#17 – The Problem

Jason Driscoll

Introduction

- Background and context
  - Semantic history (Tarski’s theory of truth)
  - Epistemic history (CTK)
- The problem
  - Benacerraf’s original formulation
  - Field’s reformulation
- Some answers
  - Gödel
  - The Combinatorialists

Setting up the Problem

- Two concerns that yield two conditions – semantic and epistemic
- "two quite distinct concerns have separately motivated accounts of the nature of mathematical truth: (1) the concern for having a homogenous semantical theory in which semantics for the propositions in mathematics parallel the semantics the semantics for the rest of the, and (2) the concern that the account of mathematical truth mesh with a reasonable epistemology"
- "It will be my general thesis that almost all accounts of the concept of mathematical truth can be identified with serving one or another of these masters *at the expense of the other*." (661)
- How can we reconcile mathematical truth in addition to knowledge?

Mathematical truth versus mathematical knowledge

- Truth here relates mainly to the truth conditions for mathematical claims
  - How our syntax and semantics allows us to demonstrate the truth of a mathematical claim
  - Semantic (related to truth, meaning, and reference)
- Knowledge relates to our ability to know said truths
  - How we can justify that we know the mathematical truths to be true
  - Epistemic (related to knowledge)

Two conditions

- “An account of knowledge that seems to work for certain empirical propositions about medium-sized physical objects but which fails to account for more theoretical knowledge is unsatisfactory—not only because it is incomplete, but because it may be incorrect as well. To think otherwise would be, among other things, to ignore the interdependence of our knowledge in different areas.” (662)
- “And similarly for accounts of truth and reference...A theory of truth for the language we speak, argue in, theorize in, mathematize in, etc., should by the same token provide similar truth conditions for similar sentences. (662)

Tarski’s Theory of Truth

- This is Benacerraf’s preferred theory of truth
- Considered the standard view
- Tarski
  - our natural language contains words like “true” and “false”
    - because of this, we can construct contradictions such as the one seen in the liar paradox
    - L   L is false
  - proscribes self-reference in language by positing an metalanguage above the object language

Tarski’s Theory of Truth

- So, we are looking for a theory of truth on which we can map mathematical truth
- “I take it that we have only one such account: Tarski’s, and that its essential feature is to define truth in terms of reference (or satisfaction) on the basis of a particular kind of syntactico-semantic analysis of the language, and thus that any putative analysis of mathematical truth must be an analysis of a concept which is a truth concept at least in Tarski’s sense.” (667)

Truth as a matter of reference

- "its essential feature is to define truth in terms of reference" (667)

Take the following example:

- 1 There are at least three large cities older than New York.
- 2 There are at least three perfect numbers greater than 17.

Each of the sentences takes the following form:

- 3 There are at least three FGs that bear R to *a*.

Standard Semantics & Platonism

1 There are at least three large cities older than New York.
2 There are at least three perfect numbers greater than 17.
3 There are at least three FGs that bear R to *a*.

- Standard semantics require that there are objects to satisfy these sentences
- 1 & 2 both take the same form 3 iff there are objects which substitute for the variables in 3 such that the properties hold
- This view is Platonist
- "One consequence of...the standard view is that logical relations are subject to uniform treatment: they are invariant with subject matter." (670)

Epistemological condition

- Aside from a good theory of truth, we need a good theory of mathematical knowledge that parallels our broader theory of knowledge
- Causal Theory of Knowledge (CTK)
  – Justified True belief (JTB)
  – Plus the condition that there must be a causal connection between the object of reference and the knowledge of said objects
- In order to justify knowledge of mathematical entities, they need to stand in a causal relationship with our knowledge
  – But on the standard view they are abstract entities
- Benacerraf additionally infers that theories of reference are also causal
  – "thus making the link to my saying knowingly that *S doubly* causal." (671)

The Problem

- “If, for example, numbers are the kinds of entities they are normally taken to be, then the connection between the truth conditions for the statements of number theory and any relevant events connected with the people who are supposed to have mathematical knowledge cannot be made out. It will be impossible to account for how anyone knows any properly number- theoretical propositions.” (673)
- The best account of truth posits the existence of abstract mathematical objects (standard semantics & Platonism) and at the same time such objects are causally removed and in tension with our best epistemic theory (CTK)
- Combining CTK with a Standard view of mathematical truth makes mathematical knowledge implausible.

The issue

- "The minimal requirement, then, is that a satisfactory account of mathematical truth must be consistent with the possibility that some such truths be knowable. To put it more strongly, the concept of mathematical truth, as explicated, must fit into an over-all account of knowledge in a way that makes it intelligible how we have the mathematical knowledge that we have. An acceptable semantics for mathematics must fit an acceptable epistemology" (667)

Field’s Reformulation

- Recall that Benacerraf assumed that CTK was the strongest theory of knowledge
- This doesn’t hold for Field, who dismisses CTK
- “The way to understand Benacerraf’s challenge, I think, is not as a challenge to our ability to *justify* our mathematical beliefs, but as a challenge to our ability to *explain the reliability of* these beliefs... Benacerraf’s challenge...is to provide an account of the mechanisms that explain how our beliefs about these remote entities can so well reflect the facts about them. The idea is that *if it appears in principle impossible to explain this*, then that tends to *undermine* the belief in mathematical entities, *despite* whatever reason we might have for believing in them (Field 25-6)

Reliabilism

- Field is concerned not with justification, but with reliability of knowledge
- Reliabilism: knowledge must have origin in reliable cognitive processes, principles, and methods
- Field is looking for an explanation of why our mathematical knowledge appears to adequately reflect facts about mathematics

Gödel’s Platonism

- Benacerraf recognizes that Gödel encountered the same problem
- Gödel appeals to intuition as the means that allow us to see mathematical truth
- “But, despite their remoteness from sense experience, we do have a perception also of the objects of set theory, as is seen from the fact that the axioms force themselves upon us as being true.” (674)
- What is still missing is the “link between our cognitive faculties and the objects known” (674)
  - violates the epistemological condition
- Field argues that Gödel has not demonstrated why intuition in this case is reliable and dismisses his conclusion on reliabilist grounds

The Combinatorialists

- What if we gave up Tarski?
- Combinatorial here refers to Hilbert, Formalism, Intuitionism, and conventionalism
  - rooted not in traditional mathematical objects, but in inscriptions and mental constructs.
- Anti-platonist
- But the Combinatorialists have no good standard for how terms are to be used
- They save their epistemology in this case by giving up mathematical truth

Conclusion

- The Benacerraf Problem presents troubles for both the platonist and anti-platonist
- On the one hand, the Platonist has trouble accessing mathematical entities causally
- On the other hand, the anti-Platonist has trouble reconciling truth based on good semantics
- Questions?