Knowledge, Truth, and Mathematics

Philosophy 405
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Class #4: Aristotle

But First...
Plato’s Epistemology

- Four different ways of believing (Republic)
- Distinction between perception and intellect (Theaetetus)
- Knowledge as recollection (Meno)

Plato’s Four Ways of Believing

Republic

- intellection or reason (forms);
- understanding (mathematics);
- belief (sensory objects);
- picture thinking or conjecture (shadows).

Plato on Perception and Intellect

Theaetetus

- Knowledge arises from making judgments.
- Judgments may be made on the basis of our perceptions.
  - Maybe otherwise, as well.
- Sensation can give us only the raw data for thought.
- There must be a mind, independent of the body, to unify and compare the input from the senses.

Knowledge in *Theaetetus*

- Socrates: Now take sound and color. Have you not, to being with, this thought which includes both at once - that they both *exist*?
- Theaetetus: I have.
- Socrates: And, further, that each of the two is *different* from the other and the *same* as itself?
- Theaetetus: Naturally.
- Socrates: And again, that both together are *two*, and each of them is *one*?
- Theaetetus: Yes.
- Socrates: And also you can ask yourself whether they are *unlike* each other or *alike*?
- Theaetetus: No doubt.
- Socrates: Then through what organ do you think all this about them both? What is common to them both cannot be apprehended either through hearing or through sight.
- ...
- Theaetetus: There is no special organ at all for [perceiving existence and nonexistence, likeness and unlikeness, sameness and difference, unity and numbers in general as applied to them, and even and odd] as there is for the others. It is clear to me that the mind in itself is its own instrument for contemplating the common terms that apply to everything (*Theaetetus* 185a-e).

Knowledge in *Meno*

Recollection

- Plato's view:
  - The slave boy, with no knowledge of mathematics, can be brought to understand a mathematical theorem by mere questioning.
  - We can see that the knowledge is inside of the slave boy, since he was only asked questions, and not taught.
- Is Socrates teaching the boy?
  - We can make statements with questions.
- A charitable reading
  - The slave boy needs to have some innate ability to recognize the mathematical truth when it is presented.
  - Does this yield innate knowledge of mathematics?

Aristotle's Criticisms of Plato

Aristotle on Plato’s Forms

- “According to the arguments from the existence of the sciences there will be Forms of all things of which there are sciences, and according to the argument that there is one attribute common to many things there will be Forms even of negations, and according to the argument that there is an object for thought even when the thing has perished, there will be Forms of perishable things; for we can have an image of these” (*Metaphysics* I.9: 990b12-14).
- There are too many forms.
  - “In seeking to grasp the causes of the things around us, the introduced others equal in number to these, as if a man who wanted to count things thought he could not do it while they were few, but tried to count them when he had added to their number” (*Metaphysics* I.9: 990b1-4).

Aristotle on Multiplicity in Mathematics

If besides the sensible solids there are to be other solids which are separate from them and prior to the sensible solids, it is plain that besides the planes [solids?] also there must be other and separate planes and points and lines; for consistency requires this. But if these exist, again besides the planes and lines and points of the mathematical solid there must be others which are separate...Again, there will be, belonging to these planes, lines, and prior to them there will have to be, by the same argument, other lines and points; and prior to these points in the prior lines there will have to be other points, though there will be no others prior to these. Now the accumulation becomes absurd... (*Metaphysics* XIII.2: 1076b12-29).

Aristotle on the Forms as Causes

- "Above all one might discuss the question what on earth the Forms contribute to sensible things, either to those that are eternal or to those that come into being and cease to be. For they cause neither movement nor any change in them...All other things cannot come from the Forms in any of the usual senses of 'from'. And to say that they are patterns and the other things share them is to use empty words and poetical metaphors. For what is it that works, looking to the Ideas? Anything can either be, or become, like another without being copied from it, so that whether Socrates exists or not a man might come to be like Socrates; and evidently this might be so even if Socrates were eternal. And there will be several patterns of the same thing, and therefore several Forms, e.g. animal and two-footed and also man himself will be Forms of man. Again, the Forms are patterns not only of sensible things, but of themselves too, e.g. the Form of genus will be a genus of Forms; therefore the same thing will be pattern and copy" (*Metaphysics* I.9: 991a9-31).
- "It must be held to be impossible that the substance and that of which it is the substance should exist apart; how, therefore, can the Ideas, being the substances of things, exist apart?" (*Metaphysics* I.9: 991b1-3).
- Note the relevance to mathematical objects, as things which exist apart.

Toward an Adjectival View of Mathematics

Criticisms of the Forms

- A2. Several patterns
- A3. Pattern and copy/third man
- A1. Empty words
  - One need not reify commonalities.
  - We can divide the world into substances and their properties.
    - People, animals, trees, and rocks are all substances.
    - Tallness and beauty are not substances, but what is said of substances.
    - The substance and its attributes must be located together.
  - Adjectival use of properties, which he will extend to mathematical objects.
    - Roundness (circularity) and twoness (from counting) are properties of primary substances, not substances themselves.
    - Mathematical terms refer to physical objects qua their shape or number.
    - Mathematics is about magnitudes, which are properties of sensible objects.

Aristotle's Own View of Mathematics

A Proto-Nominalist View

Standard Reading of Aristotle on Mathematics

shallow and incoherent

- Aristotle denies the existence of mathematical objects and the world of the forms while admitting that we have mathematical knowledge.
- “This interpretation must view Aristotle as caught in the middle of a conjuring trick: trying to offer an apparently Platonic account of mathematical knowledge while refusing to allow the objects that the knowledge is knowledge of” (Lear 161).
- My view: Aristotle’s view is incoherent, but not shallow.

Neither In The Objects Nor Separate From Them

- Aristotle argues that mathematical objects are neither in sensible objects nor separate from them.
- In other words, mathematical objects are not themselves perceivable, nor do they exist in a separate Platonic realm.
- A Neat Trick, if he can pull it off

The Argument from Division (1076a38- 1076b12)

mathematical forms do not exist in sensible things

- Points, abstract mathematical objects, are indivisible.
- If sensible bodies were made of points, then they could not be divided.
- But, bodies are divisible.
- So, they can not be made of mathematical points.
- A ball is not a sphere; a piece of paper is not a plane.

Against Separation

- Multiplying entities
- Parsimony: We would like to avoid positing a separate realm of forms and mathematical objects if possible.
  - Ockhamism

Revolutionaries and Re-Interpreters

Among anti-platonists

- Revolutionaries think that mathematical statements are false and that mathematical objects do not exist.
- Reinterpreters think that mathematical statements are true when reconstrued, and that while platonist mathematical objects do not exist, we can understand mathematical terms as shorthand for other kinds of objects.
- Aristotle is an anti-platonist, but he is a reinterpreter, not a revolutionary.

Aristotle on Re-Interpretation

Just as general propositions in mathematics are not about separate objects over and above magnitudes and numbers, but are about these, only not *as* having magnitude or being divisible, clearly it is also possible for there to be statements and proofs about perceptible magnitudes, but not *as* perceptible but as being of a certain kind (1077b18-22).

Eudoxus and Magnitudes

- Magnitudes are not just geometric lengths, as it might seem.
- They are more general; anything that can be the subject of Eudoxus's theory of proportions:
  - $(\forall x)(\forall y)(\forall z)(\forall w) \{(x : y :: z : w) \equiv$
    $(\forall v)(\forall u)[(vx > uy \supset vz > uw) \bullet (vx < uy \supset vz < uw) \bullet (vx = uy \supset vz = uw)]\}$
  - Since :: is an equivalence relation, we often replace it with '='.
  - $a{:}b{::}c{:}d \equiv ad{=}bc$
- Eudoxus developed an axiomatic treatment of proportions to handle incommensurables (irrationals).
  - E.g. the ratio of the length of a side of an isosceles right triangle to its diagonal, is incommensurable.
  - Given Eudoxus's work, the Greeks could work with incommensurable ratios without committing themselves to irrational numbers.

Magnitudes and Aristotle

- The quantifiers in Eudoxus’s theory range over magnitudes, which could be lengths, weights, volumes, areas, or times.
- They do not range over numbers, for the Greeks, though we can see that they do.
- “The generalized theory of proportion need not commit us to the existence of any special objects -magnitudes - over and above numbers and spatial magnitudes” (Lear, 167).
- We do not think there are magnitudes in addition to lengths, weights, times, and any other kind of measure to which the theory of proportions applies.
- We should not reify magnitudes.
- Mathematical objects are not substances.

Aristotle on Health

Parsimony

And it is true to say of the other sciences too, without qualification, that they deal with such and such a subject - not with what is accidental to it (e.g. not with the white, if the white thing is healthy, and the science has the healthy as its subject), but with that which is the subject of each science - with the healthy if it treats things *qua* healthy, with man if *qua* man (*Metaphysics* XIII.3: 1077b34-1078a2).

Neither in the Objects nor Separate (Again)

- There are magnitudes.
  - the subject of the theory of proportions
- There are no things that we call magnitudes.
  - just lengths, and weights, and times, and volumes of solids
- We need not think that there is a shape of the book over and above the book itself.
- There is just the book itself, considered more abstractly.
- There are mathematical objects, in the sense that the book has a shape.
- But, there are no mathematical objects separate from the sensible objects which have shapes, and other magnitudes.

Aristotle on the Soul: Matter and Form

- Every thing that we can name has matter and form.
  - The matter is, roughly, the stuff out of which it is made.
  - The form is, roughly, the shape or function of the object.
- Matter itself is mere potentiality; it is nothing in itself unless it has some form.
- Consider the difference between a lump of clay, and a similar lump made into a statue.
- We call an object by a particular name according to its form.
- Consider an eye.

Aristotle on the Soul: the Form of the Person

- All the parts of me: my heart, my lungs, my toes, have functions, and so both matter and form.
- When we put all of these pieces together, we get a person.
- We are all made out of the same kind of matter.
- But, we have different properties.
- Aristotle calls the form of a person his or her soul.
- Since the form of something is what makes it what it is, the soul includes our biological aspects, like sensation and locomotion, as well as reason.
- The soul is thus not separable from the body, though it is different from just the matter of the body.
- Aristotle is thus a monist: there is only one realm.

Mathematics and the Soul

- Just as Aristotle believes that there is a soul, but that it is just the form of the body, he believes that there are mathematical objects, as aspects of physical objects, as physical objects taken in a particular way.
- In each case, Aristotle denies that there is a separate realm.
- He is, essentially, a natural scientist about both questions.
- The soul is an aspect of a person, apart from his or her matter, but tied to his or her functions.
- Mathematical objects are just aspects of physical objects.

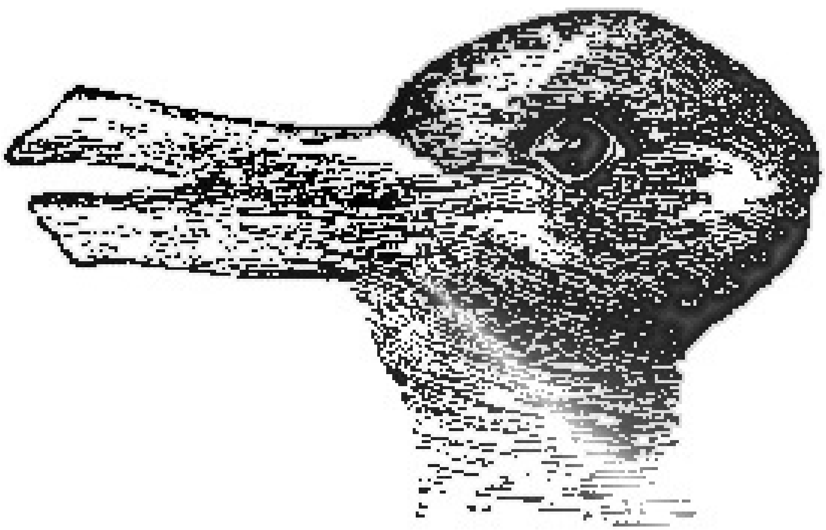
Abstraction

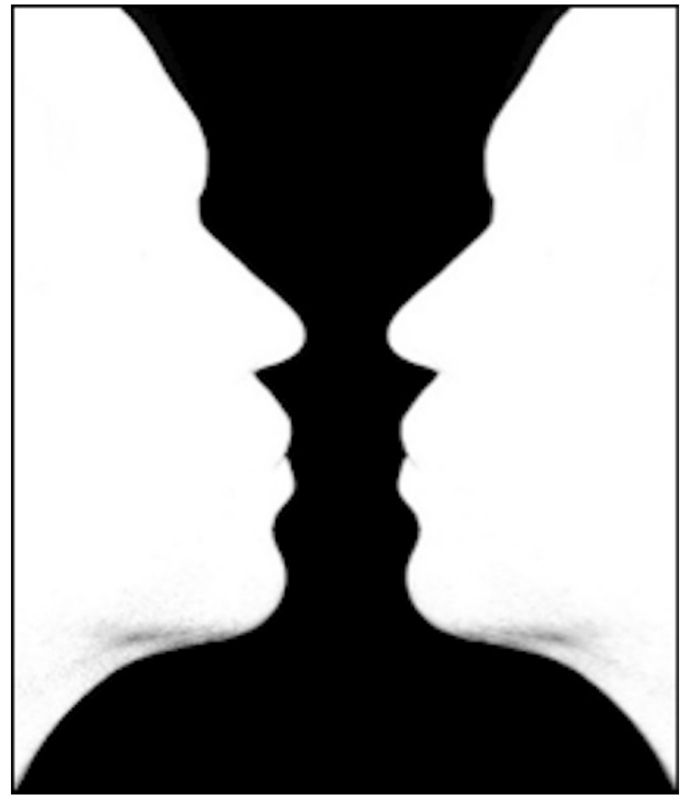
- To see sensible objects in the mathematical way, we can abstract from their sensible properties.
- The word ‘abstract’ may be used in at least two ways.
- In one way, we refer to objects outside of space-time, or outside the sensible realm, as abstract objects.
  - That is a metaphysical interpretation of ‘abstract’.
- For Aristotle, abstraction is an epistemological notion.
- Abstraction is a process we use on ordinary objects, to consider them as mathematical.
- “Of what does a demonstration hold universally? Clearly whenever after abstraction it belongs primitively - e.g. ‘two right angles’ will belong to ‘bronze isosceles triangle’, but also when being bronze and being isosceles have been abstracted” (*Posterior Analytics* I.5: 74b1).

Abstraction as Seeing-As

- The process of abstraction is a process of seeing something as some particular property it holds.
- Haman's hat (taken as a triangle) has angles that sum to 180 degrees if and only if Haman's hat is a triangle and all triangles have angles which sum to 180 degrees.
- Abstraction acts as a filter to eliminate incidental or accidental properties.

Seeing-As





Abstraction and Separation

- It is not that case Haman's hat, taken as a hat, has angles that add up to 180 degrees.
  - It is not the case that all hats have angles which sum to 180 degrees.
- So, the sensible object is not to be taken as having mathematical properties absolutely.
- The mathematical objects are not in the sensible objects.

Aristotle and Applicability

- The problem of applicability is to explain how objects in a separate realm can have any relevance to the sensible realm.
- Plato's theory of forms and mathematical objects incurs a problem of applicability.
  - How can a separate form can have any effect on or interaction with the sensible world?
  - Problem of interaction.
  - This problem led Plato to denigrate the sensible world, as a world of mere becoming, and not actual being.
- Aristotle, the natural scientist, takes a different moral from the argument.
  - We can study a perceptible triangle, like Haman's hat, as a triangle because it actually is a triangle.
  - It does not merely approach triangularity.
  - Aristotle has given us a subtle doctrine.

A Subtle Doctrine

- If [geometry's] subjects happen to be sensible, though it does not treat them *qua* sensible, the mathematical sciences will not for that reason be sciences of sensibles - nor, on the other hand, of other things separate from sensibles (*Metaphysics* XIII.3: 1078a2-4).
- It is true...to say, without qualification, that the objects of mathematics exist, and with the character ascribed to them by mathematicians...If we suppose things separated from their attributes and make any inquiry concerning them as such, we shall not for this reason be in error, any more than when one draws a line on the ground and calls it a foot long when it is not; for the error is not included in the propositions (*Metaphysics* XIII.3: 1077b31 - 1078a17).
- The best way of studying geometry is to separate the geometrical properties of objects and to posit objects that satisfy these properties alone...Though this is a fiction, it is a helpful fiction rather than a harmful one: for, at bottom, geometers are talking about existing things and properties they really have... (Lear 175).

Problems with Aristotle's Account

Platonist Intuitions

- Aristotle captures our intuitions that a separate realm can have no causal effect on the sensible world.
- But we have opposing intuitions as well.
- Mathematical statements seem to transcend the physical world.
- According to Aristotle, if matter were to disappear, there would be no more mathematics.
- The problem here is that we have opposing, inconsistent intuitions that must be resolved.
- Aristotle’s theory captures only one set of the opposing intuitions.

Physical Objects Do Not Have Mathematical Properties

- No sensible object, strictly speaking, has perfect geometric shape.
- No pizza is a perfect circle, no hat is a perfect triangle.
- The shapes that mathematicians actually study seem not to be the properties that mathematical objects actually have.
- There are not perfect examples of all the forms that the mathematician explores.
- “One unfortunate (if not damning) consequence of this account is that a natural number does not exist unless there is a collection of physical objects of that size. Similarly, a geometric object, such as a given polygon, exists only if there is a physical object that has that shape” (Shapiro 66).

Restriction

- The restrictions that Aristotle places on the existence of mathematical objects are counter-intuitive.
- Aristotle tries to avoid the problem of restriction by taking mathematical objects not to be the limits of physical bodies.
- “The mathematician, though he too treats of these [sensible] things, nevertheless does not treat of them as the limits of a natural body; nor does he consider the attributes indicated as the attributes of such bodies” (*Physics* II.2: 193b32-4).
- On the one hand, there are no separable objects.
- On the other, we take mathematical objects not to be the limits of sensible bodies, but something transcendent.
- We will return to the problem of restriction when we get to the indispensability argument.
- Aristotle’s work in mathematics sets a precedent for future empiricists in the philosophy of mathematics
  - Mill, in the 19th century
  - Quine, in the 20th century