Philosophy 203: History of Modern Western Philosophy | Hamilton College
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Kant's *Critique of Pure Reason*
Refutation of Idealism (AW 781-3)
First Antinomy (AW 792-4)

I. After the Transcendental Deduction

After the Deduction in the *Critique*, Kant explains, or transcendentally deduces, all of the particular categories.
Then, he shows in greater detail how his transcendental idealism applies to a variety of traditional philosophical problems and paradoxes, including the question of the existence of an external world, whether space and time are absolute or relational, and whether we have free will.
In some cases, Kant sides with the rationalists, claiming that we have knowledge.
For example, Kant argues for the certainty of mathematics and knowledge of an external world.
On other cases, Kant fins the rationalists' claims overly dogmatic, exceeding the limits of pure reason.

We are going to look at three of Kant's arguments from later in the *Critique*.

1. The refutation of idealism
2. Whether the universe is finite or infinite
3. The ontological argument for the existence of God

II. Refutation of Idealism

In the transcendental deduction, Kant argues that since the categories are *a priori*, then could not be derived from experience.

> Either experience makes these concepts possible, or these concepts make experience possible. The first alternative is not what happens as regards the categories (nor as regards pure sensible intuition). For they are *a priori* concepts and hence are independent of experience...The categories contain the grounds, on the part of the understanding, of the possibility of all experience as such (B167, AW 755a-b).

He thus proposes a transcendental deduction, or derivation, or discovery, of the concepts.
But, he also considers a third path.
We might think of the categories as subjective conditions for our experience that lack objective status.
They might just be necessary conditions for the way we see the world, and not conditions on how the world is.
That is, despite arguing that they are *a priori*, Kant believes that he has not shown that they are objective, that they are conditions on nature itself.

> Someone might want to propose...that the categories are...subjective predispositions for thinking that are implanted in us simultaneously with our existence; and that they were so arranged by our originator that their use harmonizes exactly with the laws of nature governing the course of experience... (B167, AW 755b).

In such a case, Kant argues, the concepts would lack necessity even if our application of those concepts were necessary.

> I could then not say that the effect is connected with the cause in the object (i.e. connected with it necessarily), but could say only that I am so equipped that I cannot think this representation otherwise than as thus connected. And this is just what the skeptic most longs for... (B168, AW 755b).

In other words, the alternative to seeing the categories as objective, since we can know nothing about the transcendental nature of the universe, is to see them as completely subjective.
The most plausible alternative to the objectivity of the categories is Humean skepticism.
But we do seem to have knowledge of the causal structure of the universe.

In the Refutation of Idealism, Kant argues that neither Berkeleyan idealism nor Humean skepticism are justified, given the conclusions of the Transcendental Analytic.

> "**Theorem** The mere, but empirically determined, consciousness of my own existence proves the existence of objects in space outside me" (B275, AW 782a).

First, Kant distinguishes between problematic idealism, which he attributes to Descartes, and dogmatic idealism, which he attributes to Berkeley.
The dogmatic idealist complains that space and time must be properties of the noumenal world.
But, since we can't know anything of the noumenal world, then we must have no knowledge of space and time.
Kant, by taking space and time to be pure forms of intuition, provides a context for rejecting dogmatic idealism.
We can take them to be objective properties without committing to knowledge of the noumenal world.
Thus, the real problem for Kant is the problematic idealist, by which term Kant refers to skeptic of the First Meditation.

> Problematic idealism...alleges that we are unable to prove by direct experience an existence apart from our own...The proof it demands must...establish that regarding external things we have not merely *imagination* but also *experience*. And establishing this surely cannot be done unless one can prove that even our *inner* experience, indubitable for Descartes, is possible only on the presupposition of *outer* experience (B275, AW 782a).

III. Tlumak on the Refutation of Idealism

The following is the version of the refutation found in Tlumak's book.

1. I am judging.
2. Some act of judging is occurring.
3. Any act of judging is an act of consciousness or awareness.
4. Acts of consciousness or awareness are representative (have a content).
5. Awareness of the instantaneous is impossible.
6. So the content of awareness is non-instantaneous.
7. Any non-instantaneous content is a successive content, that is, a series of items occurring in an order, and not all at a single instant.
8. So judgmental awareness is of a succession of items.
9. Awareness of succession implies awareness of a plurality of items as a plurality - awareness of a diversity or manifold.
10. Awareness of a plurality of items as a plurality requires that the plurality be apprehended as a numerically identical collection over the time during which the awareness is occurring.
11. This identity of the manifold over time requires that the act of awareness of this identical manifold connect up or relate the various elements which comprise it, that is, be aware of all the elements together.
12. Such a connective awareness requires that earlier items in the series be recognized together with the later items, and that all the items be recognized as belonging to this unity over time.
13. Only a persisting, identical subject of awareness can be connective; a series or collection of diverse subjects of consciousness is incapable of such connective activity.
14. So any act of judgment requires a persisting judger.
15. An identical judger must be able to be aware of his unity of consciousness.
16. But awareness of an objectless awareness itself is impossible. I can be aware of consciousness only by being aware of the object of consciousness. [Recall Hume's argument for the bundle theory of self: I never perceive myself directly.]
17. So awareness of a persisting consciousness requires awareness of a persisting object of consciousness.
18. So awareness of succession requires awareness of something persisting.
19. This something persisting cannot be an item in the series, or of the succession, since only by being aware of it can I be aware of the series.
20. This series of items (of acts of representation) constitutes my mental life.
21. So the persisting something is not part of my mental life.
22. But if something is not part of my mental life, it is existentially and attributively independent of me.
23. And since it is something which I can perceptually identify and which persists, it is re-identifiable.
24. So the persisting something required for awareness of succession, which in turn is required for judging, is an objective particular.
25. So I am aware of an objective particular.

Tlumak's version of the refutation is so good, I won't comment much further.
I just want to raise a worry about the nature of the something persisting, in steps 18-25.
I do not see why that persisting something could not be a Berkeleyan prototype, an idea to which I have intuitive access, rather than a material object or the noumenal correlate of a material object.
Kant's refutation of idealism seems, by leaving the noumenal world out of our cognition, too weak to yield a satisfying empirical realism.

IV. First Antinomy

Kant presents three antinomies, or paradoxes, to supplant his claim that reason has limits.
While some proper metaphysics can be established using synthetic *a priori* reasoning, other topics (e.g. God, free will) are beyond our ken.
Our reason, wanting answers to such questions, speculates.
The problem with such speculation is that we can argue on either side of the debate.
We can establish that the universe is infinite.
We can also establish that it is finite.
Since such antinomies can not hold, Kant sees such proofs as demonstrating that reason has exceeded its limits.
Here again, Kant has the revolutionary Hume as an influence.
We can commit such arguments to the flames.

The first antinomy concerns both the temporal and spatial finitude of the universe.
Kant argues that the universe has a beginning from the premise that an infinite series can not be completed.
If the universe existed from infinitely long ago, the present time would be the end of an infinite series.
So, there must have been some beginning.
For the spatial finitude of the universe, Kant claims that the concept of simultaneity presupposes a spatially finite universe.
If the universe were infinitely large, we could not think of all of the universe as existing simultaneously.

On the other side, Kant argues that the universe has no beginning in time from the logical impossibility of creation.
If there were a beginning point, there would have to be something before it.
But, that time would have nothing in it, since the universe has not been created yet.
So the universe would have no way to begin.
Kant's argument that the universe is spatially infinite assumes absolute space.
Imagine you were to go to the end of the universe.
Now, stick out your arm past the edge.
It seems that you could always perform this task.
Thus, the container has to be infinite.
If we take the universe to be merely the contained portion, then we have no way to think about the container, the rest of space.
So, space itself must be the infinite container.

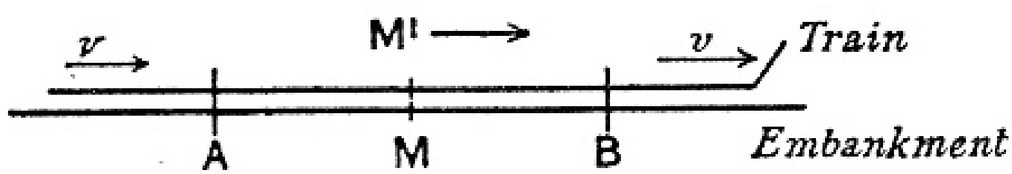
> If one wants to leave out the void, and hence space as such, as an *a priori* condition for the possibility of appearances, then the entire world of sense drops out (A433/B461, AW 794b).

Remember, space is an *a priori* form of intuition, presupposed by all possible experience.

Kant has argued, *a priori*, to both sides of a contradiction.
He concludes that pure reason has exceeded its reach.
There is no knowledge to be had of whether the universe is finite or infinite.
Like a Humean empiricist, Kant concludes that we can not know any facts of the matter.

Notice that, in making this negative argument, Kant assumes that claims about whether the universe is finite or infinite are matters for *a priori* metaphysical reasoning.
In contrast, there are some mathematical and physical facts that undermine his claims.
Kant asserts that the universe must be spatially bound because otherwise we could have no definite concept of simultaneity.
But according to the theory of relativity, simultaneity and time itself are not definite concepts anyway.
They depend on the arbitrary choice of a frame of reference.
Imagine standing on a platform waiting for a train equidistant between two signal posts, A and B.
Imagine further that lights are flashed at A and B in a way that one perceives them as flashing simultaneously.
Now, consider a train traveling in the direction from A to B, passing you at the very moment that the lights are flashed.
To a perceiver on the train, the light at B will appear before the light at A, since the frame of reference (the train) is moving toward B, and away from A.

> Events which are simultaneous with reference to the embankment are not simultaneous with respect to the train, and vice versa (relativity of simultaneity). Every reference-body (co- ordinate system) has its own particular time; unless we are told the reference-body to which the statement of time refers, there is no meaning in a statement of the time of an event (Einstein, *Relativity: The Special and General Theory*, Chapter IX).



Similarly, Kant assumes an obsolete concept of infinity.

> The true (transcendental) concept of infinity is this: that the successive synthesis of unit[s] in measuring by means of a quantum can never be completed" (A432/B460, AW 793a).

In the nineteenth century, George Cantor's work on transfinite numbers established that there are different sizes of infinity.
To count from one size of infinity to the next, we consider the smaller infinity as complete.
Today, we often define a set to be infinite if it can be put into one-one correspondence with a proper subset of itself.
For example, consider the integers and the even integers, which are a proper subset of the integers.
The integers are infinite just because we can match each one with an even integer.
See the appendix on infinity, below, or this longer, fun discussion of infinite arithmetic.

Kant's claim about the certainty of mathematics meet a similar objection.
Kant argued that we have *a priori* knowledge of Euclidean geometry arising from its role as a form of pure intuition assumed in all appearances.
But it turns out that there are various consistent non-Euclidean geometries in addition.

Euclidean geometry, instead of being the sole correct geometry, is just one of a family of mathematical theories, not mathematically favored.

In a full course on Kant, we would spend more time on the antinomies and related applications of Kant's views.
Here, we will examine just one more topic, one which we discussed at the start of the term: the ontological argument for the existence of God.

Appendix to Kant Notes on Infinity

AI. The infinite hotel

Consider the infinite hotel: a hotel with infinitely many rooms.
The hotel is fully booked.
A new guest arrives.
We can add the new guest, by shifting every current guest from Room n to Room n+1.
Then, Room 1 will be available for the arriving guests.
We can perform the same procedure repeatedly, adding single guests.
We can generalize the procedure to add any finite number of guests, m, by shifting all current guests from Room n to Room n+m.

Next, an infinite bus with an infinite number of guests arrives.
We can still accommodate them, but we need a new procedure.
We can add the infinitely many new guests by shifting every current guest from Room n to Room 2n.
Now, all the even rooms are filled, but the odd rooms are vacant.
We can put the infinite number of new guests in the odd-numbered rooms.

Next, an infinite number of infinite busloads of guests arrives.
We can still accommodate them.
Shift all current guests from Room n to Room 2^n.
Now, all the rooms that are powers of two are filled, leaving lots of empty rooms.
We can place the people on the first bus in room numbers 3^n (for n people on the bus), the people in the second bus in rooms 5^n, the people in the third bus to rooms 7^n, and so on for each (prime number)^n.
Since there are an infinite number of primes, there will be an infinite number of infinite such sequences.
And, there will be lots of empty rooms!

AII. Cardinality, size, and correspondence

The splitting headache which may arise from thinking about infinite numbers may correspond to a split between two ways to think about cardinal numbers.
We use them to measure size.
But, we also use one-one correspondence to characterize cardinal numbers.
With finite numbers, these two approaches converge.
The size of a group is the same as the correspondence between the objects in the group and some initial segment of the natural numbers.
That is, if we have five hedgehogs, we can line them up and give them each a number from one to five.

With transfinite numbers, as with the infinite hotel, the two concepts diverge.
The size of the integers seems to be bigger than the size of the even numbers.
But, they can be put into one-one correspondence with each other.

Georg Cantor, in the mid-nineteenth century, relied on the one-one correspondence notion to generate different kinds of infinite, or transfinite, numbers.
When we list the members of something, we are putting them into one-one correspondence with the natural numbers.
Cardinal numbers are the sizes of sets, the number we count to when we put the set in one-one correspondence with the natural numbers.
But, it turns out that we can not make certain lists.

For example, we can not list the real numbers.
The real numbers may be represented as their decimal expansions: non-repeating, non-terminating.
Imagine that we have such a list.
Let's represent that list abstractly:

$$
\begin{array}{l}
a_1\ a_2\ a_3\ a_4\ a_5\ a_6\ a_7 \ldots \\
b_1\ b_2\ b_3\ b_4\ b_5\ b_6\ b_7 \ldots \\
c_1\ c_2\ c_3\ c_4\ c_5\ c_6\ c_7 \ldots \\
d_1\ d_2\ d_3\ d_4\ d_5\ d_6\ d_7 \ldots \\
\ldots
\end{array}
$$

By hypothesis, the list contains all real numbers.
But, we can, for any list, demonstrate a number which does not appear on the list.
Consider the following number

$$a_1\ b_2\ c_3\ d_4\ e_5\ f_6\ g_7 \ldots$$

That number could be on the list.
Now, take each digit in that number and change it: add one to each number other than nine, and replace all nines with zeroes.
The following number is certainly not on the list.
For, it is different from the first number on the list in the first digit, different from the second number on the list in the second digit, and so on, for all numbers on the list.
If we add this new number to the list, we can form a new number that's not on the resulting list, by the same process.
Thus, all possible lists of real numbers are necessarily incomplete.
This proof is called Cantor's diagonalization argument.
It shows that the ordinary concept of size is not precisely the same as the concept of one-one correspondence.
Mathematicians tend to think of size now as one-one correspondence.
But, their use of 'size' differs from that of ordinary people, once we get to transfinites.

Numbers actually have two different functions.
Cardinal numbers measure size.
Ordinal numbers measure rank.
Let's look a bit at both, starting with the cardinals.

AIII. Cardinal arithmetic

Cardinal numbers are sets which we use to measure the sizes of sets, by one-one correspondence.
We are all familiar with many properties of cardinal numbers.
For all cardinal numbers a, b, and c, whether finite or transfinite, the following relations hold:

1. $a+b=b+a$
2. $ab=ba$
3. $a + (b + c) = (a + b) + c$
4. $a \bullet (b \bullet c) = (a \bullet b) \bullet c$
5. $a \bullet (b + c) = ab + ac$
6. $a^{(b+c)} = a^b \bullet a^c$
7. $(ab)^c = a^c \bullet b^c$
8. $(a^b)^c = a^{bc}$

But some properties of finite cardinal numbers do not hold for transfinite numbers.
Notice that $a+1=a$, when a is transfinite.
And $2a=a$ holds as well.
Even $a \bullet a=a$
We can show these all by considering a bijective mapping from one set to the other.
We showed all of these facts in the infinite hotel.

Consider one final important property which holds both of finite and transfinite numbers.

9. $2^a > a$

In set-theoretic terms, this ninth claim is that $\wp(a) > a$.
'$\wp(a)$' refers to the power set of a, the set of all subsets of a set a.
Consider a set $A = \{2, 4, 6\}$
Then $\wp(A) = \{\{2\}, \{4\}, \{6\}, \{2, 4\}, \{2, 6\}, \{4, 6\}, \{2, 4, 6\}, \varnothing\}$
In general the power set of a set with n elements will have 2^n elements.
Since sets with n members are the same size as sets with n+1 members, or with 2n members, or with n^2 members, for infinite n, we might think that sets with n members are the same size as sets with 2^n members.
For, with infinite numbers, it is not always clear that what we think of as a larger set is in fact larger.

The claim that $\wp(a) > a$ has been called Cantor's paradox.
$\wp(a)>a$ is now taken to be Cantor's theorem.
The proof of the theorem is a set-theoretic version of the diagonalization argument.
We want to show that the cardinal number C of the power set of a set is strictly larger than the cardinal number of the set itself (i.e. $C(\wp(A)) > C(A)$).
To show that fact, it suffices to show that there is no function which maps A one-one and onto its power set.
A function is called one-one if it every element of the domain maps to a different element of the range.
A function maps a set A onto another set B if the range of the function is the entire set B, i.e. if no elements of B are left out of the mapping.

Proof of Cantor's Theorem

> Assume that there is a function f: A ⇒ ℘(A)
> Consider the set B = {x | x ∈ A • x ∉ f(x)}
> B is a subset of A, since it consists only of members of A.
> So, B is an element of ℘(A), by definition of the power set.
> That means that B itself is in the range of f.
> Since, by assumption, f is one-one and onto, there must be an element of A, b, such that f(b) is B itself.
> Is b ∈ B?
> If it is, then there is a contradiction, since B is defined only to include sets which are not members of their images.
> If it is not, then there is a contradiction, since B should include all elements which are not members of their images.
> Either way, we have a contradiction.
> So, our assumption fails, and there must be no such function.
> ℘(A) > A

QED

Let's call the size of the natural numbers $\aleph_0$.
Then the real numbers, and the real plane, are the size of the power set of the natural numbers, $2^{\aleph_0}$.
We can proceed to generate larger and larger cardinals through the power set process.
Moreover, set theorists, by various ingenious methods, including addition of axioms which do not contradict the given axioms, generate even larger cardinals.

AIV. Ordinal numbers

Let's start counting.
By adding one, here, we normally mean taking the successor of 1.
So, ω+1 will be the successor of ω.
Ordinals generated in this way are called successor ordinals.
In transfinite set theory, there are also sets which are called limit elements.
We get them by taking the union of all the members of a set.
Ordinal numbers, set-theoretically, are just special kinds of sets, well-ordered sets.
A set is well-ordered if, basically, we can find an ordering relation on the set, and it has a first element.
If we consider all the sets that correspond to the finite ordinals, and combine them into a whole, we can get another well-ordered set.
This will be a new ordinal, and it will be larger than all of the ordinals in it.
So, there are two kinds of ordinals: successor ordinals and limit ordinals.

> 1, 2, 3, ... ω
> ω+1, ω+2, ω+3...2ω
> 2ω+1, 2ω+2, 2ω+3...3ω
> 4ω, 5ω, 6ω...ω^2
> $\omega^2, \omega^3, \omega^4 \ldots \omega^\omega$
> $\omega^\omega, (\omega^\omega)^\omega, ((\omega^\omega)^\omega)^\omega, \ldots \varepsilon^0$

The limit ordinals are the ones found after the ellipses on each line.
Large ordinals correspond to the large cardinals.

Notice that limit ordinals are taken as the completions of an infinite series.
Kant, in the antinomies, denied that there can be any completion of an infinite series.
But, Cantor's diagonal argument shows that there are different levels of infinity.
And, we form ordinals to represent the ranks of these different levels of infinity precisely by taking certain series to completion.