a formula of the nominalist theory which asserts the consistency of the theory in terms of space-time points. This sentence will not be derivable from the nominalist theory alone. The incompleteness of the theory debars conservativeness. Shapiro et al. take the Gödel construction to show that Field's theory is unacceptably weak. Penelope Maddy (1990) calls it "anemic," because N*, being incomplete, will necessarily omit certain consequences derivable in N+S, in which the consistency of N* is provable.

Field tries to minimize the importance of the omissions. "I suspect that the extra strength that [the platonist theory] has over [the nominalist theory] is confined to such recherché consequences..." (Field (1980) p 104) Recherché or not, the resultant theory omits this consequence.

The class of sentences provable by adding set theory to the nominalist theory is larger than the one case of the Gödel sentence.[3] Even if there are no cases where adding mathematics makes a difference to the physics, strictly construed, there are cases which make a difference to the geometry. If the geometry is taken as a physical theory, as Field must take it, these are physical differences.

Urquhart describes a version of the Banach-Tarski paradox which may be constructed in Field's theory. A region consisting of a solid ball of unit radius can be decomposed into finitely many parts and rearranged to form a solid ball of twice the radius. As a theorem of pure mathematics, this is unobjectionable. As a theorem about physical space, it is repugnant. Urquhart concludes that the problem is the breadth of Field's conservativeness claim.

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[3] See Burgess and Rosen (1997), pp 120-123.

Mathematics is not generally conservative, but that required by physical theory is. He suggests that the dispensabilist should, "Abandon any hope of a general conservative extension result with respect to mathematical theories, but only... develop such results for the mathematics actually needed in physical theory." (Urquhart (1988) pp 153-154)

The problem of incompleteness rebounds on the indispensabilist, indeed on any equation of mathematical truth with provability within a single axiomatic system. The mathematics generated in any formal, axiomatic system will be Gödel-incomplete. Formal theories, even in mathematics itself, are inappropriate loci for metaphysics.

It is, in fact, quite easy to reconstruct standard physics in ways which do not quantify over mathematical objects. For example, Field mentions the theory which consists of just the nominalist consequences of standard science. This theory can be made more attractive (i.e. recursively axiomatizable) by a Craigian reaxiomatization.[4] The debate over whether science requires numbers depends for its vitality on one's interpretation of what makes a theory attractive. "The real question then is whether an attractive nominalistic formulation of physics is possible. I say an attractive nominalistic formulation, because if no attractiveness requirement is imposed, nominalization is trivial... Obviously, such ways of obtaining nominalistic theories are of no interest." (Field (1980) p 41; similarly on p 8.)

A formal theory, e.g. in Quine's canonical form, is not constructed, or imagined, for its

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[4] For another example, one can translate any first-order theory into a language of predicate functors, removing quantification altogether. Quine first explored predicate functors, in Quine (1960c), as a way of explicating quantification in sententialist terms, replacing variables (pronouns) with sentential operators. See also Quine (1960b), Quine (1970), and Quine (1982), §45; and Bacon (1985) for deduction rules and completeness results. A summary is provided in Burgess and Rosen (1997) pp 186-7.

attractiveness. It need have no practical utility. Quine's preferred theory includes sets as the only mathematical elements, meaning that all scientific claims which refer to mathematical objects will have to be rewritten as quantifying over sets. All functions will have to be written set-theoretically. Such a theory will not even be recognizable as scientific to a physicist. It will look like first-order logic and set-theory, with a few empirical predicates.

This is no place to resolve the difficult debate over the success of dispensabilist projects to reformulate standard science. My point in this section is merely to demonstrate that the problems which arise for the indispensability argument from reliance on formal theories carry over to the dispensabilist.

§5: Further Problems with Quine's Procedure

The problems I have so far discussed may give the impression that the phenomenon at issue, difficulties in determining one's ontic commitments on the basis of regimented theories, is isolatable within the philosophy of mathematics. The problem is broader.

Skolemite puzzles about models arise within formal systems. We can generate such questions by appeal to indeterminacy of translation, but support for that doctrine seems strongest on appeal to a metaphor from the problems which arise within formal systems. Hillary Putnam (1980) argues for a broad anti-realism by appealing to problems constructing formal models of any theory. Saul Kripke's Plus/Quus example (Kripke (1982)) demonstrates difficulties for formalizing even clear and simple mathematical concepts. He, too, develops broader conclusions for our ability to know and follow rules in all areas. Without the problems from formal model theory, Kripke's puzzle is merely skeptical. In general, the problems of unintended models are either skeptical or arise from unjustifiably artificial limitations on our abilities to determine those models.

Just as Gödel's theorem cleaves truth from provability in a single formal system, the Löwenheim-Skolem theorem shows that formal models of a sufficiently strong theory can be deviant and unintended and thus do not represent our true commitments. The availability of deviant models is commonly taken to demonstrate the indeterminacy of our commitments. This indeterminacy, I claim, is merely a defect in the formal representation of our independently clear commitments.

I have merely pointed at a few of the problems which may arise from reliance on QP. Here are a couple more. Would the problem of vagueness, over which so much ink has been spilled in recent years, have as much force without the background assumption of QP? And, while the difficulties constructing a formal truth predicate are mathematically relevant, why should these be seen as a problem for philosophy?

§6: Ideal Theories

QI.1 says that we are committed to all elements over which we quantify in our best theory. The claims of science are constantly changing. What exists is roughly constant, and does not vary with our best theory. So, there are two further problems with QI. First, we will as a matter of fact suspend judgment on the claims of all our current theories, formal or informal, expecting improvements. Second and relatedly, we might expect that while awaiting a better theory, we will introduce some instrumental commitments to our current theory in order to make it practically useful.

It would thus be charitable to read QI as referring to an ideal theory. Then, QI.1 is a working hypothesis for commitment. On this reading, we can separate ontic commitments from our currently best theory, expecting our commitments to arise from a full, completed science. If we want to know what exists now, we must find an alternative way to do metaphysics. Call this method thinking.

The problem with the ideal theory interpretation is that it still links our existence claims to the construction of formal theories. Even if it avoids problems with suspended judgment and instrumental elements, it merely defers problems of regimentation. Thinking is the proper route to metaphysics. It is the basis for the construction of regimented theory, and so is prior. Rejecting QP brings us back to philosophy.

§7: Conclusions

Quine uses regimentation as a device of clarification. To this end, he urges semantic ascent, but we can adopt this device for clarification without also insisting that all our commitments be found in a single best regimented theory.

Quine notices that we regiment only when useful. "A maxim of shallow analysis prevails: expose no more logical structure than seems useful for the deduction or other inquiry at hand...[W]here it doesn't itch don't scratch." (Quine (1960a) p 160) He uses this maxim as merely a practical guide. We eschew full formalization only because we can envision what that formalization would look like, and what the yield would be. If we have ontic questions, for Quine, we have to look at the fully formal framework.

Despite the independence of philosophical issues and formal systems, we do construct formal systems with an eye to our commitments. We investigate those things we believe to exist and we do not, generally, regiment fiction. Reasoning within a formal system can, theoretically, affect our independent beliefs about what exists. It may turn out that mathematicians discover new theorems by working within a formal theory. But mathematical reasoning does not generally work this way. Regimentations are instead used as a check on fallible, informal reasoning. The benefits of mathematical regimentation may translate to the mathematized portions of science, but it is unlikely that writing science in a formal, canonical language would lead to any scientific advances.

Quine's indispensability argument can not work, for it is exactly a metaphysical conclusion which arises from the considerations of the construction of formal theory. Formal theories are generally indifferent to philosophically interesting properties. "There is no mathematical substitute for philosophy." (Kripke (1976) p 416)

One might respond to the anti-formalism of this paper by distinguishing between the commitments of a theory and the commitments of those who construct it. In light of the failure of formal theories to provide a categorical criterion for ontic commitment, the indispensabilist might then try to appeal to the elements to which those who use a theory are indispensably committed. But this is a dead end for the indispensabilist, since those who construct scientific theory need have no further commitments to mathematical objects on the basis of their role in science, once their place in the theory is interpreted instrumentally.

Quine's indispensability argument alleges that when we regiment science we find ourselves quantifying over mathematical objects, even if all we want to do is construct a formal language for empirical science. Since we are free to construct and interpret our formal theories as we wish, QI fails. We might still believe that mathematical objects exist on the basis of the utility of mathematics to science, but not because we can not excise the mathematical elements from regimentations of our theories.

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