Intuitionism

The intuitionist firmly believes in mathematics as a construction of the human mind; thus, any claim or proof that one asserts must be constructive in order for it to exist.

- In the study of mental mathematical constructions, "to exist" must be synonymous with "to be constructed" (Heyting 67).

However, mathematicians have made advances through the use of non-constructive proofs as well.

Constrictive vs. Non-Constructive Proofs

- Constructive proofs, which are the only types of proofs accepted by intuitionists are proofs that provide a direct example, or give a constructive way for producing that example.
- Non-Constructive proofs, also known as existence proofs give an indirect way of showing that a mathematical object exists, without providing a specific example.
  - i.e. Zermelo's proof that every set can be well-ordered (does not produce ordering); the claim that there exist irrational numbers x and y such that x^y is rational

L.E.J. Brouwer

- He was the the earliest and most prominent of the neo-intuitionists
- Brouwer rejects the Kantian notion that our knowledge of geometry is synthetic a priori due to non-Euclidean theories showing that geometry transcended our intuition. He asserts that geometrical knowledge stems from our arithmetic a priori and only adheres to the Kant's apriority of time rather than his apriority of space as well.

[Intuitionism] has recovered by abandoning Kant's apriority of space but adhering the more resolutely to the apriority of time. This neo-intuitionism considers the falling apart of moments of life into qualitatively different parts, to be reunited only while remaining separated by time, as the fundamental phenomenon of the human intellect, passing by abstracting from its emotional content into the fundamental phenomenon of mathematical thinking, the intuition of the bare two-oneness (Brouwer 80).

Two-Oneness

- The idea of two-oneness completes intuitionism in a sense because it gives a sound foundation to a method of belief that is solely based on construction.
- The intuition of two-oneness is the basis for our knowledge of mathematics.

"This intuition of two-oneness, the basal intuition of, mathematics creates not only the numbers one and two, but also all finite ordinal numbers, inasmuch as one of the elements of the two-oneness may be thought of as a new two-oneness, which process may be repeated indefinitely; this gives rise still further to the smallest infinite ordinal number *w* (Brouwer, 80)."

Intuitionists and LEM

- LEM – law of the excluded middle which states that:
  - For any proposition, either that proposition is true or its negation is true; the law of non-contradiction.
- The intuitionist does not accept LEM because it easily gives rise to non-constructive proofs:
- The Twin Prime Conjecture:

  I. k is the greatest prime such that k - 1 is also a prime, or k = 1 if such a number does not exist

  II. l is the greatest prime such that l - 2 is also a prime, or l = 1 if such a number does not exist (Heyting 67).

- The second assertion is not accepted by the intuitionist because of its non-constructive nature. Because it is non-constructive, it is not a legitimate proof and does not exist.

  CL1. The sequence of twin primes is either finite or infinite.

  CL2. If it is finite, then x is the larger element of the largest pair.

  CL3. If it is infinite, then x is 1.

  CLC: x is some integer.

- The law of the excluded middle is used in the first step of the classical logicist's thinking and that is not accepted by the intuitionist because in the example, it is not shown that the sequence of twin primes is finite or infinite. Therefore, constructively, we cannot show that the sequence of twin primes must be one or the other.

Is the constructive argument of the intuitionist simply a semantic argument?

- “2+2=3+1” must be read as an abbreviation for the statement: “I have effected the metal constructions indicated by “2+2” and by “3+1” and I have found that they lead to the same result (Heyting, 72).

Although this distinction made by the intuitionist seems purely semantic, it gives further support to the his belief in the constructive argument.