Reference
& Modality

FROM A LOGICAL POINT OF VIEW

9 Logico-Philosophical Essays

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belonging to the theory of meaning. We have general paradigms (7)-(9) which, though they are not definitions, yet serve to endow 'true-in-L' and 'true-in-L of' and 'names-in-L' with every bit as much clarity, in any particular application, as is enjoyed by the particular expressions of L to which we apply them. Attribution of truth in particular to 'Snow is white', for example, is every bit as clear to us as attribution of whiteness to snow. In Tarski's technical construction, moreover, we have an explicit general routine for defining truth-in-L for individual languages L which conform to a certain standard pattern and are well specified in point of vocabulary. We have indeed no similar single definition of 'true-in-L' for variable 'L'; but what we do have suffices to endow 'true-in-L', even for variable 'L', with a high enough degree of intelligibility so that we are not likely to be averse to using the idiom. No term, of course, is definable except in other terms; and the urgency of the demand for definition is proportional to the obscurity of the term.

See how unfavorably the notion of analyticity-in-L, characteristic of the theory of meaning, compares with that of truth-in-L. For the former we have no clue comparable in value to (7). Nor have we any systematic routine for constructing definitions of 'analytic-in-L', even for the various individual choices of L; definition of 'analytic-in-L' for each L has seemed rather to be a project unto itself.[11] The most evident principle of unification, linking analyticity-in-L for one choice of L with analyticity-in-L for another choice of L, is the joint use of the syllables 'analytic'.

[11] See above, pp. 32-36.

VIII
REFERENCE AND MODALITY

1

One of the fundamental principles governing identity is that of *substitutivity*—or, as it might well be called, that of *indiscernibility of identicals*. It provides that, *given a true statement of identity, one of its two terms may be substituted for the other in any true statement and the result will be true*. It is easy to find cases contrary to this principle. For example, the statements:

(1) Giorgione = Barbarelli,

(2) Giorgione was so-called because of his size

are true; however, replacement of the name 'Giorgione' by the name 'Barbarelli' turns (2) into the falsehood:

Barbarelli was so-called because of his size.

Furthermore, the statements:

(3) Cicero = Tully,

(4) 'Cicero' contains six letters

are true, but replacement of the first name by the second turns (4) false. Yet the basis of the principle of substitutivity appears quite solid; whatever can be said about the person Cicero (or Giorgione) should be equally true of the person Tully (or Barbarelli), this being the same person.

In the case of (4), this paradox resolves itself immediately. The fact is that (4) is not a statement about the person Cicero, but simply about the word 'Cicero'. The principle of substitutivity should not be extended to contexts in which the name to be supplanted occurs without referring simply to the object. Failure of substitutivity reveals merely that the occurrence to be supplanted is not *purely referential*,[1] that is, that the statement depends not only on the object but on the form of the name. For it is clear that whatever can be affirmed about the object remains true when we refer to the object by any other name.

An expression which consists of another expression between single quotes constitutes a name of that other expression; and it is clear that the occurrence of that other expression or a part of it, within the context of quotes, is not in general referential. In particular, the occurrence of the personal name within the context of quotes in (4) is not referential, not subject to the substitutivity principle. The personal name occurs there merely as a fragment of a longer name which contains, beside this fragment, the two quotation marks. To make a substitution upon a personal name, within such a context, would be no more justifiable than to make a substitution upon the term 'cat' within the context 'cattle'.

The example (2) is a little more subtle, for it is a statement about a man and not merely about his name. It was the man, not his name, that was called so and so because of his size. Nevertheless, the failure of substitutivity shows that the occurrence of the personal name in (2) is not *purely* referential. It is easy in fact to translate (2) into another statement which contains two occurrences of the name, one purely referential and the other not:

(5) Giorgione was called 'Giorgione' because of his size.

The first occurrence is purely referential. Substitution on the basis of (1) converts (5) into another statement equally true:

Barbarelli was called 'Giorgione' because of his size.

The second occurrence of the personal name is no more referential than any other occurrence within a context of quotes.

[1] Frege [3] spoke of *direct* (*gerade*) and *oblique* (*ungerade*) occurrences, and used substitutivity of identity as a criterion just as here.

It would not be quite accurate to conclude that an occurrence of a name within single quotes is *never* referential. Consider the statements:

(6) 'Giorgione played chess' is true,

(7) 'Giorgione' named a chess player,

each of which is true or false according as the quotationless statement:

(8) Giorgione played chess

is true or false. Our criterion of referential occurrence makes the occurrence of the name 'Giorgione' in (8) referential, and must make the occurrences of 'Giorgione' in (6) and (7) referential by the same token, despite the presence of single quotes in (6) and (7). The point about quotation is not that it must destroy referential occurrence, but that it can (and ordinarily does) destroy referential occurrence. The examples (6) and (7) are exceptional in that the special predicates 'is true' and 'named' have the effect of undoing the single quotes—as is evident on comparison of (6) and (7) with (8).

To get an example of another common type of statement in which names do not occur referentially, consider any person who is called Philip and satisfies the condition:

(9) Philip is unaware that Tully denounced Catiline,

or perhaps the condition:

(10) Philip believes that Tegucigalpa is in Nicaragua.

Substitution on the basis of (3) transforms (9) into the statement:

(11) Philip is unaware that Cicero denounced Catiline,

no doubt false. Substitution on the basis of the true identity:

Tegucigalpa = capital of Honduras

transforms the truth (10) likewise into the falsehood:

(12) Philip believes that the capital of Honduras is in Nicaragua.

We see therefore that the occurrences of the names 'Tully' and 'Tegucigalpa' in (9)-(10) are not purely referential.

In this there is a fundamental contrast between (9), or (10), and:

Crassus heard Tully denounce Catiline.

This statement affirms a relation between three persons, and the persons remain so related independently of the names applied to them. But (9) cannot be considered simply as affirming a relation between three persons, nor (10) a relation between person, city, and country—at least not so long as we interpret our words in such a way as to admit (9) and (10) as true and (11) and (12) as false.

Some readers may wish to construe unawareness and belief as relations between persons and statements, thus writing (9) and (10) in the manner:

(13) Philip is unaware of 'Tully denounced Catiline',

(14) Philip believes 'Tegucigalpa is in Nicaragua',

in order to put within a context of single quotes every not purely referential occurrence of a name. Church [5] argues against this. In so doing he exploits the concept of analyticity, concerning which we have felt misgivings (pp. 23-37 above); still his argument cannot be set lightly aside, nor are we required here to take a stand on the matter. Suffice it to say that there is certainly no *need* to reconstrue (9)-(10) in the manner (13)-(14). What *is* imperative is to observe merely that the contexts 'is unaware that . . .' and 'believes that . . .' *resemble* the context of the single quotes in this respect: a name may occur referentially in a statement *S* and yet not occur referentially in a longer statement which is formed by embedding *S* in the context 'is unaware that . . .' or 'believes that . . .'. To sum up the situation in a word, we may speak of the contexts 'is unaware that . . .' and 'believes that . . .' as *referentially opaque*.[2] The same is true of the contexts 'knows that . . .', 'says that . . .', 'doubts that

[2] This term is roughly the opposite of Russell's 'transparent' as he uses it in his Appendix C to *Principia*, 2d ed., vol. 1.

. . .', 'is surprised that . . .', etc. It would be tidy but unnecessary to force all referentially opaque contexts into the quotational mold; alternatively we can recognize quotation as one referentially opaque context among many.

It will next be shown that referential opacity afflicts also the so-called *modal* contexts 'Necessarily . . .' and 'Possibly . . .', at least when those are given the sense of *strict* necessity and possibility as in Lewis's modal logic.[3] According to the strict sense of 'necessarily' and 'possibly', these statements would be regarded as true:

(15) 9 is necessarily greater than 7,

(16) Necessarily if there is life on the Evening Star then there is life on the Evening Star,

(17) The number of planets is possibly less than 7,

and these as false:

(18) The number of planets is necessarily greater than 7,

(19) Necessarily if there is life on the Evening Star then there is life on the Morning Star,

(20) 9 is possibly less than 7.

The general idea of strict modalities is based on the putative notion of *analyticity* as follows: a statement of the form 'Necessarily . . .' is true if and only if the component statement which 'necessarily' governs is analytic, and a statement of the form 'Possibly . . .' is false if and only if the negation of the component statement which 'possibly' governs is analytic. Thus (15)-(17) could be paraphrased as follows:

(21) '9 > 7' is analytic,

(22) 'If there is life on the Evening Star then there is life on the Evening Star' is analytic,

(23) 'The number of planets is not less than 7' is not analytic,

and correspondingly for (18)-(20).

[3] Lewis, [1], Ch. 5; Lewis and Langford, pp. 78-89, 120-166.

That the contexts 'Necessarily . . .' and 'Possibly . . .' are referentially opaque can now be quickly seen; for substitution on the basis of the true identities:

(24) The number of planets = 9,

(25) The Evening Star = the Morning Star

turns the truths (15)-(17) into the falsehoods (18)-(20).

Note that the fact that (15)-(17) are equivalent to (21)-(23), and the fact that '9' and 'Evening Star' and 'the number of planets' occur within quotations in (21)-(23), would not of themselves have justified us in concluding that '9' and 'Evening Star' and 'the number of planets' occur irreferentially in (15)-(17). To argue thus would be like citing the equivalence of (8) to (6) and (7) as evidence that 'Giorgione' occurs irreferentially in (8). What shows the occurrences of '9', 'Evening Star', and 'the number of planets' to be irreferential in (15)-(17) (and in (18)-(20)) is the fact that substitution by (24)-(25) turns the truths (15)-(17) into falsehoods (and the falsehoods (18)-(20) into truths).

Some, it was remarked, may like to think of (9) and (10) as receiving their more fundamental expression in (13) and (14). In the same spirit, many will like to think of (15)-(17) as receiving their more fundamental expression in (21)-(23).[4] But this again is unnecessary. We would certainly not think of (6) and (7) as somehow more basic than (8), and we need not view (21)-(23) as more basic than (15)-(17). What is important is to appreciate that the contexts 'Necessarily . . .' and 'Possibly . . .' are, like quotation and 'is unaware that . . .' and 'believes that . . .', referentially opaque.

2

The phenomenon of referential opacity has just now been explained by appeal to the behavior of singular terms. But singular terms are eliminable, we know (cf. pp. 7f, 85, 166f), by paraphrase. Ultimately the objects referred to in a theory are

[4] Cf. Carnap [2], pp. 245-259.

to be accounted not as the things named by the singular terms, but as the values of the variables of quantification. So, if referential opacity is an infirmity worth worrying about, it must show symptoms in connection with quantification as well as in connection with singular terms.[5] Let us then turn our attention to quantification.

The connection between naming and quantification is implicit in the operation whereby, from 'Socrates is mortal', we infer '$(\exists x)(x$ is mortal)', that is, 'Something is mortal'. This is the operation which was spoken of earlier (p. 120) as *existential generalization*, except that we now have a singular term 'Socrates' where we then had a free variable. The idea behind such inference is that whatever is true of the object named by a given singular term is true of something; and clearly the inference loses its justification when the singular term in question does not happen to name. From:

There is no such thing as Pegasus,

for example, we do not infer:

$(\exists x)$(there is no such thing as x),

that is, 'There is something which there is no such thing as', or 'There is something which there is not'.

Such inference is of course equally unwarranted in the case of an irreferential occurrence of any substantive. From (2), existential generalization would lead to:

$(\exists x)(x$ was so-called because of its size),

that is, 'Something was so-called because of its size'. This is clearly meaningless, there being no longer any suitable antecedent for 'so-called'. Note, in contrast, that existential generalization with respect to the purely referential occurrence in (5) yields the sound conclusion:

$(\exists x)(x$ was called 'Giorgione' because of its size),

that is, 'Something was called 'Giorgione' because of its size'.

[5] Substantially this point was made by Church [3].

The logical operation of *universal instantiation* is that whereby we infer from 'Everything is itself', for example, or in symbols '$(x)(x = x)$', the conclusion that Socrates = Socrates. This and existential generalization are two aspects of a single principle; for instead of saying that '$(x)(x = x)$' implies 'Socrates = Socrates', we could as well say that the denial 'Socrates ≠ Socrates' implies '$(\exists x)(x \neq x)$'. The principle embodied in these two operations is the link between quantifications and the singular statements that are related to them as instances. Yet it is a principle only by courtesy. It holds only in the case where a term names and, furthermore, occurs referentially. It is simply the logical content of the idea that a given occurrence is referential. The principle is, for this reason, anomalous as an adjunct to the purely logical theory of quantification. Hence the logical importance of the fact that all singular terms, aside from the variables that serve as pronouns in connection with quantifiers, are dispensable and eliminable by paraphrase.[6]

We saw just now how the referentially opaque context (2) fared under existential generalization. Let us see what happens to our other referentially opaque contexts. Applied to the occurrence of the personal name in (4), existential generalization would lead us to:

(26) $(\exists x)$('x' contains six letters),

that is:

(27) There is something such that 'it' contains six letters,

or perhaps:

(28) 'Something' contains six letters.

Now the expression:

'x' contains six letters

[6] See above, pp. 7f, 13, and below, pp. 166f. Note that existential generalization as of p. 120 does belong to pure quantification theory, for it has to do with free variables rather than singular terms. The same is true of a correlative use of universal instantiation, such as is embodied in R2 of Essay V.

means simply:

The 24th letter of the alphabet contains six letters.

In (26) the occurrence of the letter within the context of quotes is as irrelevant to the quantifier that precedes it as is the occurrence of the same letter in the context 'six'. (26) consists merely of a falsehood preceded by an irrelevant quantifier. (27) is similar; its part:

'it' contains six letters

is false, and the prefix 'there is something such that' is irrelevant. (28), again, is false—if by 'contains six' we mean 'contains exactly six'.

It is less obvious, and correspondingly more important to recognize, that existential generalization is unwarranted likewise in the case of (9) and (10). Applied to (9), it leads to:

$(\exists x)$(Philip is unaware that x denounced Catiline),

that is:

(29) Something is such that Philip is unaware that it denounced Catiline.

What is this object, that denounced Catiline without Philip's having become aware of the fact? Tully, that is, Cicero? But to suppose this would conflict with the fact that (11) is false.

Note that (29) is not to be confused with:

Philip is unaware that $(\exists x)(x$ denounced Catiline),

which, though it happens to be false, is quite straightforward and in no danger of being inferred by existential generalization from (9).

Now the difficulty involved in the apparent consequence (29) of (9) recurs when we try to apply existential generalization to modal statements. The apparent consequences:

(30) $(\exists x)(x$ is necessarily greater than 7),

(31) $(\exists x)$(necessarily if there is life on the Evening Star then there is life on x)

of (15) and (16) raise the same questions as did (29). What is this number which, according to (30), is necessarily greater than 7? According to (15), from which (30) was inferred, it was 9, that is, the number of planets; but to suppose this would conflict with the fact that (18) is false. In a word, to be necessarily greater than 7 is not a trait of a number, but depends on the manner of referring to the number. Again, what is the thing x whose existence is affirmed in (31)? According to (16), from which (31) was inferred, it was the Evening Star, that is, the Morning Star; but to suppose this would conflict with the fact that (19) is false. Being necessarily or possibly thus and so is in general not a trait of the object concerned, but depends on the manner of referring to the object.

Note that (30) and (31) are not to be confused with:

Necessarily $(\exists x)(x > 7)$,

Necessarily $(\exists x)$(if there is life on the Evening Star then there is life on x),

which present no problem of interpretation comparable to that presented by (30) and (31). The difference may be accentuated by a change of example: in a game of a type admitting of no tie it is necessary that some one of the players will win, but there is no one player of whom it may be said to be necessary that he win.

We had seen, in the preceding section, how referential opacity manifests itself in connection with singular terms; and the task which we then set ourselves at the beginning of this section was to see how referential opacity manifests itself in connection rather with variables of quantification. The answer is now apparent: if to a referentially opaque context of a variable we apply a quantifier, with the intention that it govern that variable from outside the referentially opaque context, then what we commonly end up with is unintended sense or nonsense of the type (26)–(31). In a word, we cannot in general properly *quantify into* referentially opaque contexts.

The context of quotation and the further contexts '. . . was so called', 'is unaware that . . .', 'believes that . . .', 'Necessarily . . .', and 'Possibly . . .' were found referentially opaque in the preceding section by consideration of the failure of substitutivity of identity as applied to singular terms. In the present section these contexts have been found referentially opaque by a criterion having to do no longer with singular terms, but with the miscarriage of quantification. The reader may feel, indeed, that in this second criterion we have not really got away from singular terms after all; for the discrediting of the quantifications (29)-(31) turned still on an expository interplay between the singular terms 'Tully' and 'Cicero', '9' and 'the number of planets', 'Evening Star' and 'Morning Star'. Actually, though, this expository reversion to our old singular terms is avoidable, as may now be illustrated by re-arguing the meaninglessness of (30) in another way. Whatever is greater than 7 is a number, and any given number x greater than 7 can be uniquely determined by any of various conditions, some of which have '$x > 7$' as a *necessary* consequence and some of which do not. One and the same number x is uniquely determined by the condition:

(32) $$x = \sqrt{x} + \sqrt{x} + \sqrt{x} \neq \sqrt{x}$$ x = 9

and by the condition:

(33) There are exactly x planets,

but (32) has '$x > 7$' as a necessary consequence while (33) does not. *Necessary* greaterness than 7 makes no sense as applied to a *number* x; necessity attaches only to the connection between '$x > 7$' and the particular method (32), as opposed to (33), of specifying x. ?

Similarly, (31) was meaningless because the sort of thing x which fulfills the condition:

(34) If there is life on the Evening Star then there is life on x,

namely, a physical object, can be uniquely determined by any of various conditions, not all of which have (34) as a necessary consequence. *Necessary* fulfillment of (34) makes no sense as applied to a physical object x; necessity attaches, at best, only to the connection between (34) and one or another particular means of specifying x.

The importance of recognizing referential opacity is not easily overstressed. We saw in §1 that referential opacity can obstruct substitutivity of identity. We now see that it also can interrupt quantification: quantifiers outside a referentially opaque construction need have no bearing on variables inside it. This again is obvious in the case of quotation, as witness the grotesque example:

$$(\exists x)(\text{'}six\text{' contains '}x\text{'}).$$

3

We see from (30)-(31) how a quantifier applied to a modal sentence may lead simply to nonsense. Nonsense is indeed mere absence of sense, and can always be remedied by arbitrarily assigning some sense. But the important point to observe is that granted an understanding of the modalities (through uncritical acceptance, for the sake of argument, of the underlying notion of analyticity), and given an understanding of quantification ordinarily so called, we do not come out automatically with any meaning for quantified modal sentences such as (30)-(31). This point must be taken into account by anyone who undertakes to work out laws for a quantified modal logic.

The root of the trouble was the referential opacity of modal contexts. But referential opacity depends in part on the ontology accepted, that is, on what objects are admitted as possible objects of reference. This may be seen most readily by reverting for a while to the point of view of §1, where referential opacity was explained in terms of failure of interchangeability of names which name the same object. Suppose now we were to repudiate all objects which, like 9 and the planet Venus, or Evening Star, are nameable by names which fail of interchangeability in modal contexts. To do so would be to sweep away all examples indicative of the opacity of modal contexts.

But what objects would remain in a thus purified universe? An object x must, to survive, meet this condition: if S is a statement containing a referential occurrence of a name of x, and S' is formed from S by substituting any different name of x, then S and S' not only must be alike in truth value as they

stand, but must stay alike in truth value even when 'necessarily' or 'possibly' is prefixed. Equivalently: putting one name of x for another in any analytic statement must yield an analytic statement. Equivalently: any two names of x must be synonymous.[7]

Thus the planet Venus as a material object is ruled out by the possession of heteronymous names 'Venus', 'Evening Star', 'Morning Star'. Corresponding to these three names we must, if modal contexts are not to be referentially opaque, recognize three objects rather than one—perhaps the Venus-concept, the Evening-Star-concept, and the Morning-Star-concept.

Similarly 9, as a unique whole number between 8 and 10, is ruled out by the possession of heteronymous names '9' and 'the number of the planets'. Corresponding to these two names we must, if modal contexts are not to be referentially opaque, recognize two objects rather than one; perhaps the 9-concept and the number-of-planets-concept. These concepts are not numbers, for the one is neither identical with nor less than nor greater than the other.

The requirement that any two names of x be synonymous might be seen as a restriction not on the admissible objects x, but on the admissible vocabulary of singular terms. So much the worse, then, for this way of phrasing the requirement; we have here simply one more manifestation of the superficiality of treating ontological questions from the vantage point of singular terms. The real insight, in danger now of being obscured, was rather this: necessity does not properly apply to the fulfillment of conditions by *objects* (such as the ball of rock which is Venus, or the number which numbers the planets), apart from special ways of specifying them. This point was most conveniently brought out by consideration of singular terms, but it is not abrogated by their elimination. Let us now review the matter from the point of view of quantification rather than singular terms.

[7] See above, p. 32. Synonymy of names does not mean merely naming the same thing; it means that the statement of identity formed of the two names is analytic.

From the point of view of quantification, the referential opacity of modal contexts was reflected in the meaninglessness of such quantifications as (30)–(31). The crux of the trouble with (30) is that a number x may be uniquely determined by each of two conditions, for example, (32) and (33), which are not necessarily, that is, analytically, equivalent to each other. But suppose now we were to repudiate all such objects and retain only objects x such that *any two conditions uniquely determining x are analytically equivalent.* All examples such as (30)–(31), illustrative of the referential opacity of modal contexts, would then be swept away. It would come to make sense in general to say that there is an object which, independently of any particular means of specifying it, is necessarily thus and so. It would become legitimate, in short, to quantify into modal contexts.

Our examples suggest no objection to quantifying into modal contexts as long as the values of any variables thus quantified are limited to *intensional objects*. This limitation would mean Meaning? allowing, for purposes of such quantification anyway, not classes but only class-concepts or attributes, it being understood that two open sentences which determine the same class still determine distinct attributes unless they are analytically equivalent. It would mean allowing, for purposes of such quantification, not numbers but only some sort of concepts which are related to the numbers in a many-one way. Further it would mean allowing, for purposes of such quantification, no concrete objects but only what Frege [3] called senses of names, and Carnap [3] and Church have called individual concepts. It is a drawback of such an ontology that the principle of individuation of its entities rests invariably on the putative notion of synonymy, or analyticity.

Actually, even granted these dubious entities, we can quickly see that the expedient of limiting the values of variables to them is after all a mistaken one. It does not relieve the original difficulty over quantifying into modal contexts; on the contrary, examples quite as disturbing as the old ones can be adduced within the realm of intensional objects. For, where

A is any intensional object, say an attribute, and 'p' stands for an arbitrary true sentence, clearly

(35) $$A = (\imath x)[p \,.\, (x = A)].$$

Yet, if the true sentence represented by 'p' is not analytic, then neither is (35), and its sides are no more interchangeable in modal contexts than are 'Evening Star' and 'Morning Star', or '9' and 'the number of the planets'.

Or, to state the point without recourse to singular terms, it is that the requirement lately italicized — "any two conditions uniquely determining x are analytically equivalent" — is not assured merely by taking x as an intensional object. For, think of 'Fx' as any condition uniquely determining x, and think of 'p' as any nonanalytic truth. Then '$p \,.\, Fx$' uniquely determines x but is not analytically equivalent to 'Fx', even though x be an intensional object.

It was in my 1943 paper that I first objected to quantifying into modal contexts, and it was in his review of it that Church proposed the remedy of limiting the variables thus quantified to intensional values. This remedy, which I have just now represented as mistaken, seemed all right at the time. Carnap [3] adopted it in an extreme form, limiting the range of his variables to intensional objects throughout his system. He did not indeed describe his procedure thus; he complicated the picture by propounding a curious double interpretation of variables. But I have argued[8] that this complicating device has no essential bearing and is better put aside.

By the time Church came to propound an intensional logic of his own [6], he perhaps appreciated that quantification into modal contexts could not after all be legitimized simply by limiting the thus quantified variables to intensional values. Anyway his departures are more radical. Instead of a necessity operator attachable to sentences, he has a necessity predicate attachable to complex names of certain intensional objects called propositions. What makes this departure more serious than it sounds

[8] In a criticism which Carnap generously included in his [3], pp. 196f.

is that the constants and variables occurring in a sentence do not, without special provision, recur in the name of the corresponding proposition. Church makes such provision by introducing a primitive function that applies to intensional objects and yields their extensions as values. The interplay, usual in modal logic, between occurrences of expressions outside modal contexts and recurrences of them inside modal contexts, is mediated in Church's system by this function. Perhaps we should not call it a system of modal logic; Church generally did not. Anyway let my continuing discussion be understood as relating to modal logics only in the narrower sense, where the modal operator attaches to sentences.

Church [4] and Carnap tried — unsuccessfully, I have just argued — to meet my criticism of quantified modal logic by restricting the values of their variables. Arthur Smullyan took the alternative course of challenging my criticism itself. His argument depends on positing a fundamental division of names into proper names and (overt or covert) descriptions, such that proper names which name the same object are always synonymous. (Cf. (38) below.) He observes, quite rightly on these assumptions, that any examples which, like (15)–(20) and (24)–(25), show failure of substitutivity of identity in modal contexts, must exploit some descriptions rather than just proper names. Then, taking a leaf from Russell [2], he explains the failure of substitutivity by differences in the structure of the contexts, in respect of what Russell called the scopes of the descriptions.[9] As stressed in the preceding section, however, referential opacity remains to be reckoned with even when descriptions and other singular terms are eliminated altogether.

Nevertheless, the only hope of sustaining quantified modal logic lies in adopting a course that resembles Smullyan's, rather than Church [4] and Carnap [3], in this way: it must overrule my objection. It must consist in arguing or deciding that quantification into modal contexts makes sense even though any

[9] Unless a description fails to name, its scope is indifferent to extensional contexts. But it can still matter to intensional ones.

value of the variable of such a quantification be determinable by conditions that are not analytically equivalent to each other. The only hope lies in accepting the situation illustrated by (32) and (33) and insisting, despite it, that the object x in question is necessarily greater than 7. This means adopting an invidious attitude toward certain ways of uniquely specifying x, for example (33), and favoring other ways, for example (32), as somehow better revealing the "essence" of the object. Consequences of (32) can, from such a point of view, be looked upon as necessarily true of the object which is 9 (and is the number of the planets), while some consequences of (33) are rated still as only contingently true of that object.

Evidently this reversion to Aristotelian essentialism (cf. p. 22) is required if quantification into modal contexts is to be insisted on. An object, of itself and by whatever name or none, must be seen as having some of its traits necessarily and others contingently, despite the fact that the latter traits follow just as analytically from some ways of specifying the object as the former traits do from other ways of specifying it. In fact, we can see pretty directly that any quantified modal logic is bound to show such favoritism among the traits of an object; for surely it will be held, for each thing x, on the one hand that

(36) $\text{necessarily } (x = x)$

and on the other hand that

(37) $\sim \text{necessarily } [p \cdot (x = x)]$,

where 'p' stands for an arbitrary contingent truth.

Essentialism is abruptly at variance with the idea, favored by Carnap, Lewis, and others, of explaining necessity by analyticity (cf. p. 143). For the appeal to analyticity can pretend to distinguish essential and accidental traits of an object only relative to how the object is specified, not absolutely. Yet the champion of quantified modal logic must settle for essentialism.

Limiting the values of his variables is neither necessary nor sufficient to justify quantifying the variables into modal contexts. Limiting their values can, however, still have this purpose in conjunction with his essentialism: if he wants to limit his essentialism to special sorts of objects, he must correspondingly limit the values of the variables which he quantifies into modal contexts.

The system presented in Miss Barcan's pioneer papers on quantified modal logic differed from the systems of Carnap and Church in imposing no special limitations on the values of variables. That she was prepared, moreover, to accept the essentialist presuppositions seems rather hinted in her theorem:

(38) $(x)(y)\{(x = y) \supset [\text{necessarily } (x = y)]\}$,

for this is as if to say that some at least (and in fact at most; cf. '$p \,.\, Fx$') of the traits that determine an object do so necessarily. The modal logic in Fitch [1] follows Miss Barcan on both points. Note incidentally that (38) follows directly from (36) and a law of substitutivity of identity for variables:

$$(x)(y)[(x = y \,.\, Fx) \supset Fy] \,.$$

The upshot of these reflections is meant to be that the way to do quantified modal logic, if at all, is to accept Aristotelian essentialism. To defend Aristotelian essentialism, however, is not part of my plan. Such a philosophy is as unreasonable by my lights as it is by Carnap's or Lewis's. And in conclusion I say, as Carnap and Lewis have not: so much the worse for quantified modal logic. By implication, so much the worse for unquantified modal logic as well; for, if we do not propose to quantify across the necessity operator, the use of that operator ceases to have any clear advantage over merely quoting a sentence and saying that it is analytic.

4

The worries introduced by the logical modalities are introduced also by the admission of attributes (as opposed to classes). The idiom 'the attribute of being thus and so' is referentially opaque, as may be seen, for example, from the fact that the true statement:

(39) The attribute of exceeding 9 = the attribute of exceeding 9

goes over into the falsehood:

> The attribute of exceeding the number of the planets = the attribute of exceeding 9

under substitution according to the true identity (24). Moreover, existential generalization of (39) would lead to:

(40) $(\exists x)$(the attribute of exceeding x = the attribute of exceeding 9)

which resists coherent interpretation just as did the existential generalizations (29)–(31) of (9), (15), and (16). Quantification of a sentence which contains the variable of quantification within a context of the form 'the attribute of . . .' is exactly on a par with quantification of a modal sentence.

Attributes, as remarked earlier, are individuated by this principle: two open sentences which determine the same class do not determine the same attribute unless they are analytically equivalent. Now another popular sort of intensional entity is the *proposition*. Propositions are conceived in relation to statements as attributes are conceived in relation to open sentences: two statements determine the same proposition just in case they are analytically equivalent. The foregoing strictures on attributes obviously apply equally to propositions. The truth:

(41) The proposition that $9 > 7$ = the proposition that $9 > 7$

goes over into the falsehood:

> The proposition that the number of the planets > 7 = the proposition that $9 > 7$.

under substitution according to (24). Existential generalization of (41) yields a result comparable to (29)–(31) and (40).

Most of the logicians, semanticists, and analytical philosophers who discourse freely of attributes, propositions, or logical modalities betray failure to appreciate that they thereby imply a metaphysical position which they themselves would scarcely condone. It is noteworthy that in *Principia Mathematica*, where attributes were nominally admitted as entities, all actual contexts occurring in the course of formal work are such as could be fulfilled as well by classes as by attributes. All actual contexts are *extensional* in the sense of page 30 above. The authors of *Principia Mathematica* thus adhered in practice to a principle of extensionality which they did not espouse in theory. If their practice had been otherwise, we might have been brought sooner to an appreciation of the urgency of the principle.

We have seen how modal sentences, attribute terms, and proposition terms conflict with the nonessentialist view of the universe. It must be kept in mind that those expressions create such conflict only when they are quantified into, that is, when they are put under a quantifier and themselves contain the variable of quantification. We are familiar with the fact (illustrated by (26) above) that a quotation cannot contain an effectively free variable, reachable by an outside quantifier. If we preserve a similar attitude toward modalities, attribute terms, and proposition terms, we may then make free use of them without any misgivings of the present urgent kind.

What has been said of modality in these pages relates only to strict modality. For other sorts, for example, physical necessity and possibility, the first problem would be to formulate the notions clearly and exactly. Afterward we could investigate whether such modalities, like the strict ones, cannot be quantified into without precipitating an ontological crisis. The question concerns intimately the practical use of language. It concerns, for example, the use of the contrary-to-fact conditional within a quantification; for it is reasonable to suppose that the contrary-to-fact conditional reduces to the form 'Necessarily, if p then q' in some sense of necessity. Upon the contrary-to-fact conditional depends in turn, for example, this definition of solubility in water: To say that an object is soluble in water is to say that it would dissolve if it were in water. In discussions of physics, naturally, we need quantifications containing the clause 'x is soluble in water', or the equivalent in words; but, according to the definition suggested, we should then have to admit within quantifications the expression 'if x were in water then x would dissolve', that is, 'necessarily if x is in water then

x dissolves'. Yet we do not know whether there is a suitable sense of 'necessarily' into which we can so quantify.[10]

Any way of imbedding statements within statements, whether based on some notion of "necessity" or, for example, on a notion of "probability" as in Reichenbach, must be carefully examined in relation to its susceptibility to quantification. Perhaps the only useful modes of statement composition susceptible to unrestricted quantification are the truth functions. Happily, no other mode of statement composition is needed, at any rate, in mathematics; and mathematics, significantly, is the branch of science whose needs are most clearly understood.

Let us return, for a final sweeping observation, to our first test of referential opacity, namely, failure of substitutivity of identity; and let us suppose that we are dealing with a theory in which (a) *logically* equivalent formulas are interchangeable in all contexts *salva veritate* and (b) the logic of classes is at hand.[11] For such a theory it can be shown that *any* mode of statement composition, other than the truth functions, is referentially opaque. For, let ϕ and ψ be any statements alike in truth value, and let $\Phi(\phi)$ be any true statement containing ϕ as a part. What is to be shown is that $\Phi(\psi)$ will also be true, unless the context represented by 'Φ' is referentially opaque. Now the class named by $\hat{\alpha}\phi$ is either V or Λ, according as ϕ is true or false; for remember that ϕ is a statement, devoid of free α. (If the notation $\hat{\alpha}\phi$ without recurrence of α seems puzzling, read it as $\hat{\alpha}(\alpha = \alpha \,.\, \phi)$.) Moreover ϕ is logically equivalent to $\hat{\alpha}\phi = \text{V}$. Hence, by (a), since $\Phi(\phi)$ is true, so is $\Phi(\hat{\alpha}\phi = \text{V})$. But $\hat{\alpha}\phi$ and $\hat{\alpha}\psi$ name one and the same class, since ϕ and ψ are alike in truth value. Then, since $\Phi(\hat{\alpha}\phi = \text{V})$ is true, so is $\Phi(\hat{\alpha}\psi = \text{V})$ unless the context represented by 'Φ' is referentially opaque. But if $\Phi(\hat{\alpha}\psi = \text{V})$ is true, then so in turn is $\Phi(\psi)$, by (a).

[10] For a theory of disposition terms, like 'soluble', see Carnap [5].
[11] See above, pp. 27, 87.