Jack Messerly

Formalism

Late 19th Century Mathematics

•Enormous developments and advancements in various fields of mathematics in the late 1800's

•Bolzano
•Weierstrauss
•Cauchy
•Cantor
•Dedekind
•Frege

Paradoxes

- "Thus, thanks to the Herculean collaboration of Frege, Dedekind, and Cantor, the infinite was made kind and enjoyed a reign of great triumph...But reaction was not lacking. It took in fact a very dramatic form...In the joy of discovering new and important results, mathematicians paid too little attention to the validity of their deductive methods. For, simply as a result of employing definitions and deductive methods which had become customary, contradictions began gradually to appear. These contradictions, the so-called paradoxes of set theory, thought at first scattered, became progressively more acute and more serious... Confronted by these paradoxes, Dedekind and Frege completely abandoned their point of view and retreated... Cantor's doctrine, too, was attacked on all sides. So violent was this reaction that even the most ordinary methods of mathematics were threatened and their employment was on the verge of being declared illicit" (Hilbert, 190).

Schools of Thought

- Logisicm (Frege)

- Intuitionism (Kant)

- Formalism (Hilbert)

The Infinite

•The infinite is not found in the world, so we cannot deal with it in terms of scientific theories

•But the infinite (Cantor's transfinites, for example) are very useful in mathematics.

•So how can we defend the infinite in mathematics when so many paradoxes have arisen?

"Just think, the definitions and deductive methods which everyone learns, teaches, and uses in mathematics, the paragon of truth and certitude, lead to absurdities! If mathematical thinking is defective, where are we to find truth and certitude? There is, however, a completely satisfactory way of avoiding the paradoxes without betraying our science" (Hilbert, 191).

1. Carefully investigate deductive methods. Strengthen them and make them useful.
2. Establish in mathematics deductions that are as sound as those in elementary number theory, where there are no paradoxes.

Hilbert's Formalism

- Term Formalism
  - Mathematical terms refer to symbols and characters
- Game Formalism
  - Ideal terms account for universality but are meaningless
    - Mathematics: a combination of real and ideal elements
- Deductive Formalism
  - Mathematics consists of deductions within consistent systems
    - Sound
    - Complete
      - Gödel's Incompleteness Theorems