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The Thought: A Logical Inquiry Author(s): Gottlob Frege Source: *Mind*, New Series, Vol. 65, No. 259 (Jul., 1956), pp. 289-311 Published by: Oxford University Press on behalf of the Mind Association Stable URL: <u>http://www.jstor.org/stable/2251513</u> Accessed: 19/05/2011 13:41

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[July, 1956

MIND

A QUARTERLY REVIEW

OF

PSYCHOLOGY AND PHILOSOPHY

I.--THE THOUGHT : A LOGICAL INQUIRY

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BY GOTTLOB FREGE

[Translators' Note: This essay was first published in the Beiträge zur Philosophie des Deutschen Idealismus for 1918-19, and was the first of two connected essays, the other being 'Die Verneinung', which has been translated into English by Mr. P. T. Geach, and appears in his and Mr. M. Black's Translations from the Philosophical Writings of Gottlob Frege. A. M. and Marcelle Quinton, Oxford.]

THE word "true" indicates the aim of logic as does "beautiful" that of aesthetics or "good" that of ethics. All sciences have truth as their goal; but logic is also concerned with it in a quite different way from this. It has much the same relation to truth as physics has to weight or heat. To discover truths is the task of all sciences; it falls to logic to discern the laws of truth. The word "law" is used in two senses. When we speak of laws of morals or the state we mean regulations which ought to be obeyed but with which actual happenings are not always in conformity. Laws of nature are the generalization of natural occurrences with which the occurrences are always in accordance. It is rather in this sense that I speak of laws of truth. This is, to be sure, not a matter of what happens so much as of what is. Rules for asserting, thinking, judging, inferring, follow from the laws of truth. And thus one can very well speak of laws of thought too. But there is an imminent danger here of mixing different things up. Perhaps the expression "law of thought" is interpreted by analogy with " law of nature " and the generalization of thinking as a mental occurrence is meant by it. A law of thought in this sense would be a psychological law. And so one might come to believe that logic deals with the mental process of thinking and the psychological laws in accordance

with which it takes place. This would be a misunderstanding of the task of logic, for truth has not been given the place which is its due here. Error and superstition have causes just as much as genuine knowledge. The assertion both of what is false and of what is true takes place in accordance with psychological laws. A derivation from these and an explanation of a mental process that terminates in an assertion can never take the place of a proof of what is asserted. Could not logical laws also have played a part in this mental process? I do not want to dispute this, but when it is a question of truth possibility is not enough. For it is also possible that something not logical played a part in the process and deflected it from the truth. We can only decide this after we have discerned the laws of truth: but then we will probably be able to do without the derivation and explanation of the mental process if it is important to us to decide whether the assertion in which the process terminates is justified. In order to avoid this misunderstanding and to prevent the blurring of the boundary between psychology and logic, I assign to logic the task of discovering the laws of truth, not of assertion or thought. The meaning of the word " true " is explained by the laws of truth.

But first I shall attempt to outline roughly what I want to call true in this connexion. In this way other uses of our word may be excluded. It is not to be used here in the sense of "genuine" or "veracious", nor, as it sometimes occurs in the treatment of questions of art, when, for example, truth in art is discussed, when truth is set up as the goal of art, when the truth of a work of art or true feeling is spoken of. The word "true" is put in front of another word in order to show that this word is to be understood in its proper, unadulterated sense. This use too lies off the path followed here; that kind of truth is meant whose recognition is the goal of science.

Grammatically the word "true" appears as an adjective. Hence the desire arises to delimit more closely the sphere in which truth can be affirmed, in which truth comes into the question at all. One finds truth affirmed of pictures, ideas, statements, and thoughts. It is striking that visible and audible things occur here alongside things which cannot be perceived with the senses. This hints that shifts of meaning have taken place. Indeed ! Is a picture, then, as a mere visible and tangible thing, really true, and a stone, a leaf, not true ? Obviously one would not call a picture true unless there were an intention behind it. A picture must represent something. Furthermore, an idea is not called true in itself but only with

respect to an intention that it should correspond to something. It might be supposed from this that truth consists in the correspondence of a picture with what it depicts. Correspondence is a relation. This is contradicted, however, by the use of the word "true", which is not a relation-word and contains no reference to anything else to which something must correspond. If I do not know that a picture is meant to represent Cologne Cathedral then I do not know with what to compare the picture to decide on its truth. A correspondence, moreover, can only be perfect if the corresponding things coincide and are, therefore, not distinct things at all. It is said to be possible to establish the authenticity of a banknote by comparing it stereoscopically with an authentic one. But it would be ridiculous to try to compare a gold piece with a twenty-mark note stereoscopically. would only be possible to compare an idea with a thing if the thing were an idea too. And then, if the first did correspond perfectly with the second, they would coincide. But this is not at all what is wanted when truth is defined as the correspondence of an idea with something real. For it is absolutely essential that the reality be distinct from the idea. But then there can be no complete correspondence, no complete truth. So nothing at all would be true; for what is only half true is untrue. Truth cannot tolerate a more or less. But yet? Can it not be laid down that truth exists when there is correspondence in a certain respect ? But in which ? For what would we then have to do to decide whether something were true? We should have to inquire whether it were true that an idea and a reality, perhaps, corresponded in the laid-down respect. And then we should be confronted by a question of the same kind and the game could begin again. So the attempt to explain truth as correspondence collapses. And every other attempt to define truth collapses For in a definition certain characteristics would have to too. be stated. And in application to any particular case the question would always arise whether it were true that the characteristics were present. So one goes round in a circle. Consequently, it is probable that the content of the word "true" is unique and indefinable.

When one ascribes truth to a picture one does not really want to ascribe a property which belongs to this picture altogether independently of other things, but one always has something quite different in mind and one wants to say that that picture corresponds in some way to this thing. "My idea corresponds to Cologne Cathedral" is a sentence and the question now arises of the truth of this sentence. So what is improperly called the truth of pictures and ideas is reduced to the truth of sentences. What does one call a sentence ? A series of sounds; but only when it has a sense, by which is not meant that every series of sounds that has sense is a sentence. And when we call a sentence true we really mean its sense is. From which it follows that it is for the sense of a sentence that the question of truth arises in general. Now is the sense of a sentence an idea ? In any case being true does not consist in the correspondence of this sense with something else, for otherwise the question of truth would reiterate itself to infinity.

Without wishing to give a definition, I call a thought something for which the question of truth arises. So I ascribe what is false to a thought just as much as what is true.¹ So I can say : the thought is the sense of the sentence without wishing to say as well that the sense of every sentence is a thought. The thought, in itself immaterial, clothes itself in the material garment of a sentence and thereby becomes comprehensible to us. We say a sentence expresses a thought.

A thought is something immaterial and everything material and perceptible is excluded from this sphere of that for which the question of truth arises. Truth is not a quality that corresponds with a particular kind of sense-impression. So it is sharply distinguished from the qualities which we denote by the words "red", "bitter", "lilac-smelling". But do we not see that the sun has risen and do we not then also see that this is true ? That the sun has risen is not an object which emits rays that reach my eyes, it is not a visible thing like the sun itself. That the sun has risen is seen to be true on the basis of sense-impressions. But being true is not a material, perceptible property. For being magnetic is also recognized on the basis of senseimpressions of something, though this property corresponds as little as truth with a particular kind of sense-impressions. So far these properties agree. However, we need sense-impressions in order to recognize a body as magnetic. On the other hand,

¹ In a similar way it has perhaps been said 'a judgment is something which is either true or false '. In fact I use the word 'thought' in approximately the sense which 'judgment' has in the writings of logicians. I hope it will become clear in what follows why I choose 'thought'. Such an explanation has been objected to on the ground that in it a distinction is drawn between true and false judgments which of all possible distinctions among judgments has perhaps the least significance. I cannot see that it is a logical deficiency that a distinction is given with the explanation. As far as significance is concerned, it should not by any means be judged as trifling if, as I have said, the word 'true' indicates the aim of logic.

when I find that it is true that I do not smell anything at this moment, I do not do so on the basis of sense-impressions.

It may nevertheless be thought that we cannot recognize a property of a thing without at the same time realizing the thought that this thing has this property to be true. So with every property of a thing is joined a property of a thought, namely, that of truth. It is also worthy of notice that the sentence "I smell the scent of violets" has just the same content as the sentence "it is true that I smell the scent of violets". So it seems, then, that nothing is added to the thought by my ascribing to it the property of truth. And yet is it not a great result when the scientist after much hesitation and careful inquiry, can finally say "what I supposed is true"? The meaning of the word "true" seems to be altogether unique. May we not be dealing here with something which cannot, in the ordinary sense, be called a quality at all ? In spite of this doubt I want first to express myself in accordance with ordinary usage, as if truth were a quality, until something more to the point is found.

In order to work out more precisely what I want to call thought, I shall distinguish various kinds of sentences.¹ One does not want to deny sense to an imperative sentence, but this sense is not such that the question of truth could arise for it. Therefore I shall not call the sense of an imperative sentence a thought. Sentences expressing desires or requests are ruled out in the same way. Only those sentences in which we communicate or state something come into the question. But I do not count among these exclamations in which one vents one's feelings, groaning, sighing, laughing, unless it has been decided by some agreement that they are to communicate something. But how about interrogative sentences? In a word-question we utter an incomplete sentence which only obtains a true sense through the completion for which we ask. Word-questions are accordingly left out of consideration here. Sentence-questions are a different matter. We expect to hear "yes" or "no". The answer "yes" means the same as an indicative sentence, for in it the thought that was already completely contained in the interrogative sentence is laid down as true. So a sentencequestion can be formed from every indicative sentence. An exclamation cannot be regarded as a communication on this

 1 I am not using the word 'sentence' here in a purely grammatical sense where it also includes subordinate clauses. An isolated subordinate clause does not always have a sense about which the question of truth can arise, whereas the complex sentence to which it belongs has such a sense.

account, since no corresponding sentence-question can be formed. An interrogative sentence and an indicative one contain the same thought; but the indicative contains something else as well, namely, the assertion. The interrogative sentence contains something more too, namely a request. Therefore two things must be distinguished in an indicative sentence: the content, which it has in common with the corresponding sentence-question, and the assertion. The former is the thought, or at least contains the thought. So it is possible to express the thought without laying it down as true. Both are so closely joined in an indicative sentence that it is easy to overlook their separability. Consequently we may distinguish:

- (1) the apprehension of a thought—thinking,
- (2) the recognition of the truth of a thought—judgment,¹
- (3) the manifestation of this judgment-assertion.

We perform the first act when we form a sentence-question. An advance in science usually takes place in this way, first a thought is apprehended, such as can perhaps be expressed in a sentencequestion, and, after appropriate investigations, this thought is finally recognized to be true. We declare the recognition of truth in the form of an indicative sentence. We do not have to use the word "true" for this. And even when we do use it the real assertive force lies, not in it, but in the form of the indicative sentence and where this loses its assertive force the word " true " cannot put it back again. This happens when we do not speak seriously. As stage thunder is only apparent thunder and a stage fight only an apparent fight, so stage assertion is only apparent assertion. It is only acting, only fancy. In his part the actor asserts nothing, nor does he lie, even if he says something of whose falsehood he is convinced. In poetry we have the case of thoughts being expressed without being actually put forward as true in spite of the form of the indicative sentence, although it may be suggested to the hearer to make an assenting judgment himself. Therefore it must still always be asked, about what is presented in the form of an indicative sentence.

¹ It seems to me that thought and judgment have not hitherto been adequately distinguished. Perhaps language is misleading. For we have no particular clause in the indicative sentence which corresponds to the assertion, that something is being asserted lies rather in the form of the indicative. We have the advantage in German that main and subordinate clauses are distinguished by the word-order. In this connexion it is noticeable that a subordinate clause can also contain an assertion and that often neither main nor subordinate clause express a complete thought by themselves but only the complex sentence does. whether it really contains an assertion. And this question must be answered in the negative if the requisite seriousness is lacking. It is irrelevant whether the word "true" is used here. This explains why it is that nothing seems to be added to a thought by attributing to it the property of truth.

An indicative sentence often contains, as well as a thought and the assertion, a third component over which the assertion does not extend. This is often said to act on the feelings, the mood of the hearer or to arouse his imagination. Words like "alas" and "thank God" belong here. Such constituents of sentences are more noticeably prominent in poetry, but are seldom wholly absent from prose. They occur more rarely in mathematical, physical, or chemical than in historical expositions. What are called the humanities are more closely connected with poetry and are therefore less scientific than the exact sciences which are drier the more exact they are, for exact science is directed toward truth and only the truth. Therefore all constituents of sentences to which the assertive force does not reach do not belong to scientific exposition but they are sometimes hard to avoid, even for one who sees the danger connected with them. Where the main thing is to approach what cannot be grasped in thought by means of guesswork these components have their justification. The more exactly scientific an exposition is the less will the nationality of its author be discernible and the easier will it be to translate. On the other hand, the constituents of language, to which I want to call attention here. make the translation of poetry very difficult, even make a complete translation almost always impossible, for it is in precisely that in which poetic value largely consists that languages differ most.

It makes no difference to the thought whether I use the word "horse" or "steed" or "cart-horse" or "mare". The assertive force does not extend over that in which these words differ. What is called mood, fragrance, illumination in a poem, what is portrayed by cadence and rhythm, does not belong to the thought.

Much of language serves the purpose of aiding the hearer's understanding, for instance the stressing of part of a sentence by accentuation or word-order. One should remember words like "still" and "already" too. With the sentence "Alfred has still not come" one really says "Alfred has not come" and, at the same time, hints that his arrival is expected, but it is only hinted. It cannot be said that, since Alfred's arrival is not expected, the sense of the sentence is therefore false. The word

"but" differs from "and" in that with it one intimates that what follows is in contrast with what would be expected from what preceded it. Such suggestions in speech make no difference to the thought. A sentence can be transformed by changing the verb from active to passive and making the object the subject at the same time. In the same way the dative may be changed into the nominative while "give" is replaced by "receive". Naturally such transformations are not indifferent in every respect; but they do not touch the thought, they do not touch what is true or false. If the inadmissibility of such transformations were generally admitted then all deeper logical investigation would be hindered. It is just as important to neglect distinctions that do not touch the heart of the matter as to make distinctions which concern what is essential. But what is essential depends on one's purpose. To a mind concerned with what is beautiful in language what is indifferent to the logician can appear as just what is important.

Thus the contents of a sentence often go beyond the thoughts expressed by it. But the opposite often happens too, that the mere wording, which can be grasped by writing or the gramophone does not suffice for the expression of the thought. The present tense is used in two ways : first, in order to give a date, second, in order to eliminate any temporal restriction where timelessness or eternity is part of the thought. Think, for instance, of the laws of mathematics. Which of the two cases occurs is not expressed but must be guessed. If a time indication is needed by the present tense one must know when the sentence was uttered to apprehend the thought correctly. Therefore the time of utterance is part of the expression of the thought. If someone wants to say the same today as he expressed yesterday using the word "today", he must replace this word with "vesterday". Although the thought is the same its verbal expression must be different so that the sense, which would otherwise be affected by the differing times of utterance, is readjusted. The case is the same with words like "here" and "there". In all such cases the mere wording, as it is given in writing, is not the complete expression of the thought, but the knowledge of certain accompanying conditions of utterance, which are used as means of expressing the thought, are needed for its correct apprehension. The pointing of fingers, hand movements, glances may belong here too. The same utterance containing the word "I" will express different thoughts in the mouths of different men, of which some may be true, others false.

The occurrence of the word "I" in a sentence gives rise to some questions.

Consider the following case. Dr. Gustav Lauben says, "I have been wounded ". Leo Peter hears this and remarks some days later, "Dr. Gustav Lauben has been wounded". Does this sentence express the same thought as the one Dr. Lauben uttered himself? Suppose that Rudolph Lingens were present when Dr. Lauben spoke and now hears what is related by Leo Peter. If the same thought is uttered by Dr. Lauben and Leo Peter then Rudolph Lingens, who is fully master of the language and remembers what Dr. Lauben has said in his presence, must now know at once from Leo Peter's report that the same thing is under discussion. But knowledge of the language is a separate thing when it is a matter of proper names. It may well be the case that only a few people associate a particular thought with the sentence "Dr. Lauben has been wounded ". In this case one needs for complete understanding a knowledge of the expression "Dr. Lauben". Now if both Leo Peter and Rudolph Lingens understand by "Dr. Lauben" the doctor who lives as the only doctor in a house known to both of them, then they both understand the sentence "Dr. Gustav Lauben has been wounded " in the same way, they associate the same thought with it. But it is also possible that Rudolph Lingens does not know Dr. Lauben personally and does not know that he is the very Dr. Lauben who recently said "I have been wounded ". In this case Rudolph Lingens cannot know that the same thing is in question. I say, therefore, in this case: the thought which Leo Peter expresses is not the same as that which Dr. Lauben uttered.

Suppose further that Herbert Garner knows that Dr. Gustav Lauben was born on 13th September, 1875 in N.N. and this is not true of anyone else; against this, suppose that he does not know where Dr. Lauben now lives nor indeed anything about him. On the other hand, suppose Leo Peter does not know that Dr. Lauben was born on 13th September 1875, in N.N. Then as far as the proper name "Dr. Gustav Lauben" is concerned, Herbert Garner and Leo Peter do not speak the same language, since, although they do in fact refer to the same man with this name, they do not know that they do so. Therefore Herbert Garner does not associate the same thought with the sentence "Dr. Gustav Lauben has been wounded" as Leo Peter wants to express with it. To avoid the drawback of Herbert Garner's and Leo Peter's not speaking the same language, I am assuming that Leo Peter uses the proper name "Dr. Lauben" and Herbert Garner, on the other hand, uses the proper name "Gustav Lauben". Now it is possible that Herbert Garner takes the sense of the sentence "Dr. Lauben has been wounded" to be true while, misled by false information, taking the sense of the sentence "Gustav Lauben has been wounded" to be false. Under the assumptions given these thoughts are therefore different.

Accordingly, with a proper name, it depends on how whatever it refers to is presented. This can happen in different ways and every such way corresponds with a particular sense of a sentence containing a proper name. The different thoughts which thus result from the same sentence correspond in their truth-value, of course; that is to say, if one is true then all are true, and if one is false then all are false. Nevertheless their distinctness must be recognized. So it must really be demanded that a single way in which whatever is referred to is presented be associated with every proper name. It is often unimportant that this demand should be fulfilled but not always.

Now everyone is presented to himself in a particular and primitive way, in which he is presented to no-one else. So, when Dr. Lauben thinks that he has been wounded, he will probably take as a basis this primitive way in which he is presented to himself. And only Dr. Lauben himself can grasp thoughts determined in this way. But now he may want to communicate with others. He cannot communicate a thought which he alone can grasp. Therefore, if he now says "I have been wounded", he must use the "I" in a sense which can be grasped by others, perhaps in the sense of "he who is speaking to you at this moment", by doing which he makes the associated conditions of his utterance serve for the expression of his thought.¹

Yet there is a doubt. Is it at all the same thought which first that man expresses and now this one ?

A person who is still untouched by philosophy knows first of all things which he can see and touch, in short, perceive with the

¹ I am not in the happy position here of a mineralogist who shows his hearers a mountain crystal. I cannot put a thought in the hands of my readers with the request that they should minutely examine it from all sides. I have to content myself with presenting the reader with a thought, in itself immaterial, dressed in sensible linguistic form. The metaphorical aspect of language presents difficulties. The sensible always breaks in and makes expression metaphorical and so improper. So a battle with language takes place and I am compelled to occupy myself with language although it is not my proper concern here. I hope I have succeeded in making clear to my readers what I want to call a thought. senses, such as trees, stones and houses, and he is convinced that another person equally can see and touch the same tree and the same stone which he himself sees and touches. Obviously no thought belongs to these things. Now can he, nevertheless, stand in the same relation to a person as to a tree ?

Even an unphilosophical person soon finds it necessary to recognize an inner world distinct from the outer world, a world of sense-impressions, of creations of his imagination, of sensations, of feelings and moods, a world of inclinations, wishes and decisions. For brevity I want to collect all these, with the exception of decisions, under the word "idea".

Now do thoughts belong to this inner world? Are they ideas? They are obviously not decisions. How are ideas distinct from the things of the outer world? First :

Ideas cannot be seen or touched, cannot be smelled, nor tasted, nor heard.

I go for a walk with a companion. I see a green field, I have a visual impression of the green as well. I have it but I do not see it.

Secondly: ideas are had. One has sensations, feelings, moods, inclinations, wishes. An idea which someone has belongs to the content of his consciousness.

The field and the frogs in it, the sun which shines on them are there no matter whether I look at them or not, but the senseimpression I have of green exists only because of me, I am its bearer. It seems absurd to us that a pain, a mood, a wish should rove about the world without a bearer, independently. An experience is impossible without an experient. The inner world presupposes the person whose inner world it is.

Thirdly: ideas need a bearer. Things of the outer world are however independent.

My companion and I are convinced that we both see the same field; but each of us has a particular sense-impression of green. I notice a strawberry among the green strawberry leaves. My companion does not notice it, he is colour-blind. The colourimpression, which he receives from the strawberry, is not noticeably different from the one he receives from the leaf. Now does my companion see the green leaf as red, or does he see the red berry as green, or does he see both as of one colour with which I am not acquainted at all? These are unanswerable, indeed really nonsensical, questions. For when the word "red" does not state a property of things but is supposed to characterize sense-impressions belonging to my consciousness, it is only applicable within the sphere of my consciousness. For it is

impossible to compare my sense-impression with that of someone else. For that it would be necessary to bring together in one consciousness a sense-impression, belonging to one consciousness, with a sense-impression belonging to another consciousness. Now even if it were possible to make an idea disappear from one consciousness and, at the same time, to make an idea appear in another consciousness, the question whether it were the same idea in both would still remain unanswerable. It is so much of the essence of each of my ideas to be the content of my consciousness, that every idea of another person is, just as such, distinct from mine. But might it not be possible that my ideas, the entire content of my consciousness might be at the same time the content of a more embracing, perhaps divine, consciousness? Only if I were myself part of the divine consciousness. But then would they really be my ideas, would I This oversteps the limits of human underbe their bearer? standing to such an extent that one must leave its possibility out of account. In any case it is impossible for us as men to compare another person's ideas with our own. I pick the strawberry, I hold it between my fingers. Now my companion sees it too, this very same strawberry; but each of us has his own idea. No other person has my idea but many people can see the same thing. No other person has my pain. Someone can have sympathy for me but still my pain always belongs to me and his sympathy to him. He does not have my pain and I do not have his sympathy.

Fourthly: every idea has only one bearer; no two men have the same idea.

For otherwise it would exist independently of this person and independently of that one. Is that lime-tree my idea? By using the expression "that lime-tree" in this question I have really already anticipated the answer, for with this expression I want to refer to what I see and to what other people can also look at and touch. There are now two possibilities. If my intention is realized when I refer to something with the expression "that lime-tree" then the thought expressed in the sentence "that lime-tree is my idea" must obviously be negated. But if my intention is not realized, if I only think I see without really seeing, if on that account the designation "that limetree" is empty, then I have gone astray into the sphere of fiction without knowing it or wanting to. In that case neither the content of the sentence "that lime-tree is my idea" nor the content of the sentence "that lime-tree is not my idea" is true, for in both cases I have a statement which lacks an object. So

then one can only refuse to answer the question for the reason that the content of the sentence "that lime-tree is my idea" is a piece of fiction. I have, naturally, got an idea then, but I am not referring to this with the words "that lime-tree". Now someone may really want to refer to one of his ideas with the words "that lime-tree". He would then be the bearer of that to which he wants to refer with those words, but then he would not see that lime-tree and no-one else would see it or be its bearer.

I now return to the question: is a thought an idea? If the thought I express in the Pythagorean theorem can be recognized by others just as much as by me then it does not belong to the content of my consciousness, I am not its bearer; yet I can, nevertheless, recognize it to be true. However, if it is not the same thought at all which is taken to be the content of the Pythagorean theorem by me and by another person, one should not really say "the Pythagorean theorem" but "my Pythagorean theorem ", " his Pythagorean theorem " and these would be different; for the sense belongs necessarily to the sentence. Then my thought can be the content of my consciousness and his thought the content of his. Could the sense of my Pythagorean theorem be true while that of his was false? I said that the word "red" was applicable only in the sphere of my consciousness if it did not state a property of things but was supposed to characterize one of my sense-impressions. Therefore the words "true" and "false", as I understand them, could also be applicable only in the sphere of my consciousness, if they were not supposed to be concerned with something of which I was not the bearer, but were somehow appointed to characterize the content of my consciousness. Then truth would be restricted to the content of my consciousness and it would remain doubtful whether anything at all comparable occurred in the consciousness of others.

If every thought requires a bearer, to the contents of whose consciousness it belongs, then it would be a thought of this bearer only and there would be no science common to many, on which many could work. But I, perhaps, have my science, namely, a whole of thought whose bearer I am and another person has his. Each of us occupies himself with the contents of his own consciousness. No contradiction between the two sciences would then be possible and it would really be idle to dispute about truth, as idle, indeed almost ludicrous, as it would be for two people to dispute whether a hundred-mark note were genuine, where each meant the one he himself had in his pocket and understood the word "genuine" in his own particular sense. If someone takes thoughts to be ideas, what he then recognizes to be true is, on his own view, the content of his consciousness and does not properly concern other people at all. If he were to hear from me the opinion that a thought is not an idea he could not dispute it, for, indeed, it would not now concern him.

So the result seems to be ; thoughts are neither things of the outer world nor ideas.

A third realm must be recognized. What belongs to this corresponds with ideas, in that it cannot be perceived by the senses, but with things, in that it needs no bearer to the contents of whose consciousness to belong. Thus the thought, for example, which we expressed in the Pythagorean theorem is timelessly true, true independently of whether anyone takes it to be true. It needs no bearer. It is not true for the first time when it is discovered, but is like a planet which, already before anyone has seen it, has been in interaction with other planets.¹

But I think I hear an unusual objection. I have assumed several times that the same thing that I see can also be observed by other people. But how could this be the case, if everything were only a dream? If I only dreamed I was walking in the company of another person, if I only dreamed that my companion saw the green field as I did, if it were all only a play performed on the stage of my consciousness, it would be doubtful whether there were things of the outer world at all. Perhaps the realm of things is empty and I see no things and no men, but have only ideas of which I myself am the bearer. An idea, being something which can as little exist independently of me as my feeling of fatigue, cannot be a man, cannot look at the same field together with me, cannot see the strawberry I am holding. It is quite incredible that I should really have only my inner world instead of the whole environment, in which I am supposed to move and to act. And yet it is an inevitable consequence of the thesis that only what is my idea can be the object of my awareness. What would follow from this thesis if it were true? Would there then be other men? It would certainly be possible but I should know nothing of it. For a man cannot be my idea, consequently, if our thesis were true, he also cannot be an object of my awareness. And so the ground would be removed from under any process of thought in which

¹ One sees a thing, one has an idea, one apprehends or thinks a thought. When one apprehends or thinks a thought one does not create it but only comes to stand in a certain relation, which is different from seeing a thing or having an idea, to what already existed beforehand.

I might assume that something was an object for another person as for myself, for even if this were to happen I should know nothing of it. It would be impossible for me to distinguish that of which I was the bearer from that of which I was not. In judging something not to be my idea I would make it the object of my thinking and, therefore, my idea. On this view, is there a green field? Perhaps, but it would not be visible to me. For if a field is not my idea, it cannot, according to our thesis, be an object of my awareness. But if it is my idea it is invisible, for ideas are not visible. I can indeed have the idea of a green field, but this is not green for there are no green ideas. Does a shell weighing a hundred kilogrammes exist, according to this view? Perhaps, but I could know nothing of it. If a shell is not my idea then, according to our thesis, it cannot be an object of my awareness, of my thinking. But if a shell were my idea, it would have no weight. I can have an idea of a heavy shell. This then contains the idea of weight as a part-idea. But this part-idea is not a property of the whole idea any more than Germany is a property of Europe. So it follows:

Either the thesis that only what is my idea can be the object of my awareness is false, or all my knowledge and perception is limited to the range of my ideas, to the stage of my consciousness. In this case I should have only an inner world and I should know nothing of other people.

It is strange how, upon such reflections, the opposites collapse into each other. There is, let us suppose, a physiologist of the senses. As is proper for a scholarly scientist, he is, first of all, far from supposing the things he is convinced he sees and touches to be his ideas. On the contrary, he believes that in senseimpressions he has the surest proof of things which are wholly independent of his feeling, imagining, thinking, which have no need of his consciousness. So little does he consider nervefibres and ganglion-cells to be the content of his consciousness that he is, on the contrary, rather inclined to regard his consciousness as dependent on nerve-fibres and ganglion-cells. He establishes that light-rays, refracted in the eye, strike the visual nerve-endings and bring about a change, a stimulus, there. Some of it is transmitted through nerve-fibres and ganglion-cells. Further processes in the nervous system are perhaps involved, colour-impressions arise and these perhaps join themselves to what we call the idea of a tree. Physical, chemical and physiological occurrences insert themselves between the tree and my idea. These are immediately connected with my consciousness but, so it seems, are only occurrences in my nervous system and

every spectator of the tree has his particular occurrences in his particular nervous system. Now the light-rays, before they enter my eye, may be reflected by a mirror and be spread further as if they came from a place behind the mirror. The effects on the visual nerves and all that follows will now take place just as they would if the light-rays had come from a tree behind the mirror and had been transmitted undisturbed to the eye. So an idea of a tree will finally occur even though such a tree does not exist at all. An idea, to which nothing at all corresponds, can also arise through the bending of light, with the mediation of the eye and the nervous system. But the stimulation of the visual nerves need not even happen through light. If lightning strikes near us we believe we see flames, even though we cannot see the lightning itself. In this case the visual nerve is perhaps stimulated by electric currents which originate in our body in consequence of the flash of lightning. If the visual nerve is stimulated by this means, just as it would be stimulated by lightrays coming from flames, then we believe we see flames. It just depends on the stimulation of the visual nerve, it is indifferent how that itself comes about.

One can go a step further still. This stimulation of the visual nerve is not actually immediately given, but is only a hypo-We believe that a thing, independent of us, stimulates thesis. a nerve and by this means produces a sense-impression, but, strictly speaking, we experience only the end of this process which projects into our consciousness. Could not this senseimpression, this sensation, which we attribute to a nerve-stimulation, have other causes also, as the same nerve-stimulation can arise in different ways? If we call what happens in our consciousness idea, then we really experience only ideas but not their causes. And if the scientist wants to avoid all mere hypothesis, then only ideas are left for him, everything resolves into ideas, the light-rays, nerve-fibres and ganglion-cells from which he started. So he finally undermines the foundations of Is everything an idea? Does everything his own construction. need a bearer, without which it could have no stability? I have considered myself as the bearer of my ideas, but am I not an idea myself? It seems to me as if I were lying in a deck-chair, as if I could see the toes of a pair of waxed boots, the front part of a pair of trousers, a waistcoat, buttons, part of a jacket, in particular sleeves, two hands, the hair of a beard, the blurred outline of a nose. Am I myself this entire association of visual impressions, this total idea? It also seems to me as if I see a chair over there. It is an idea. I am not actually much dif-

ferent from this myself, for am I not myself just an association of sense-impressions, an idea? But where then is the bearer of these ideas? How do I come to single out one of these ideas and set it up as the bearer of the rest? Why must it be the idea which I choose to call "I"? Could I not just as well choose the one that I am tempted to call a chair? Why, after all, have a bearer for ideas at all? But this would always be something essentially different from merely borne ideas, something independent, needing no extraneous bearer. If everything is idea, then there is no bearer of ideas. And so now, once again, I experience a change into the opposite. If there is no bearer of ideas then there are also no ideas, for ideas need a bearer without which they cannot exist. If there is no ruler, there are also no subjects. The dependence, which I found myself induced to confer on the experience as opposed to the experient, is abolished if there is no more bearer. What I called ideas are then independent objects. Every reason is wanting for granting an exceptional position to that object which I call "I".

But is that possible? Can there be an experience without someone to experience it? What would this whole play be without an onlooker? Can there be a pain without someone who has it? Being experienced is necessarily connected with pain, and someone experiencing is necessarily connected with being experienced. But there is something which is not my idea and yet which can be the object of my awareness, of my thinking. I am myself of this nature. Or can I be part of the content of my consciousness while another part is, perhaps, an idea of the moon? Does this perhaps take place when I judge that I am looking at the moon? Then this first part would have a consciousness and part of the content of this consciousness would be I myself once more. And so on. Yet it is surely inconceivable that I should be boxed into myself in this way to infinity, for then there would not be only one I but infinitely many. I am not my own idea and if I assert something about myself, e.g. that I do not feel any pain at this moment, then my judgment concerns something which is not a content of my consciousness, is not my idea, that is me myself. Therefore that about which I state something is not necessarily my idea. But, someone perhaps objects, if I think I have no pain at the moment, does not the word 'I' nevertheless correspond with something in the content of my consciousness and is that not an idea? That may be. 'A certain idea in my consciousness may be associated with the idea of the word 'I'. But then it is an idea among other ideas and I am its bearer as I am the bearer of

the other ideas. I have an idea of myself but I am not identical with this idea. What is a content of my consciousness, my idea, should be sharply distinguished from what is an object of my thought. Therefore the thesis that only what belongs to the content of my consciousness can be the object of my awareness, of my thought, is false.

Now the way is clear for me to recognize another person as well as to be an independent bearer of ideas. I have an idea of him but I do not confuse it with him himself. And if I state something about my brother I do not state it about the idea that I have of my brother.

The invalid who has a pain is the bearer of this pain, but the doctor in attendance who reflects on the cause of this pain is not the bearer of the pain. He does not imagine he can relieve the pain by anaesthetizing himself. An idea in the doctor's mind may very well correspond to the pain of the invalid but that is not the pain and not what the doctor is trying to remove. The doctor might consult another doctor. Then one must distinguish: first, the pain whose bearer is the invalid, second, the first doctor's idea of this pain, third, the second doctor's idea of this pain. This idea does indeed belong to the content of the second doctor's consciousness, but it is not the object of his reflection, it is rather an aid to reflection, as a drawing can be such an aid perhaps. Both doctors have the invalid's pain, which they do not bear, as their common object of thought. It can be seen from this that not only a thing but also an idea can be the common object of thought of people who do not have the idea.

So, it seems to me, the matter becomes intelligible. If man could not think and could not take something of which he was not the bearer as the object of his thought he would have an inner world but no outer world. But may this not be based on a mistake? I am convinced that the idea I associate with the words 'my brother' corresponds to something that is not my idea and about which I can say something. But may I not be making a mistake about this? Such mistakes do happen. We then, against our will, lapse into fiction. Indeed ! By the step with which I secure an environment for myself I expose myself to the risk of error. And here I come up against a further distinction between my inner and outer worlds. I cannot doubt that I have a visual impression of green but it is not so certain that I see a lime-leaf. So, contrary to widespread views, we find certainty in the inner world while doubt never altogether leaves us in our excursions into the outer world. It is difficult in many cases, nevertheless, to distinguish probability from certainty here, so we can presume to judge about things in the outer world. And we must presume this even at the risk of error if we do not want to succumb to far greater dangers.

In consequence of these last considerations I lay down the following: not everything that can be the object of my understanding is an idea. I, as a bearer of ideas, am not myself an idea. Nothing now stands in the way of recognizing other people to be bearers of ideas as I am myself. And, once given the possibility, the probability is very great, so great that it is in my opinion no longer distinguishable from certainty. Would there be a science of history otherwise? Would not every precept of duty, every law otherwise come to nothing? What would be left of religion? The natural sciences too could only be assessed as fables like astrology and alchemy. Thus the reflections I have carried on, assuming that there are other people besides myself who can take the same thing as the object of their consideration, of their thinking, remain essentially unimpaired in force.

Not everything is an idea. Thus I can also recognize the thought, which other people can grasp just as much as I, as being independent of me. I can recognize a science in which many people can be engaged in research. We are not bearers of thoughts as we are bearers of our ideas. We do not have a thought as we have, say, a sense-impression, but we also do not see a thought as we see, say, a star. So it is advisable to choose a special expression and the word 'apprehend' offers itself for the purpose. A particular mental capacity, the power of thought, must correspond to the apprehension ¹ of thought. In thinking we do not produce thoughts but we apprehend them. For what I have called thought stands in the closest relation to truth. What I recognize as true I judge to be true quite independently of my recognition of its truth and of my thinking about it. That someone thinks it has nothing to do with the truth of a 'Facts, facts, facts' cries the scientist if he wants to thought. emphasise the necessity of a firm foundation for science. What is a fact? A fact is a thought that is true. But the scientist will surely not recognise something which depends on men's varying states of mind to be the firm foundation of science.

¹The expression 'apprehend' is as metaphorical as 'content of consciousness'. The nature of language does not permit anything else. What I hold in my hand can certainly be regarded as the content of my hand but is all the same the content of my hand in quite a different way from the bones and muscles of which it is made and their tensions, and is much more extraneous to it than they are. The work of science does not consist of creation but of the discovery of true thoughts. The astronomer can apply a mathematical truth in the investigation of long past events which took place when on earth at least no one had yet recognized that truth. He can do this because the truth of a thought is time-less. Therefore that truth cannot have come into existence with its discovery.

Not everything is an idea. Otherwise psychology would contain all the sciences within it or at least it would be the highest judge over all the sciences. Otherwise psychology would rule over logic and mathematics. But nothing would be a greater misunderstanding of mathematics than its subordination to psychology. Neither logic nor mathematics has the task of investigating minds and the contents of consciousness whose bearer is a single person. Perhaps their task could be represented rather as the investigation of the mind, of the mind not of minds.

The apprehension of a thought presupposes someone who apprehends it, who thinks. He is the bearer of the thinking but not of the thought. Although the thought does not belong to the contents of the thinker's consciousness yet something in his consciousness must be aimed at the thought. But this should not be confused with the thought itself. Similarly Algol itself is different from the idea someone has of Algol.

The thought belongs neither to my inner world as an idea nor yet to the outer world of material, perceptible things.

This consequence, however cogently it may follow from the exposition, will nevertheless not perhaps be accepted without opposition. It will, I think, seem impossible to some people to obtain information about something not belonging to the except by sense-perception. Sense-perception inner world indeed is often thought to be the most certain, even to be the sole, source of knowledge about everything that does not belong to the inner world. But with what right? For sense-impressions are necessary constituents of sense-perceptions and are a part of the inner world. In any case two men do not have the same, though they may have similar, sense-impressions. These alone do not disclose the outer world to us. Perhaps there is a being that has only sense-impressions without seeing or touching things. To have visual impressions is not to see things. How does it happen that I see the tree just there where I do see it? Obviously it depends on the visual impressions I have and on the particular type which occur because I see with two eves. A particular image arises, physically speaking, on each of the

Another person sees the tree in the same place. two retinas. He also has two retinal images but they differ from mine. We must assume that these retinal images correspond to our impressions. Consequently we have visual impressions, not only not the same, but markedly different from each other. And yet we move about in the same outer world. Having visual impressions is certainly necessary for seeing things but not sufficient. What must still be added is non-sensible. And yet this is just what opens up the outer world for us; for without this nonsensible something everyone would remain shut up in his inner world. So since the answer lies in the non-sensible, perhaps something non-sensible could also lead us out of the inner world and enable us to grasp thoughts where no sense-impressions were involved. Outside one's inner world one would have to distinguish the proper outer world of sensible, perceptible things from the realm of the nonsensibly perceptible. We should need something non-sensible for the recognition of both realms but for the sensible perception of things we should need senseimpressions as well and these belong entirely to the inner world. So that in which the distinction between the way in which a thing and a thought is given mainly consists is something which is attributable, not to both realms, but to the inner world. Thus I cannot find this distinction to be so great that on its account it would be impossible for a thought to be given that did not belong to the inner world.

The thought, admittedly, is not something which it is usual to The world of the real is a world in which this acts on call real. • that, changes it and again experiences reactions itself and is changed by them. All this is a process in time. We will hardly recognize what is timeless and unchangeable as real. Now is the thought changeable or is it timeless? The thought we express by the Pythagorean theorem is surely timeless, eternal, unchangeable. But are there not thoughts which are true today but false in six months time? The thought, for example, that the tree there is covered with green leaves, will surely be false in six months time. No, for it is not the same thought at all. The words 'this tree is covered with green leaves ' are not sufficient by themselves for the utterance, the time of utterance is involved as well. Without the time-indication this gives we have no complete thought, *i.e.* no thought at all. Only a sentence supplemented by a time-indication and complete in every respect expresses a thought. But this, if it is true, is true not only today or tomorrow but timelessly. Thus the present tense in ' is true ' does not refer to the speaker's present but is, if the expression

be permitted, a tense of timelessness. If we use the mere form of the indicative sentence, avoiding the word 'true', two things must be distinguished, the expression of the thought and the assertion. The time-indication that may be contained in the sentence belongs only to the expression of the thought, while the truth, whose recognition lies in the form of the indicative sentence, is timeless. Yet the same words, on account of the variability of language with time, take on another sense, express another thought; this change, however, concerns only the linguistic aspect of the matter.

And yet! What value could there be for us in the eternally unchangeable which could neither undergo effects nor have effect on us? Something entirely and in every respect inactive would be unreal and non-existent for us. Even the timeless, if it is to be anything for us, must somehow be implicated with the temporal. What would a thought be for me that was never apprehended by me? But by apprehending a thought I come into a relation to it and it to me. It is possible that the same thought that is thought by me today was not thought by me yesterday. In this way the strict timelessness is of course annulled. But one is inclined to distinguish between essential and inessential properties and to regard something as timeless if the changes it undergoes involve only its inessential properties. A property of a thought will be called inessential which consists in, or follows from the fact that, it is apprehended by a thinker.

How does a thought act? By being apprehended and taken to be true. This is a process in the inner world of a thinker which can have further consequences in this inner world and which, encroaching on the sphere of the will, can also make itself noticeable in the outer world. If, for example, I grasp the thought which we express by the theorem of Pythagoras, the consequence may be that I recognise it to be true and, further, that I apply it, making a decision which brings about the acceleration of masses. Thus our actions are usually prepared by thinking and judgment. And so thought can have an indirect influence on the motion of masses. The influence of one person on another is brought about for the most part by thoughts. One communicates a thought. How does this happen? One brings about changes in the common outside world which, perceived by another person, are supposed to induce him to apprehend a thought and take it to be true. Could the great events of world history have come about without the communication of thoughts? And yet we are inclined to regard thoughts as unreal because they appear to be without influence

on events, while thinking, judging, stating, understanding and the like are facts of human life. How much more real a hammer appears compared with a thought. How different the process of handing over a hammer is from the communication of a thought. The hammer passes from one control to another, it is gripped, it undergoes pressure and on account of this its density, the disposition of its parts, is changed in places. There is nothing of all this with a thought. It does not leave the control of the communicator by being communicated, for after all a person has no control over it. When a thought is apprehended, it at first only brings about changes in the inner world of the apprehender, yet it remains untouched in its true essence, since the changes it undergoes involve only inessential properties. There is lacking here something we observe throughout the order of nature : reciprocal action. Thoughts are by no means unreal but their reality is of quite a different kind from that of things. And their effect is brought about by an act of the thinker without which they would be ineffective, at least as far as we can see. And yet the thinker does not create them but must take them as they are. They can be true without being apprehended by a thinker and are not wholly unreal even then, at least if they could be apprehended and by this means be brought into operation.