# Knowledge, Truth, and Mathematics

Philosophy 405
Russell Marcus
Hamilton College, Spring 2014
Class #3: Plato

# Plato Summary

Knowledge must be of eternal objects. (*Phaedo, Republic*)

The world we receive in sense perception, the sensible world, is constantly changing. (*Timaeus, Phaedo*)

So, we can not have knowledge of the world we receive in sense perception. (*Theaetetus*)

Our best explanation of the changes in the sensible world involves the interactions of eternal forms. (*Phaedo, Republic*)

So, there is a sensible realm and an intelligible realm. (*Republic*)

We do not receive mathematical objects via sense perception, so they must belong to the intelligible realm. (*Republic*)

Mathematical objects undergo some sorts of changes, so they can not be perfectly eternal and unchanging forms. (*Republic*)

The intelligible realm is known via recollection. (*Phaedo, Meno*)

# Plato's Metaphysics

# The World of Being and the World of Becoming

- *Timaeus*

- The world of being is eternal and true.
  - ‣ Stable objects for knowledge
  - ‣ Apprehended by reason

- The world of becoming is created and transient.
  - ‣ What we perceive with sensation
  - ‣ In flux
  - ‣ shadows on the wall of the cave

# Plato and Pythagoras

- "The world has been framed in the likeness of that which is apprehended by reason and mind and is unchangeable..."
- The world of becoming has some constant foundation.
- For the Pythagoreans, this foundation was mathemtical.
- For Plato, it is dominated by the forms.

# Four Arguments for the Forms

- P1. The argument from the sciences
  - ‣ the objects of a science must be stable and constant

- P2. The one over many
  - ‣ forms as universals, as explanations of commonalities
  - ‣ If two things are both tall, there is some property that they share, tallness.
  - ‣ On a narrow view, only some commonalities have corresponding forms.
  - ‣ On a broad view, all commonalities lead to forms.
  - ‣ It is typical to say that early Plato accepted the broad view, but later Plato moved to a narrower view.

- P3. The object for thought
  - ‣ James Brown, Unicorns

- P4.  As Causes

# Forms, Mathematical Objects, and Universality

- Forms are the unchanging realities which compose the real world.
    - ‣ Truth, good, beauty, justice, etc.
    - ‣ Particular objects have their properties as a result of participating in forms.
- Forms seem to be both universals and particulars.
- Unlike objects in the world of becoming, the forms never participate in both of opposite forms.
    - ‣ They are uniform.
- Mathematical objects, like the forms, never participate in opposites.
    - ‣ Things are odd or even because of their participation in particular forms.
    - ‣ Three can never be even.
    - ‣ Although it is not a form itself of odd, it is not itself the opposite of even.
- So, mathematical objects are like the forms in their eternal existence and properties.

# Forms, Mathematical Objects, and Particularity

- Mathematical objects are unlike the forms in being particulars.
  - ‣ We can not make a number odd, or square, by performing an addition or other operation.
  - ‣ Mathematical objects have their own natures.

- "Suppose...that we add one to one. You would surely avoid saying that the cause of our getting two is the addition, or in the case of a divided unit, the division. You would loudly proclaim that you know of no other way in which any given object can come into being except by participation in the reality peculiar to its appropriate universal, and that in the cases which I have mentioned you recognize no other cause for the coming into being of two than participation in duality, and that whatever is to become two must participate in this, and whatever is to become one must participate in unity" (*Phaedo* 101b-c).

# Criticism of Geometers

## Book VII of the *Republic*

- "[Geometry] is in direct contradiction with the language employed in it by its adepts...Their language is most ludicrous, though they cannot help it, for they speak as if they were doing something and as if all their words were directed toward action. For all their talk is of squaring and applying and adding and the like, whereas in fact the real object of the entire study is pure knowledge" (*Republic* 527a).

- When we talk about adding, or squaring, or performing other mathematical operations, it looks as if we are changing some object.

- Mathematical objects must be stable and unchanging.

- So, the active language of mathematicians is misleading.

# Pure and Applied Mathematics

- Some mathematicians focus on processes or actions with mathematical objects.
  - ‣ engineers, and scientists
  - ‣ applications of mathematics
- Some mathematicians focus on the eternal natures of mathematical objects.
  - ‣ Pure mathematicians, examining the eternal mathematical truths
- In the *Philebus*, Plato calls the former ordinary men and the latter philosophers.
- The true philosopher, the person of highest contemplation of truth, is distinguished from the mathematician in the *Republic*.
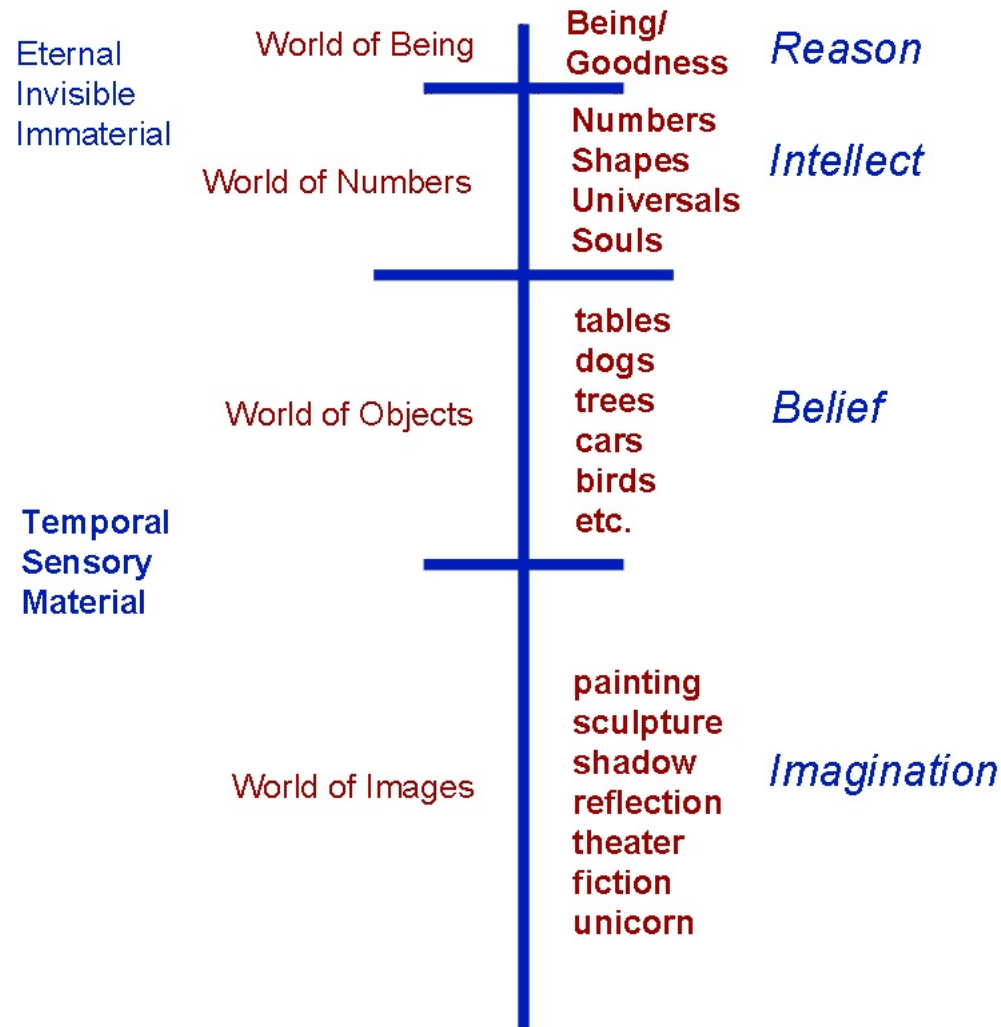
# Mathematics as Secondary to Philosophy

- Distinguish Understanding from Intellect
  - ‣ Understanding grasps mathematics.
  - ‣ Intellect is for the forms.
- Two arguments to the conclusion that mathematics is not of the highest order of reasoning.
- 1. The geometer uses diagrams, whereas the dialectician (or philosopher) uses only ideas.
  - ‣ Since diagrams are only perceivable through the senses, they are associated with the world of becoming, rather than the world of being.
- 2. The mathematician makes assumptions, but does not trace everything back to first principles.
  - ‣ The geometer has to start with assumptions, and can never get rid of them.
  - ‣ "Students of geometry and reckoning and such subjects first postulate the odd and the even and the various figures and three kinds of angles and other things akin to these in each branch of a science, regard them as known, and, treating them as absolute assumptions, do not deign to render any further account of them to themselves or others, taking it for granted that they are obvious to everybody" (*Republic*, 510 c).

# The Exalted Philosopher

- The philosopher works without indispensable assumptions.

- "By the other section of the intelligible I mean that which the reason itself lays hold of by the power of dialectic, treating its assumptions not as absolute beginnings but literally as hypotheses, underpinnings, footings, and springboards so to speak, to enable it to rise to that which requires no assumption and is the starting point of all, and after attaining to that again taking hold of the first dependencies from it, so to proceed downward to the conclusion, making no sue whatever of any object of sense but only of pure ideas moving on through ideas to ideas and ending with ideas (*Republic* 511b-c).

# The Divided Line
## A Picture of Plato's Ontology

Eternal
Invisible
Immaterial

Temporal
Sensory
Material

World of Being — Being/ Goodness — *Reason*

World of Numbers — Numbers Shapes Universals Souls — *Intellect*

World of Objects — tables dogs trees cars birds etc. — *Belief*

World of Images — painting sculpture shadow reflection theater fiction unicorn — *Imagination*

# The Divided Line

- Are mathematical objects images of the forms?

- Plato elevates arithmetic above geometry in the second selection from the *Republic*.

- Two kinds of mathematical objects?
  - ▸ ideal numbers and geometrical ideas, forms of mathematical objects
  - ▸ mathematical numbers, and geometrical figures, which would pattern themselves after the ideal numbers and geometric ideas

- The latter objects are the objects of *logistike*, which was the ancient Greek science of calculation.

- The former are closer to the forms.

# Plato's Epistemology

## *Republic*

- intellection or reason (forms);
- understanding (mathematics);
- belief (sensory objects);
- picture thinking or conjecture (shadows).

# Knowledge in *Theaetetus*

- Socrates: Now take sound and color.  Have you not, to being with, this thought which includes both at once - that they both *exist*?

- Theaetetus: I have.

- Socrates: And, further, that each of the two is *different* from the other and the *same* as itself?

- Theaetetus: Naturally.

- Socrates: And again, that both together are *two*, and each of them is *one*?

- Theaetetus: Yes.

- Socrates: And also you can ask yourself whether they are *unlike* each other or *alike*?

- Theaetetus: No doubt.

- Socrates: Then through what organ do you think all this about them both?  What is common to them both cannot be apprehended either through hearing or through sight.

- ...

- Theaetetus: There is no special organ at all for [perceiving existence and nonexistence, likeness and unlikeness, sameness and difference, unity and numbers in general as applied to them, and even and odd] as there is for the others.  It is clear to me that the mind in itself is its own instrument for contemplating the common terms that apply to everything (*Theaetetus* 185a-e).

# Plato on Perception and Intellect

## *Theaetetus*

- Knowledge is making judgments.
- Judgments may be made on the basis of our perceptions.
  - ‣ Maybe otherwise, as well.
- Sensation can give us only the raw data for thought.
- There must be a mind, independent of the body, to unify and compare the input from the senses.

# Knowledge in *Meno*

## Recollection

- Plato's view:
  - The slave boy, with no knowledge of mathematics, can be brought to understand a mathematical theorem by mere questioning.
  - We can see that the knowledge is inside of the slave boy, since he was only asked questions, and not taught.

- Is Socrates teaching the boy?
  - We can make statements with questions.

- A charitable reading
  - The slave boy needs to have some innate ability to recognize the mathematical truth when it is presented.
  - Does this yield innate knowledge of mathematics?

# Aristotle on Plato's Forms

■ "According to the arguments from the existence of the sciences there will be Forms of all things of which there are sciences, and according to the argument that there is one attribute common to many things there will be Forms even of negations, and according to the argument that there is an object for thought even when the thing has perished, there will be Forms of perishable things; for we can have an image of these" (*Metaphysics* I.9: 990b12-14).

■ There are too many forms.

  ▸ "In seeking to grasp the causes of the things around us, the introduced others equal in number to these, as if a man who wanted to count things thought he could not do it while they were few, but tried to count them when he had added to their number" (*Metaphysics* I.9: 990b1-4).

# Aristotle on Multiplicity in Mathematics

If besides the sensible solids there are to be other solids which are separate from them and prior to the sensible solids, it is plain that besides the planes [solids?] also there must be other and separate planes and points and lines; for consistency requires this.  But if these exist, again besides the planes and lines and points of the mathematical solid there must be others which are separate...Again, there will be, belonging to these planes, lines, and prior to them there will have to be, by the same argument, other lines and points; and prior to these points in the prior lines there will have to be other points, though there will be no others prior to these.  Now the accumulation becomes absurd... (*Metaphysics* XIII.2: 1076b12-29).

# Aristotle on the Forms as Causes

- Above all one might discuss the question what on earth the Forms contribute to sensible things, either to those that are eternal or to those that come into being and cease to be.  For they cause neither movement nor any change in them...All other things cannot come from the Forms in any of the usual senses of 'from'.  And to say that they are patterns and the other things share them is to use empty words and poetical metaphors.  For what is it that works, looking to the Ideas?  Anything can either be, or become, like another without being copied from it, so that whether Socrates exists or not a man might come to be like Socrates; and evidently this might be so even if Socrates were eternal.  And there will be several patterns of the same thing, and therefore several Forms, e.g. animal and two-footed and also man himself will be Forms of man.  Again, the Forms are patterns not only of sensible things, but of themselves too, e.g. the Form of genus will be a genus of Forms; therefore the same thing will be pattern and copy (*Metaphysics* I.9: 991a9-31).

- It must be held to be impossible that the substance and that of which it is the substance should exist apart; how, therefore, can the Ideas, being the substances of things, exist apart? (*Metaphysics* I.9: 991b1-3).

- Note the relevance to mathematical objects, as things which exist apart.

# Toward an Adjectival View of Mathematics

## From More Criticisms of the Forms

A2. Several patterns

A3. Pattern and copy/third man

A1. Empty words
- ▸ One need not reify commonalities.
- ▸ We can divide the world into substances and their properties.
  - – People, animals, trees, and rocks are all substances.
  - – Tallness and beauty are not substances, but what is said of substances.
  - – The substance and its attributes must be located together.
- ▸ Adjectival use of properties, which he will extend to mathematical objects.
  - – Roundness (circularity) and twoness (from counting) are properties of primary substances, not substances themselves.
  - – Mathematical terms refer to physical objects qua their shape or number.
  - – Mathematics is about magnitudes, which are properties of sensible objects.