

Knowledge, Truth, and Mathematics

Philosophy 405
Russell Marcus
Hamilton College, Spring 2014

Class #9a: Finishing Modern Empiricism

Business

- This is the last class that I will lead for a while.
 - ▶ Genesis on Mill
 - ▶ Austin on Cantor
 - ▶ Jason on Logicism
 - ▶ Jack on Formalism
 - ▶ Shaq on Intuitionism
 - (The big three)
- Today:
 - ▶ Finish the empiricists
 - ▶ Say a little about Kant's synthetic *a priori*.



The Empiricist's Problem with Mathematics

- The empiricist claims that all knowledge arises from sense experience.
- Mathematical objects are not sensible.
- It is difficult to see how sense experience can support claims about mathematical objects.
- Furthermore, many mathematical claims are universal in nature.
- But, our experience is limited, and finite.
- Again, it is difficult to see how sense experience can support universal mathematical claims.
- The Rationalist's Solutions
 - Descartes: Innate ideas
 - Leibniz: Intuitive knowledge

Two Options for the Empiricist

- The nominalist (e.g. Berkeley) denies that we have mathematical knowledge.
- Other empiricists try to account for some mathematical knowledge using only our sense experience.
 - ▶ Reclamation
 - ▶ I do not doubt but it will be easily granted that the *knowledge* we have of *mathematical truths* is not only certain, but *real knowledge*, and not the bare empty vision of vain insignificant *chimeras* of the brain. And yet, if we will consider, we shall find that it is only of our own *ideas* (Locke, *Essay*, IV.4.6, p 8).
 - ▶ All of the objects of human reason or inquiry may naturally be divided into two kinds, namely, *relations of ideas* and *matters of fact*. Of the first kind are the sciences of geometry, algebra, and arithmetic, and, in short, every affirmation which is either intuitively or demonstratively certain. *That the square of the hypotenuse is equal to the square of the two sides* is a proposition which expresses a relation between these figures. *That three times five is equal to the half of thirty* expresses a relation between these numbers. Propositions of this kind are discoverable by the mere operation of thought, without dependence on what is anywhere existent in the universe. Though there never were a circle or triangle in nature, the truths demonstrated by Euclid would forever retain their certainty and evidence (Hume, *Enquiry* IV.1, p 3).

Reclamation

- The reclamation project, has two fronts.
- Restriction: give up some of the general principles supposedly known innately.
- Reinterpretation: take mathematical claims to refer to non-mathematical objects.
 - Two sets of tools for the reclamation project.
 - Sensation, and any ideas which can be attributed to our sense experience.
 - Psychological capacities of our minds, including memory and the ability to reflect on our ideas.
 - Principle of contradiction
 - Doctrine of abstract ideas

Locke on Abstract Ideas

- We abstract the mathematical properties of objects from their specific properties: the chalk, the slight curve in one side, the location on the board.
- We ignore some properties and focus on others like triangularity.
- “Words become general by being made the signs of general ideas: and ideas become general, by separating from them the circumstances of time and place, and any other ideas that may determine them to this or that particular existence. By this way of abstraction they are made capable of representing more individuals than one; each of which having in it a conformity to that abstract idea, is (as we call it) of that sort” (Locke, *Essay* III.3.6, p 3).
- “By the same way that they come by the general name and idea of man, they easily advance to more general names and notions. For, observing that several things that differ from their idea of man, and cannot therefore be comprehended under that name, have yet certain qualities wherein they agree with man, by retaining only those qualities, and uniting them into one idea, they have again another and more general idea; to which having given a name they make a term of a more comprehensive extension: which new idea is made, not by any new addition, but only as before, by leaving out the shape, and some other properties signified by the name man, and retaining only a body, with life, sense, and spontaneous motion, comprehended under the name animal” (Locke, *Essay* III.3.8, p 3).
- This generality yields the certainty of mathematics, since mathematical claims are only about our abstract ideas, and not about the external world.

Mathematics and Morality

- Ethical ideas are, like mathematical ones, based on abstractions and also liable to certainty.
- “For certainty being but the perception of the agreement or disagreement of our *ideas*; and demonstration nothing but the perception of such agreement, by the intervention of other *ideas* or mediums, our moral *ideas*, as well as mathematical, being archetypes themselves, and so adequate and complete *ideas*; all the agreement or disagreement which we shall find in them will produce real knowledge, as well as in mathematical figures” (Locke, *Essay* IV.4.7, p 8).
- “All the discourses of the mathematicians about the squaring of a circle, conic sections, or any other part of mathematics, *do not concern* the *existence* of any of those figures, but their demonstrations, which depend on their *ideas*, are the same, whether there is any square or circle existing in the world or not. In the same manner the truth and certainty of *moral* discourses abstract from the lives of men and the existence of those virtues in the world of which they treat” (Locke, *Essay* IV.4.8, p 8).

Berkeley's Against Abstract Ideas

- Locke claimed that we have an abstract idea which stands for all triangles, whether scalene, isosceles, or equilateral.
- Berkeley claims that we have no abstract ideas.
- We can see that we have no abstract ideas by mere reflection on our mental states.
 - ▶ “If any man has the faculty of framing in his mind such an idea of a triangle as is here described, it is in vain to pretend to dispute him out of it, nor would I go about it. All I desire is that the reader would fully and certainly inform himself whether he has such an idea or not. And this, methinks, can be no hard task for anyone to perform. What is more easy than for anyone to look a little into his own thoughts, and there try whether he has, or can attain to have, an idea that shall correspond with the description that is... given [by Locke] of the general idea of a triangle, which is *neither oblique nor rectangle, equilateral, equicrural nor scalenon, but all and none of these at once?*” (Berkeley, *Principles* Introduction §13, p 466).
- This claim is the core of Berkeley's argument against abstract ideas.
 - ▶ An idea of chair it would have to apply to all chairs.
 - ▶ Some chairs are black, others are blue, or green.
 - ▶ An idea which corresponds to all of these is impossible.
 - ▶ No image will do as the idea of man, for it would have to be an image of a short man and a tall man, of a hairy man, and of a bald man.

Empiricism Without Abstract Ideas

Using Particular Terms Generally

- Berkeley claims that we use particular terms generally and so there is no need to pretending to form abstract ideas.
 - ▶ “A word becomes general by being made the sign, not of an abstract general idea, but of several particular ideas, any one of which it indifferently suggests to the mind” (*Principles* Introduction §11, AW 442a).
- Hume agrees.
 - ▶ “The image in the mind is only that of a particular object, though the application of it in our reasoning be the same as if it were universal” (Hume, *Treatise* I.1.7, p 5).
 - ▶ “A particular idea becomes general by being annexed to a general term, that is, to a term which, from a customary conjunction, has a relation to many other particular ideas and readily recalls them in the imagination” (Hume, *Treatise* I.1.7, p 6).

Berkeley Denies Mathematical (and Scientific) Knowledge

- An ability to speak generally is fundamental to mathematics and empirical science, where universal claims are ubiquitous.
- But while taking particulars to stand for other particulars avoids a commitment to abstract ideas, it may not succeed in supporting knowledge of those universal claims.
- The empiricist engaged in the reclamation project needs some account of our knowledge of mathematical objects which does not appeal to innate ideas, in the light of Descartes's master argument for nativism.
- Berkeley argues that no such account is possible.
- Since we can have no ideas of mathematical objects, we have no real mathematical knowledge, despite the security of our inferences.
 - ▶ “The theories, therefore, in arithmetic...can be supposed to have nothing at all for their object. Hence we may see how entirely the science of numbers is subordinate to practice and how jejune and trifling it becomes when considered as a matter of mere speculation” (Berkeley, *Principles* §120).

Hume Pursues Reclamation

- We can take objects to be of the same sort if they have any properties in common.
 - ▶ All (Euclidean) triangles have their angle sums in common, so they are the same sort of triangles.
 - ▶ But they do not have their side lengths in common, so they are not all scalene, etc.
- We use symbols, like numerical inscriptions.
 - ▶ One particular idea or word can lead us to think of many different ones, as when the first notes of a song give us the whole tune.
 - ▶ We can recall different component aspects of a general term, depending on the appropriate context.
 - ▶ These psychological capacities may be unexplained or inexplicable, but they are also undeniable.
 - ▶ “Nothing is more admirable than the readiness with which the imagination suggests its ideas and presents them at the very instant in which they become necessary or useful” (Hume, *Treatise* I.1.7, pp 6-7).
- Just as Hume re-interpreted ‘cause’ to be a mental phenomenon, and explains inductions to be psychological habits, he explains general terms as arising from habits of use.
 - ▶ “If ideas be particular in their nature and at the same time finite in their number, it is only by custom they can become general in their representation and contain an infinite number of other ideas under them” (Hume, *Treatise* I.1.7, p 7).

Nominalism and Conceptualism

- Locke's view seems to straddle nominalism and conceptualism.
 - ▶ “Universality does not belong to things themselves, which are all of them particular in their existence, even those words and *ideas* which in their signification are general. When therefore we quit particulars, the generals that rest are only creatures of our own making, their general nature being nothing but the capacity they are put into by the understanding of signifying or representing many particulars. For the signification they have is nothing but a relation that, by the mind of man, is added to them” (Locke, III.3.11).
- Hume's more clearly a nominalist.
 - ▶ Despite claiming that mathematical truths are legitimate relations of ideas, Hume denies that there are any mathematical objects.
- In contemporary language, we can say that Locke and Hume are sentence (or propositional) realists, while remaining object (or metaphysical) nominalists about mathematics.
 - ▶ Mathematical sentences are true.
 - ▶ They do not denote any real objects.

Rationalism and Empiricism

- Descartes and Leibniz gave us certainty about mathematics, which seemed to inform also everything else, including science.
 - That view seemed implausible, and relied on innateness.
- Locke and Berkeley tried to remove innate ideas.
 - They fell upon the rocks of abstract ideas.
 - Berkeley and Locke fail to separate mathematics as a distinct domain untouched by the skepticism which Hume shows is the inevitable consequence of empiricism.
- The relations of ideas/matters of fact distinction helps Hume avoid both the skepticism which infects Locke's account and the nihilism which affects Berkeley's account.
 - Hume still falls on the attempt to derive all ideas from sense impressions.
 - Hume argues that our knowledge of geometry depends on its use in science, and that the objects of geometry are the same as scientific objects.
 - So, he seems to be missing something about the nature of mathematics which is independent of science.
 - Also, geometry becomes impugned along with all ideas, derived from impressions.

Heirs

- In the twentieth century, the views we have seen reappear in updated forms
- Hume's view becomes formalism, sort of (Jack)
- Berkeley's view becomes fictionalism (Genesis, also Jack)
- The rationalism of Descartes and Leibniz is present in Frege's logicism (Jason)
- Locke's conceptualism (via Kant) becomes intuitionism (Shaq)