The Eleatic and the Indispensabilist

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Abstract:

The debate over whether we should believe that mathematical objects exist quickly leads to the question of what criterion for determining what to believe exists we should use. Quine claimed that we should believe in the existence of mathematical objects because of their indispensable role in scientific theory. Eleatics argue that only objects with causal properties exist. Mark Colyvan's recent defense of Quine's indispensability argument and its criterion against some contemporary eleatics presents an intriguing new argument. I show that Colyvan's argument is not decisive against the eleatic.

§1: Introduction

Quine argued that we should believe in the existence of mathematical objects because of their indispensable role in scientific theory. In contrast, philosophers with various motivations have argued for an eleatic principle. The eleatic principle states, approximately, that all that exists are the objects in the causal realm. The eleatic principle is ordinarily invoked by its proponents to argue that we should not believe in the existence of mathematical objects. So, the debate over whether we should believe that mathematical objects exist leads to the question of which criterion for determining what to believe exists we should use.

Recently, Mark Colyvan has defended Quine's indispensability argument, and its criterion, against some contemporary eleatics. Colyvan presents an intriguing argument in favor of the indispensability argument, one which is allegedly independent of the debate over mathematical objects. I first sketch the indispensabilist and eleatic positions, framing the debate so that it avoids question-begging. Then, I show that Colyvan's argument is not decisive against the eleatic. In the end, I sketch a position that captures some of the advantages of each side.

§2. Quine's Argument

Quine's indispensability argument proceeds as follows.[1]

QIA:
- QIA.1: We should believe the (single, holistic) theory which best accounts for our sense experience.
- QIA.2: If we believe a theory, we must believe in its ontic commitments.
- QIA.3: The ontic commitments of any theory are the objects over which that theory first-order quantifies.
- QIA.4: The theory which best accounts for our sense experience first-order quantifies over mathematical objects.
- QIA.C: We should believe that mathematical objects exist.

The theory to which QIA refers and which purportedly yields mathematical objects is designed to explain our sense experience, as of the sugar maple on my front lawn. My best theory refers to some objects which we can not sense directly, like sub-atomic particles, and to some objects which we can not sense at all, like sets. Some of the objects posited by my theory are constituents of ordinary objects like maple trees. Other objects to which my theory refers are posited for more formal or technical reasons. Proponents of QIA do not argue for a Pythagorean view, that trees are made out of sets. Rather, their argument is that we need sets in order to regiment the best theory of maple syrup.

QIA depends essentially on Quine's method for determining the ontic commitments of a theory. First, QIA.1-2, we choose a best theory. Then, QIA.3, we regiment that theory in first-order logic with identity. Last, QIA.4, we examine the domain of quantification of the theory to see what objects the theory needs to come out as true. Quine's method for determining our

[1] Quine nowhere presents a detailed indispensability argument, though he alludes to one in many places. Among them: Quines 1939, 1948, 1951, 1955, 1958, 1960, 1978, and 1986.

commitments applies to any theory. Theories which refer to trees, electrons, and numbers, and theories which refer to ghosts, caloric, and God, are equally amenable of Quine's general procedure.

There are a variety of possible responses to the Quinean argument. One possibility is to deny QIA.4.[2] Another possibility is to deny Quine's method for determining the commitments of a theory.

§3: The Eleatic Principle

Eleatics, like David Armstrong, propose an alternative to Quine's method. The eleatic principle, which has its roots in Plato's *Sophist*[3], is notoriously difficult to formulate precisely. Armstrong, in different places, emphasizes both causal activity and spatio-temporal location.

> Against the suggestion that the world might contain...such things as possibilities, timeless propositions and "abstract" classes, I argued that these latter entities had no causal power; and that if they had no power there was no good reason to postulate them (Armstrong 1978b: 46).

> The world is nothing but a single spatio-temporal system (Armstrong 1978a 1: 126).

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[2] Field 1980 spurred a wide range of projects aimed at denying QIA.4 by rewriting either mathematical or scientific theory to dispense with quantification over mathematical objects. Burgess and Rosen 1997 elegantly compiles many of these dispensabilist strategies. We can easily eliminate quantification over mathematical objects from first-order theories of standard science, using a variety of tricks which yield ugly, unwieldy theories. So, the Quinean must emphasize the 'best' in both QIA.1 and QIA.4. I grant QIA.4, and set aside the questions of whether dispensabilist projects succeed, for the purposes of this paper.

[3] "I am proposing as a mark to distinguish real things that they are nothing but power" (*Sophist* 247e, Cornford translation).

The following paradigmatic formulation is sufficient for my purposes in this paper:

EP: Only those things which are causally active are real.[4]

The eleatic principle is difficult to formulate precisely in part because necessary and sufficient conditions are always hard to come by, and in part because it relies on a concept, causation, which is itself notoriously unclear. Oddie 1982 denies the viability of EP on the basis of the first problem; he shows that various formulations all fall to counter-examples. Despite problems with the concept of causation, it is not at all unclear how EP is intended as an alternative to QIA. QIA says that we should believe in mathematical objects, since they are ineliminable elements of our best theory. EP says that we should not believe in mathematical objects, even if they are included in our best theory, because they have no causal connection to maple syrup (or anything else).

The eleatic need not deny the indispensabilist's claims, at QIA.1 and QIA.2, that we should believe our best scientific theory and the posits that it makes. S/he need not deny the confirmation holism that underlies those premises. Nor need the eleatic deny QIA.4, the claim that mathematics is ineliminable from that theory. The central disagreement between the eleatic and the indispensabilist concerns how we read the posits from the theory. The eleatic claims that to be is to be causally active; our theory contains both real posits (exemplified by trees and electrons) and instrumental ones (e.g. the square root of two). The indispensabilist claims that to be is to be the value of a variable, and that any distinction between real and instrumental posits in

[4] Compare to Oddie 1982: 286 and Azzouni 2004b: 150.

our best theory is arbitrary.

EP does not make an arbitrary distinction between real and unreal posits. The eleatic has a principled way of distinguishing between real and instrumental elements of our theory, deriving directly from the abstractness of mathematical objects. We are causally isolated from them. Mark Balaguer calls the fact that we are unable to interact with mathematical objects the "principle of causal isolation," or PCI, which is another version of the eleatic principle. He uses PCI to reject the indispensability argument, and to defend EP. The indispensabilist is forced to reject the commonsensical PCI. "The Quine-Putnam argument should be construed as an argument not for platonism or the truth of mathematics but, rather, for the *falsity of PCI*" (Balaguer 1998: 110).

The eleatic argues that there can be no empirical evidence for the existence of mathematical objects. Armstrong asserts that science can accept objects that help to explain the behavior of ordinary objects, but denies that mathematical objects can do this. They are merely heuristic devices which lack causal efficacy. "If any entities outside the [spatio-temporal] system are postulated, but have no effect on the system, there is no compelling reason to postulate them" (Armstrong 1980: 154).

§4: Defending EP

Our question is whether to believe that mathematical objects exist. We have, in our sights, two competing principles for determining what to believe exists. The mathematical realist will be tempted by QIA, while the mathematical anti-realist will be tempted by EP. To decide between them, it would be nice, and it would avoid question-begging, if we had reasons independent of mathematics to adopt one or another of the principles.

Azzouni defends EP by arguing that QIA commits us to objects we do not really believe exist. He describes instances in which existential quantifications within science should be seen as merely instrumental. The users of scientific theories are not committed to centers of mass, quasi-particles, and mathematical objects.

Azzouni 1997b considers a system of two masses connected by a spring, moving in a gravitational field. The separate motions of the masses are too complicated to calculate, but we can describe the system if we consider it in terms of its center of mass, which is not located on the springs, and its reduced mass.

Quasi-particles are posits used to replace one intractable many-body problem in condensed-matter physics with many one-body problems, using Fermi Liquid theory. Scientists introduce quasi-particles aware that a fictionalization is involved. "[I]t's not that physicists are failing to ask whether or not they're committed to the entities introduced in this way. *They already take themselves* not to be so committed. That's why, for example, such 'particles' are called quasi-particles" (Azzouni 1997b: 195).

Azzouni suggests that we cleave ontic commitment from the existential quantifier, while maintaining the quantifier's inferential role. If we want to clarify our commitments within formal scientific theory, Azzouni suggests minting a predicate to be read as 'is physically real'.[5] Azzouni's principle underlying ascriptions of the predicate is that we have thick epistemic access

[5] See Azzouni 2004a: 383; and much of Azzouni 2004b, especially Chapter 4.

to anything physically real.

Penelope Maddy cites, to a similar end, skepticism surrounding atoms in the early stages of atomic theory.[6] Before the experiments which yielded more direct evidence of the existence of atoms, scientists hedged their bets about these elements. Atomic theory was accepted, it could be naturally taken as quantifying over atoms, but scientists did not really believe that the atoms existed.

> [T]hough atomic theory was well-confirmed by almost any philosopher's standard as early as 1860, some scientists remained skeptical until the turn of the century - when certain ingenious experiments provided so-called "direct verification" - and even the supporters of atoms felt this early skepticism to be scientifically justified. This is not to say that the skeptics necessarily recommended the removal of atoms from, say, chemical theory; they did however, hold that only the directly verifiable consequences of atomic theory should be believed, whatever the explanatory power or the fruitfulness or the systemic advantages of thinking in terms of atoms. In other words, the confirmation provided by experimental success extended only so far into the atomic-based chemical theory *T*, not to the point of confirming its statements about the existence of atoms (Maddy 1992: 280-1).

Maddy cites other examples of false assumptions in science: taking water waves to be infinitely deep, and treating matter as continuous in fluid dynamics. According to Maddy, it is accepted scientific practice to separate our actual commitments from those made by our best theories. "If we remain true to our naturalistic principles, we must allow a distinction to be drawn between parts of a theory that are true and parts that are merely useful. We must even allow that the merely useful parts might in fact be indispensable" (Maddy 1992: 281).

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[6] Maddy's concern is to withhold truth to sentences of the theory, while Azzouni's concern is to avoid commitments to entities. For the purposes of this paper, the distinction is irrelevant.

Armstrong, Azzouni, and Balaguer agree that we can distinguish between the mathematical and non-mathematical content of our theories. Otherwise, we would not know which references are to be taken literally, and which are instrumental. In explaining physical phenomena, say the eleatics, we only commit to the non-mathematical, physical content of the explanation, even if it refers to mathematical objects along the way. We know as we construct our theory the kinds of things to which we are committed. Our explanation of why my hand does not pass through a wall may refer to mathematical objects, but the subjects of the explanation are hands and walls, and not mathematical objects.

I do not take Azzouni and Maddy's examples to be decisive in favor of EP. Still, they are the kinds of examples we are seeking in that they do not beg the question of the existence of mathematical objects. In the next two sections, I examine three examples which favor QIA, and show that they are unsuccessful.

§5: Colyvan's Defense of QIA

Mark Colyvan, in Colyvan 2001, defends the indispensability argument, and Quine's method, against the eleatic. He argues that we are committed by physical theory to non-causal, non-mathematical entities which play indispensable explanatory roles.[7] If we admit non-causal, non-mathematical objects, then the eleatic principle fails, independently of what we think about mathematical objects. The door is wide open to admit mathematical objects, as well. And,

[7] Indispensability arguments must present some goal for which commitment is indispensable. For Quine, this goal was the construction of scientific theory. Colyvan focuses on scientific explanation, as Armstrong did. The examples play the same role in both domains.

Colyvan argues, there are good reasons to admit non-causal, non-mathematical objects. Thus, according to Colyvan, the principled distinction which supports EP is wrong.

Colyvan presents three examples. The first example concerns the bending of light. Colyvan argues that the best explanation of light bending around large objects is geometric, rather than causal. "It's not that something *causes* the light to deviate from its usual path; it's simply that light travels along space-time geodesics and that the curvature of space-time is greater around massive objects" (Colyvan 2001: 47-8). Large masses covary with curvatures in space-time, but it is not clear, on a causal picture, which causes which. "Simple covariance doesn't guarantee that one of the factors causes the other" (ibid). Furthermore, according to the non-Minkowski vacuum solutions to the Einstein equation, there are empty, yet curved space-times. On the causal picture, these curvatures are uncaused, and thus unexplained.

Colyvan's second example concerns the existence of two antipodes in the Earth's atmosphere with exactly the same pressure and temperature at the same time. The causal explanation, which refers to atmospheric conditions, suffices only to describe the existence of the antipodes, and does not explain why they must exist. The existence of antipodes is guaranteed by a topological theorem. The proof of this theorem provides the remainder of the explanation, and is non-causal.

Lastly, Colyvan asks us to consider the Fitzgerald-Lorentz contraction. A body in motion contracts, relative to an inertial reference frame, in the direction of motion. Minkowski's explanation of this contraction relies on equations in four dimensions, representing the space-time manifold. Colyvan calls this, "A purely geometric explanation of the contraction, featuring such non-causal entities as the Minkowski metric and other geometric properties of Minkowski space" (Colyvan 2001: 51).

§6: Response to Colyvan

To evaluate Colyvan's examples, recall that we are looking for non-causal entities other than mathematical objects which play an explanatory role. For, his argument was that since we need non-causal non-mathematical elements in our best theory, the eleatic principle is shown false, independently of the contentious mathematical case. If Colyvan's examples were to show only that mathematical elements of our best theory were indispensable, then using those examples to choose QIA over EP would be just like using a bare preference for mathematical realism to choose QIA over EP, as I described at the beginning of §4.

Colyvan's first example, the geodesics, either begs the question, or is insufficient. If we take the geodesics as pure mathematical objects, Colyvan begs the question by presenting a geometric object as explanatory. If we take geodesics to be physical entities, then we should see them as properties of physical space-time, as opposed to objects of pure geometry. In this case, we may naturally see masses as causing curvatures.

Colyvan rejects the causal interpretation. "[A]ny account that permits mass to *cause* the curvature of space-time is unintuitive to say the least" (Colyvan 2001: 48). The unintuitiveness, for Colyvan, may arise from thinking of space-time as abstract, or relationalist. If we think of it substantivally, the causal explanation is not problematic. Indeed, any non-causal explanation of the curving geodesics near massive objects would make those curves seem entirely accidental.

Without massive objects in or near their paths, light rays travel in straight lines.

The case of an empty, yet curved, space-time only reinforces the claim that we do not need a non-causal, non-mathematical explanation. The curvature of space-time is not an event which can be explained in terms of antecedent conditions, say. We can take it, with the substantivalist, to be a property of an object, or collection of objects: space-time points or regions. Or, we can take an empty yet curved manifold as a pure geometric object. In neither case do we need to posit non-causal, non-mathematical objects.

In Colyvan's second case, the antipodes, we must again make a pure/applied distinction regarding the topological theorem. The pure mathematical theorem does not guarantee that these antipodes have the same temperature and pressure. We need bridge principles which apply this theorem to the Earth and its weather patterns. Once we add these bridge principles, the proof which guarantees the antipodes may naturally be regarded as a causal explanation. For, the bridge principles will refer to causal structures within the Earth's atmosphere, and it is these which explain the existence of the antipodes. This explanation will, as Colyvan notes, refer to non-causal entities such as continuous functions and spheres, but these are mathematical objects. We are looking for non-mathematical, yet non-causal, elements.

A similar response applies to the contraction example. The equations which explain the contraction are supposed to make indispensable reference to non-causal entities. But the equations apply to the physical world, and thus explain the contraction of a physical body in motion, only if coupled with bridge principles which explain their applicability. The physical objects, along with mathematical objects, provide the explanation. It is a perennially interesting question why mathematical objects are so useful to physical theories. But, the indispensability of mathematics may, as I mentioned in §3, be granted by the eleatics without weakening their argument against the indispensabilists.

In no case has Colyvan shown that a non-causal entity, other than a mathematical object, plays an essential role in scientific explanation. The eleatic, ex hypothesi, need not show that mathematical entities can be removed from explanations in the physical world. Thus, Colyvan's argument does not show that QIA should be preferred to EP.

§7: Beyond Eleatics and Indispensabilists

I claimed, at the end of §4, that the Azzouni/Maddy examples were not decisive against QIA. I examined three examples in which the eleatic purports to deny the existence of an object over which a best theory quantifies:

1. Centers of mass
2. Quasi-particles
3. Atoms, prior to experimental verification

It is fairly easy to see how a defender of QIA could deny that we should believe in the existence of centers of mass and quasi-particles. They are introduced as idealizing fictions, to make intractable calculations possible. Engineers and experimental physicists require such idealizations. But, says the defender of QIA, our best theory is not the one we use for practical purposes. Our best theory will be the one in which we regiment our most sincere, and our most austere, commitments. Similar remarks can be made against Maddy's examples of false assumptions within science, like taking water waves to be infinitely deep. Any theory which takes the Caribbean Sea to be infinitely deep is clearly not a theory we should use to reveal our ontic commitments. Maddy's case of the atoms is a bit trickier. While recent scholarship argues that Mach, who is often cited as one of the atom skeptics, was not as skeptical as he is ordinarily portrayed,[8] Maddy's claim that we often remain skeptical of the referents of even our best theory does not depend on this one case. Still, the Quinean will just accuse the atom skeptic of intellectual dishonesty: you may not deny that which your theory demands.

Quine's method for determining our ontic commitments is supposed to be intellectually sincere in that it avoids the double-talk of denying that mathematical objects exist while using them ineliminably in scientific theory. But if the defender of QIA responds to examples like those of Azzouni and Maddy by retreating to an ideal best theory, s/he is presuming an eleatic principle. This point should not be underestimated. The central reason to denigrate a theory which refers to such instrumental posits is precisely because we know that they lack causal powers. It is not clear that the defender of QIA is really avoiding double-talk.

To decide between EP and QIA on the basis of the Azzouni/Maddy cases, one would need to be much more specific about the nature of, and criteria for, a best theory. The criteria for comparing and assessing theories is beyond the range of this paper. My goal has been to establish that EP is not out of the running, at least not on the basis of Colyvan's examples. But I do not mean to have defended EP. In §3, I put aside Oddie's worries about specifying the eleatic principle more precisely. Important distinctions often elude specification. But, worries about

[8] See Banks 2004.

causation, and the compatibility of differing formulations, can not be put aside forever. It may be that neither EP nor QIA are, in the end, completely defensible.

Both the eleatic and the indispensabilist capture different important intuitions about science and mathematics. We should pursue a position that captures some of the motivation behind EP and some of the motivation behind QIA, but please the defenders of neither view. EP seems reasonable when applied within empirical science. It allows us to rid ourselves of centers of mass and infinitely deep water waves, as well as ghosts, without presuming an idealized version of science that avoids quantifying over them. EP also explains the indispensabilist's appeal to ideal theories. On the other hand, EP denies the existence of mathematical objects. Part of the motivation for the eleatic principle, in fact, is a desire to deny the existence of abstract entities, though Armstrong defends the existence of universals, which he takes to be causally effective.

In contrast, the defender of QIA captures the fact that mathematical objects are essential to our most austere theories, to any scientific theory which is practically useful or even recognizable as science. There is nothing in EP which prevents us from emphasizing it within empirical science, but limiting its scope to such theories. It can do its work on centers of mass and ghosts. Nothing entails that EP must extend to formal theories, like mathematics. There is no fundamental incompatibility between the EP, taken as a criterion for determining the commitments of scientific theory, and mathematical realism, which posits an independent realm. The eleatic principle merely tells us how to read the claims of empirical science. There may be independent reasons to accept mathematical axioms as well as those of empirical science and to believe that mathematical objects exist. To put the matter crudely, the proponent of EP in empirical science may be either a realist or a nominalist about mathematical objects. Denying the existence of mathematical objects on the basis of EP begs the question of the legitimacy of mathematics in its own right.

References

Armstrong, David. 1980. *The Nature of Mind*. Ithaca: Cornell University Press.

Armstrong, David. 1978b. *A Theory of Universals: Universals and Scientific Realism, Volume II*. Cambridge: Cambridge University Press.

Armstrong, David. 1978a. *Nominalism and Realism: Universals and Scientific Realism, Volume I*. Cambridge: Cambridge University Press.

Azzouni, Jody. 2004b. *Deflating Existential Consequence: A Case for Nominalism*. New York: Oxford University Press.

Azzouni, Jody. 2004a. "Theory, Observation, and Scientific Realism'." *British Journal for the Philosophy of Science* 55: 371-392.

Azzouni, Jody. 1997a. "Thick Epistemic Access: Distinguishing the Mathematical from the Empirical." *The Journal of Philosophy* 94: 472-484.

Azzouni, Jody. 1997b. "Applied Mathematics, Existential Commitment, and the Quine-Putnam Indispensability Thesis." *Philosophia Mathematica (3)* 5: 193-209.

Balaguer, Mark. 1998. *Platonism and Anti-Platonism in Mathematics*. New York: Oxford University Press.

Banks, E.C. 2004. "The Philosophical Roots of Ernst Mach's Economy of Thought." *Synthese* 139.1: 23-53.

Burgess, John, and Gideon Rosen. 1997. *A Subject with No Object*. New York: Oxford.

Cartwright, Nancy. 1983. *How the Laws of Physics Lie*. Oxford: Clarendon Press.

Colyvan, Mark. 2001. *The Indispensability of Mathematics*. Oxford University Press.

Field, Hartry. 1980. *Science Without Numbers*. Princeton: Princeton University Press.

Maddy, Penelope. 1992. "Indispensability and Practice." *The Journal of Philosophy* 89: 275-289.

Oddie, Graham. 1982. "Armstrong on the Eleatic Principle and Abstract Entities." *Philosophical Studies* 41: 285-95.

Quine, W.V. 1986. "Reply to Charles Parsons." In Hahn, Lewis Edwin and Paul Arthur Schilpp (eds.). 1986. *The Philosophy of W.V. Quine*. La Salle: Open Court.

Quine, W.V. 1978. "Success and the Limits of Mathematization." In Quine (1981a).

Quine, W.V. 1960. *Word & Object*. Cambridge: The MIT Press.

Quine, W.V. 1958. "Speaking of Objects." In *Ontological Relativity and Other Essays*. New York: Columbia University Press.

Quine, W.V. 1955. "Posits and Reality." In his *The Ways of Paradox*. Cambridge: Harvard University Press, 1976.

Quine, W.V.O. 1951. "Two Dogmas of Empiricism." Reprinted in *From a Logical Point of View*. Cambridge: Harvard University Press, 1980.

Quine, W.V. 1948. "On What There Is." Reprinted in *From a Logical Point of View*. Cambridge: Harvard University Press, 1980.

Quine, W.V. 1939. "Designation and Existence." In Feigl and Sellars, *Readings in Philosophical Analysis*, Appleton-Century-Crofts, Inc., New York: 1940.

Van Fraassen, Bas C. 1980. *The Scientific Image*. Oxford: Clarendon Press.