Philosophy 405: Knowledge, Truth and Mathematics
Spring 2016
Tuesdays and Thursdays: 1pm - 2:15pm
Philosophy Seminar Room

Hamilton College
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Syllabus

Course Description and Overview

Mathematics has long had a prominent place in philosophy. Plato's students were implored to excel in mathematics; a sign over the door to his Academy said, "Let no one enter who is ignorant of geometry." Aristotle wrote, "Mathematics has come to be the whole of philosophy for modern thinkers" (*Metaphysics* I.9: 992a32).

Some prominent philosophers in the early modern period were also important mathematicians, including Descartes, who developed analytic geometry, and Leibniz, who developed the calculus. In the late nineteenth and early twentieth centuries, philosophers including Frege and Russell made advances in the foundations of mathematics proper. In recent years, many philosophers have made contributions to set theory and mathematical logic, independently of their philosophical work.

In the other direction, mathematicians from Euclid forward have contributed to philosophy. Cantor's work on transfinite numbers transformed the philosopher's concept of infinity, which had played a central role in philosophical debate about God and the origins of the universe for millennia. Other philosophical topics like necessity and contingency have received mathematical treatment which has changed the way philosophers argue about these concepts. Indeed, some mathematicians, like Hilbert, Gödel, von Neumann, and Tarski, are central philosophical figures.

Even philosophers who have not contributed to mathematics have made mathematical insights central to their work. Berkeley tried to debunk the calculus on philosophical grounds. Kant's transcendental idealism begins with the question of what the structure of our reasoning must be in order to yield mathematical certainty.

Still, some philosophers who spend time with mathematics deny that the relationship of mathematics to philosophy is particularly close. Wittgenstein wrote that philosophy, "Leaves mathematics as it is, and no mathematical discovery can advance it." Kripke implored, "There is no mathematical substitute for philosophy."

This course is divided into two parts. The first part, covering roughly the first eight weeks of the term, is an historical survey of philosophical questions about mathematics. Do we have *a priori* knowledge of necessary truths? Is our knowledge of mathematics empirical? Do we have mathematical knowledge at all? The readings in the first part of the course, covered mainly chronologically, range from ancient philosophy into the twentieth century. We will survey a range of views about the status of mathematics and our intellectual capacities.

The second half of the course focuses generally on a debate among mathematical platonists, those who believe that mathematical objects exist or that mathematical statements are true. On one side, the indispensabilists believe that all knowledge must be grounded in sense experience. Since we have no direct sense experience of mathematical objects, our beliefs about them are justified indirectly, by their uses in scientific theories which are constructed to explain or account for our observations.

Opposing the indispensabilists, autonomy platonists believe that the robustness of our mathematical beliefs shows that we have cognitive capacities beyond mere perception, capacities which ground our mathematical beliefs. Toward the end of the course, we will look at recent work on one of those purported capacities, mathematical intuition, including some of my own work on the debate.

Texts

Most readings are available in my forthcoming *An Historical Introduction to the Philosophy of Mathematics* (HIPM). Some will come from my monograph, *Autonomy Platonism and the Indispensability Argument* (AP and the IA). You will have access to those books. Other readings will appear on the course website.

Two excellent secondary sources are :

James Robert Brown. *Philosophy of Mathematics: An Introduction to the World of Proofs and Pictures*, 2nd edition. New York: Routledge, 2008.

Stewart Shapiro. *Thinking About Mathematics: The Philosophy of Mathematics*. New York: Oxford, 2000.

On-Line Resources

The website for this course is:

http://www.thatmarcusfamily.org/philosophy/Course_Websites/Math_S16/Course_Home.html

The course website includes an html syllabus, readings, assignments, class notes, handouts, and other links. Limited material will be available on Blackboard.

Assignments and Grading

Your responsibilities for this course include the following, with their contributions to your grade calculation in parentheses:

1. The readings listed below.
2. Twenty reading précis (10%)
3. Two or three seminar presentations/papers (30%; 10% each)
4. Term paper (8-12 pages) (40%)
5. Final exam (20%)

Readings are to be completed before the class indicated. Each chapter of *An Historical Introduction to the Philosophy of Mathematics* includes both primary sources and introductory commentary by the editors. The primary sources are required reading.

Reading précis are distillations of some argument in an assigned reading, 100–150 words. A précis is a skeletal version of an argument, the argument in its most efficient form. In preparing for most classes, you should write one précis before class. You may choose to write about an entire reading or to focus on a portion of the reading. If there is more than one reading, choose one on which to focus. You need not complete précis for the two classes in which you are presenting a seminar paper. Writing précis should not be a burden. Just do a little writing as you read.

The first ten précis are due on Friday, March 11, at 4pm. The last ten précis are due on Friday, May 6, at 4pm. You will mainly be graded on the completion of twenty précis, rather than their quality. I expect that the précis will be useful to you in preparing both for classes and for the final exam.

Two or three times during the term, you will lead the class in a **presentation** of a **seminar paper**. The paper (750-1200 words) is due to be submitted for a grade by 4pm three days after your presentation. (So, papers presented on a Tuesday are due on Friday; papers presented on Thursday are due on Sunday.) Each chapter of *An Historical Introduction to the Philosophy of Mathematics* contains Themes to Explore, which may be useful topics for a seminar paper. You may be creative with your presentation. Your grade for the

seminar paper will depend on both the paper and your presentation of it.

Your **term papers** will be completed in four stages. A one–to-two-paragraph abstract of your paper with a proposed bibliography is due on Thursday, March 3. A précis of your argument with an annotated bibliography is due on Thursday, March 31. A full draft of your term paper is due on Tuesday, April 19. The final draft is due on Tuesday, May 3. I hope that you will be able to use one or more of your seminar papers as part of your term paper. See the Paper Assignment handout for various options for paper topics. I will be happy to meet with you to discuss your topic, in advance. Failure to hand in a draft, or handing in an insufficient draft, will reduce your final paper grade by two steps (e.g. from B+ to B-).

The **final exam** will be on Wednesday, May 11, from 7pm to 10pm. Preparatory questions will be posted on the course website.

Grades on assignments will be posted on Blackboard, along with a running total, which I call your grade calculation. Your grade calculation is a guide for me to use in assigning you a final grade. There are no rules binding how I translate your grade calculation into a letter grade. The Hamilton College key for converting letter grades into percentages is not a tool for calculating your final grade. I welcome discussion of the purposes and methods of grading, as well as my own grading policies.

Both the **Writing Center** and the **Oral Communications Center** have an astoundingly wonderful set of resources to help you write and speak more effectively.

The Hamilton College Honor Code will be enforced.

Accessibility and Diversity of Learning Styles

Your well-being and success in this course are important to me. Different people learn best in different ways. Please come talk with me about how best to balance your individual needs and learning style with my expectations for the course. If you are eligible for testing accommodations, please also see Allen Harrison, Associate Dean of Students for Multicultural Affairs and Accessibility Services in the Office of the Dean of Students, Elihu Root House.

Contacting Me

My office hours for the Spring 2016 term are 10:30am–noon, Tuesdays and Thursdays. My office is 202 College Hill Road, Room 210. My email address is rmarcus1@hamilton.edu.

Schedule

*Readings marked with a * are optional.*

Class	Date	Topic	Primary Readings, to be completed before class
1	Tuesday, January 19	Mathematics and the Philosophy of Mathematics	Brown, Chapter 1
2	Thursday, January 21	Pythagoras and the Pythagoreans	HIPM 1: Pythagoras, Parmenides, and Zeno's Paradoxes
3	Tuesday, January 26	Plato's Platonism	HIPM 2: Plato *from* HIPM 3: Criticisms of Plato's Theory of Forms

Class	Date	Topic	Primary Readings, to be completed before class
4	Thursday, January 28	Aristotle's Qua	HIPM 3: Aristotle *Lear, "Aristotle's Philosophy of Mathematics"
5	Tuesday, February 2	Mathematics, Clarity, and Distinctness	HIPM 4: The Rationalists, through p 131 *Kline, "Coordinate Geometry" *Kline, "The Mathematization of Science"
6	Thursday, February 4	Innate Ideas	HIPM 4: The Rationalists *Kline, "The Creation of the Calculus"
7	Tuesday, February 9	Lockean Conceptualism	HIPM 5: The Empiricists, to p 194
8	Thursday, February 11	Hume's Distinction	HIPM 5: The Empiricists, pp 202–end
9–10	Tuesday, February 16–Thursday, February 18	The Synthetic *A Priori*	HIPM 6: Kant *Kitcher, "Kant and the Foundations of Mathematics"
11	Tuesday, February 23	Cantor's Paradise	HIPM 8: Cantor's Transfinites *Tiles, "Numbering the Continuum"
12–13	Thursday, February 25–Tuesday, March 1	Logicism	HIPM 9: Logicism *Russell, "On Our Knowledge of General Principles" *Russell, "How *A Priori* Knowledge is Possible"
14	Thursday, March 3 **Term Paper Abstracts**	Formalism and Incompleteness	HIPM 10: Formalism
15	Tuesday, March 8	Intuitionism	HIPM 11: Intuitionism
16	Thursday, March 10	Conventionalism	HIPM 12: Conventionalism
Friday March 11		**First 10 Reading Précis are due**	
Spring Break			
17	Tuesday, March 29	The Problem	HIPM 15: The Benacerraf Problem
18	Thursday, March 31 **Term Paper Précis due**	Two Dogmas of Empiricism	Quine, "Two Dogmas of Empiricism" *Grice and Strawson, "In Defense of a Dogma"
19	Tuesday, April 5	The Indispensability Argument	HIPM 16: Quine, Quine, and Putnam Quine on Recreation *Marcus, *AP and the IA,* Chapter 2: The Quinean Indispensability Argument
20	Thursday, April 7	Dispensabilism	HIPM 16: Field, from *Science without Numbers* *MacBride, "Listening to Fictions: A Study of Fieldian Nominalism"

Class	Date	Topic	Primary Readings, to be completed before class
21	Tuesday, April 12	The Weasel	Leng, "What's Wrong with Indispensability? (Or, the Case for Recreational Mathematics)" *Marcus, *AP and the LA*, Chapter Four: The Weasel
22	Thursday, April 14	Autonomy Platonism I	Marcus, *AP and the LA*, Chapter One: Platonism: An Overview *HIPM 20: Contemporary Apriorism
23	Tuesday, April 19 **Term Paper Draft Due**	The Unfortunate Consequences	Marcus, *AP and the LA*, Chapter Five: The Unfortunate Consequences
24	Thursday, April 22	Intuition in Mathematics I	Chudnoff, "Awareness of Abstract Objects"
25	Tuesday, April 26	Intuition in Mathematics II	Chudnoff, "Intuition in Mathematics"
26	Thursday, April 28	Autonomy Platonism II	Marcus, "The Indispensabilist and the Autonomist"
27	Tuesday, May 3 **Term Paper Due**	Reflective Equilibrium	Goodman, "The New Riddle of Induction," §1–§2 (pp 59–65) Rawls, *A Theory of Justice*, §§3–4, 9 Russell, "Logical Atomism", to top of p 326
28	Thursday, May 5	Autonomy Platonism III	Marcus, *AP and the LA*, Chapter 11: Circles and Justification

Friday, May 6, 4pm: Final ten précis due

Wednesday, May 11, 7-10pm: Final Exam

Deadline Summary

Thursday, March 3: Term paper abstracts and bibliography
Friday, March 11, 4pm: First ten reading précis
Thursday, March 31: Term paper précis and annotated bibliography
Tuesday, April 19: First draft of term paper
Tuesday, May 3: Final draft of term paper
Friday, May 6, 4pm: Final ten précis due
Wednesday, May 11, 7-10pm: Final Exam

Seminar papers are due by 4pm three days after your presentation.