Autonomy Platonism

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Knowledge, Truth and Mathematics

Final Projects

- Drafts to everyone today, now.
- First critics must send their summaries and criticisms/questions (500-1000 words) to the author of the paper, its second critic, and me by **Sunday May 11**.
- Second critics should send their further comments and questions (which could be a bit shorter) to the authors of the first paper, its first critic, and me by noon on **Tuesday, May 13**.
- Our Evening: Wednesday, May 14, 7pm. Here?
- Assignments of Critics to Authors
  - Austin, "Quine and the Problem of Evaluation"
    - First Critic: Jack
    - Second Critic: Genesis
  - Genesis, "A Progressive Shift Towards Human Complexity"
    - First Critic: Jason
    - Second Critic: Jack
  - Jack, "A Look at The Interpretations of Quine's Response to Carnap's Logical Empiricism"
    - First Critic: Shaq
    - Second Critic: Jason
  - Jason, "The Benacerraf Dilemma and Hartry Field's Reformulation: Can we reduce Field's reformulation to Benacerraf's access problem?"
    - First Critic: Austin
    - Second Critic: Shaq
  - Shaq, "The Radical Empiricist, the Neo-Intuitionist and the Logicist"
    - First Critic: Genesis
    - Second Critic: Austin

Toward Autonomy Platonism and the Indispensability Argument

- Two Autonomy Platonisms
  - Balaguer’s FBP (Wrap-up)
  - My intuition-based autonomy platonism (IBAP?)

Plenitudinous Platonism

- Every consistent set of mathematical axioms truly describes a universe of mathematical objects
- Solves Benacerraf's dilemma by demanding only knowledge of consistency for access.

Two Objections to FBP

1. Too many objects in too many true theories
   - We seek answers to open questions
2. The denial of necessity
   - We don't know how we could determine whether our world has abstracta.
   - The hypothesis is useless.
   - "It just doesn't seem to me that there is any interesting sense in which 'There exists an empty set' is necessary but 'There exists a purple hula hoop' is not" (Balaguer 1998: 44-45).

On Purple Hula Hoops and Sets

- That there is a purple hula hoop depends on facts about the world over which we have some control.
  - We have interactions in events that result in the creation of hula hoops.
  - We can put together a plan for the eradication of hula hoops.
  - We can explain what contingent facts are contingent on.
- We can't say anything about what differences could yield the existence or non-existence of mathematical objects.
  - Nothing we do or could do has any effects on the existence or non-existence of mathematical objects.
- Mathematical objects exist in all possible worlds.
  - True mathematical claims are true in all possible worlds.
  - False ones are false in all possible worlds.

The Utility of Necessity

- FBP says that every consistent mathematical theory truly describes a mathematical universe.
- This is very close to saying that the theorems of mathematics, when true, are necessarily true, and that mathematical objects exist necessarily.
- Balaguer wants to leave open the possibility of worlds without mathematical objects.
- But the necessity of mathematics could help the FBPist explain why consistency entails truth.
- Also, one might merely wish to account for the commonsense belief that there is a difference between what might have been different and what could not have been different.

Explaining Necessity?

- Balaguer says that the necessitarian needs an explanation.
- It is difficult to see what kind of explanation for the necessity of mathematical claims one could provide.
- Causal explanations are out.
- Abstract objects are not governed by the laws of physics and so can have no explanations of the sort we provide for contingent non-existence claims.
- Purely mathematical explanations generally yield conditional claims: on the basis of certain axioms or assumptions, certain other theorems follow.
- We can explain why it is impossible for there to be a set such that its power set is the same size as itself, for example, only on the assumption of the existence of sets.
- It seems as if this is just the kind of question that doesn't have a good answer,.
  - 'Why is there something rather than nothing?'

From FBP to Intuition-Based Autonomy Platonism

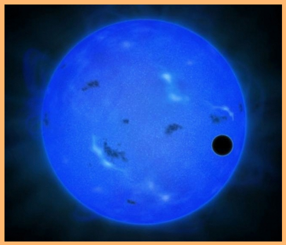
- FBP attempts to account for mathematical knowledge on the basis of merely our pre-theoretic apprehension of consistency.
- The autonomy platonist who wishes to explain our focus on the standard model will appeal to more contentious epistemic capacities.
  - mathematical intuition
- Some philosophers are skeptical of the prospects for such a view.
  - “One *might* adopt the ontological position that there are multiple ‘universes of sets’ and hold that nevertheless we have somehow mentally singled out one such universe of sets, even though anything we say that is true of it will be true of many others as well. But since it is totally obscure how we could have mentally singled out one such universe, I take it that this is not an option any plenitudinous platonist would want to pursue” (Field 1998b: 335.)

Platonism and Intuition

- Platonism in the philosophy of mathematics may be expressed either as a semantic thesis (PS) or as a related ontological thesis (PO).
  - PS Some existential mathematical sentences are true and others are false. Universal and conditional mathematical claims may be non-vacuously true or false.
  - PO There are abstract mathematical objects, possibly including (but not limited to) sets, numbers, and spaces.
- The traditional platonist view faces epistemological challenges.
  - Critics of PO complain that it is impossible to justify knowledge of mathematical objects because of their isolation from us.
  - Critics of PS complain that it is impossible for us to know any mathematical propositions since whatever makes them true or false is inaccessible to us.
- Responses
  - rejecting PS and PO
  - re-interpreting mathematical claims to refer to non-mathematical things like concrete objects
  - defending an indirect account of our knowledge of mathematics via our knowledge of scientific theories or explanations
- Or we can appeal to a special faculty for perceiving mathematical objects or apprehending mathematical claims called mathematical intuition.
- The central problem with mathematical intuition is that it seems to be a mysterious or spooky faculty, inconsistent with the best accounts of ourselves.

Intuition

- A belief-forming process which may yield belief in propositions.
- Intuitions are immediate inclinations to belief.
- The ‘aha’ feeling
- They may take as their subjects concepts and objects, including modal properties, which are unavailable to sense experience.
- We can intuit
  - That the square of two is four
  - That the locus of all points equidistant from a given point is a circle
  - That one doesn’t know that that is a barn
  - That what they have on Twin Earth is not water



Axioms and Theorems

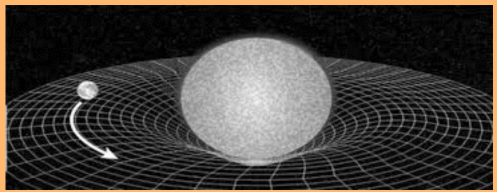
- We can justify mathematical beliefs in part by appeals to the derivability of theorems from axioms.
- We also need an account of our knowledge of the axioms.
- Simple appeals to the immediacy and obviousness of the axioms are unsatisfying and often misleading.
  - Some axioms are neither immediate nor obvious.
  - Mathematical theories are variously axiomatizable.
  - Equivalent axiomatizations have distinct virtues.
- Our most secure mathematical beliefs may not be our best axioms.
- We need some account of our beliefs in some mathematical propositions if we are going to use inferential tools to derive others.

Thick Mathematical Intuition

- Descartes
- Kant and then mathematical intuitionists of the early twentieth century
  - We develop mathematics by reflecting on our pure forms of intuition.
    - Mathematical concepts are constructed in intuition.
    - space (for geometry)
    - and time (for arithmetic, which also requires spatial intuition)
  - “Let the geometrician take up [the questions of what relation the sum of a triangle’s angles bears to a right angle]. He at once begins by constructing a triangle. Since he knows that the sum of two right angles is exactly equal to the sum of all the adjacent angles which can be constructed from a single point on a straight line, he prolongs one side of his triangle and obtains two adjacent angles, which together are equal to two right angles. He then divides the external angle by drawing a line parallel to the opposite side of the triangle, and observes that he has thus obtained an external adjacent angle which is equal to an internal angle - and so on. In this fashion, though a chain of inference guided throughout by intuition, he arrives at a fully evident and universally valid solution of the problem “(*Critique* A716-7/B744-5).

Platonism and Kantian Intuition

- Kantian invocations of intuition are in the service of a conceptualist account of mathematics.
  - Mathematical propositions are constructed in thought.
  - Mathematical objects are mental objects.
- Such a view is best classified as reinterpretive, taking apparent references to mathematical objects really to be references to mental acts.
- Kant's use of 'intuition' in mathematics is thus not compatible with PO, which takes the objects of mathematical claims to be abstract.
- It follows from Kant's view that any mathematical intuition is secure.
  - thickness
  - Indefeasible beliefs about the Euclidean structure of space



Gödel

- Mathematical intuition of objects, on analogy with perception
  - "[D]espite their remoteness from sense experience, we do have something like a perception also of the objects of set theory, as is seen from the fact that the axioms force themselves upon us as being true. I don't see any reason why we should have less confidence in this kind of perception, i.e. in mathematical intuition, than in sense perception, which induces us to build up physical theories and to expect that future sense perceptions will agree with them, and, moreover, to believe that a question not decidable now has meaning and may be decided in the future" (Gödel 1964: 268).
- Incompatible with standard constraints on epistemology.
- We have no extra-sensory organs for seeing mathematical objects.
- Too thick

Katz

- “In the formal sciences, it is common to refer to seeing that something is the case as “intuition” and to take such immediate apprehension as a source of basic mathematical knowledge... The notion of intuition that is relevant to our rationalist epistemology is that of an immediate, i.e. noninferential, purely rational apprehension of the structure of an abstract object, that is, an apprehension that involves absolutely no connection to anything concrete” (Katz 1998: 43-44).
- Replaces Gödel’s appeal to “something like a perception” with apprehension.
- Mathematical intuition is contiguous with mathematical reasoning generally.
- Intuition is our capacity to grasp mathematical truths too basic to be derived.
  - “We can say that reason is rationality in application to deductive structures and intuition is the same faculty in application to elements of such structures. We can think of intuition as reason in the structurally degenerate case” (Katz 1990: 381).
- Maybe too thick too?

Philosophical Intuition

- Sosa: "At t, it is intuitive to S that p iff (a) if at t S were merely to understand fully enough the proposition that p (absent relevant perception, introspection, and reasoning), then S would believe that p; (b) at t, S does understand the proposition that p; and (c) the proposition that p is abstract" (Sosa 1998: 259).
- Bealer: "We do not mean a magical power or inner voice or special glow or any other mysterious quality. When you have an intuition that A, it *seems* to you that A... a genuine kind of conscious episode" (Bealer 1998: 207).
- Cohen: "The term "intuition" here is not being used in the sense of Spinoza, Bergson, or Husserl. It does not describe a cognitive act that is somehow superior to sensory perception. Nor, on the other hand, does it refer merely to hunches that are subsequently checkable by sensory perception or by calculation. Nor does this kind of intuition entail introspection, since it may just be implicit in a spoken judgment. Its closest analogue is an intuition of grammatical well-formedness. In short, an intuition that *p* is here just an immediate and untutored inclination, without evidence or inference, to judge that *p*" (Cohen 1981: 318).

What Intuition is Not, Part I

From Bealer

- Belief
  - The set-theoretic comprehension axiom may seem true (i.e. we may have an intuition that it is true) even though we have over-riding beliefs that show it to be false..
- Commonsense opinions or judgments
  - category error
  - They may lead to beliefs or opinions, but they are not themselves beliefs.
  - They are cognitive experiences.
- Spontaneous inclinations to belief
  - Inclinations to believe are not episodic in the way in which intuitions, which have phenomenal content, are.
- The raising to consciousness of nonconscious background beliefs
  - I have many more of them than I have intuitions or even possible intuitions.
  - We often have intuitions which lead us to utterly new beliefs, as when I follow a proof of a new theorem.
  - "If I am to have an intuition about numbers, then above and beyond a mere inclination, something else must happen - a sui generis cognitive episode must occur. Inclinations to believe are simply not episodic in this way" (Bealer 1998: 209).

What Intuition is Not, Part II

- Guesses and hunches
  - different phenomenal character
  - We give up our guesses and hunches when presented with contrary data.
  - If it seems to me that there are five coins in your pocket, and you pull out six, I can still hold that it seemed to me that there were five even though that seeming was not accurate.
  - Intuitions are linked to seeming, not to guessing.
- Merely linguistic intuitions
  - Closer
  - But not all rational intuitions are linguistic ones.
  - Propositions like ‘if P then not-not-P’, hold for any language.

The Phenomenology of Intuition

- Mathematical intuition is a seeming.
- It can accompany some mathematical beliefs, but does not accompany more complex ones.
- "It does not *seem* to me that 252=625; this is something I learned from calculations or a table. Note how this differs, phenomenologically, from what happens when one has an intuition. After a moment's reflection on the question, it just *seems* to you that, if P or Q, then it is not the case that both not P and not Q. Likewise, upon considering [the Gettier-style case of mistaking poodles for sheep] it just *seems* to you that the person in the example would not know that there is a sheep in the pasture. Nothing comparable happens in the case of the proposition that 252=625" (Bealer 1998: 210-1).

Sense Experience and Belief

- When I have a sense experience, I might form a belief based on that experience.
- On seeing an apple in my hand I may form a belief that there is an apple in my hand.
- I might not form that belief.
- I might for example be considering whether I am in a dream state.

Intuition and Belief

- When I have an intuition of a basic mathematical fact, I might form a belief based on that intuition.
- I might have an intuition that seven and five equal twelve.
- That intuition will ordinarily lead to or confirm my belief that seven and five are twelve.
- In skeptical cases, I might withhold my belief; I might wonder whether arithmetical claims are merely fictional.

Thin Mathematical Intuition

- I call mathematical intuition thin because of the gap between the intuition and the beliefs which may be formed on its basis.
  - First I have an intuition.
  - Then I think about what it shows.
- A mathematical intuition can lead us to recognize a mathematical proposition as true or false.
  - 7+5=12
  - unrestricted axiom of comprehension
- Intuition mediates our recognition of the modal character of the proposition.
  - We have an intuition not only that a sum holds, say, but that it must.
  - This recognition contributes to the phenomenal content of an intuition.
  - It feels as if the content of a particular intuition has modal character.
  - I recognize that an apple could have been green and may not be ripe.
  - The sum of seven and five can only be twelve.
- The modal character of my mathematical beliefs arises directly from the nature of the objects of those beliefs.
  - independent of sense experience
  - immutable

Apriority

- We can see that mathematical intuition is an *a priori* method of belief formation by considering the content and modal character of the beliefs we acquire by intuition.
  - distinct from from the processes of our acquisition and justification of beliefs about ordinary objects
- '*a priori*' does not entail that the content of the beliefs acquired by intuition are free from error.
  - necessarily true, if true
  - necessarily false, if false
- We are lamentably constructed so that we sometimes take false claims for true ones.
- There are non-*a priori* methods of mathematical belief acquisition, too.
  - Reading about the proof of a theorem (testimony)
  - Looking at lists of mathematical facts, like multiplication tables

Mysticism?

- Appeals to mathematical intuition are widely derided, often with concern about mystical capacities of perception.
  - “Someone *could* try to explain the reliability of these initially plausible mathematical judgments by saying that we have a special faculty of mathematical intuition that allows us direct access to the mathematical realm. I take it though that this is a desperate move...” (Field 1989: 28).
- Insofar as we think of intuition as an extra-sensory perception, such a move is indeed desperate.
  - We have no such mystical ability.
- Fortunately the notion of intuition on which I am relying is not like those.
  - E.g. I do not presume that mathematical intuition is direct access or infallible.
- Still, the question at which Field hints, whether and to what degree our mathematical intuitions are reliable, is important.

Is Thin Intuition Reliable?

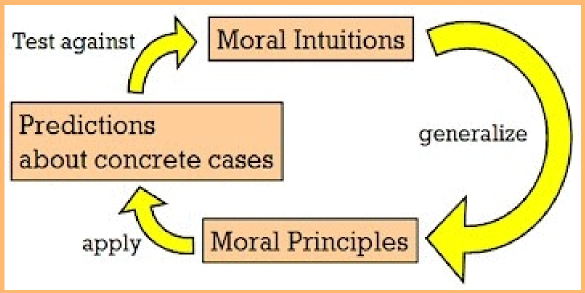
- Some intuitions are more reliable than others.
  - Some sense experiences are more clear than others.
  - We can discover that some intuitive claims that we take to be true are wrong.
  - Gödel and the continuum hypothesis
- It is conceivable that our mathematical intuitions could turn out to be highly unreliable.
- Since our theories of mathematics are constructed to account for those intuitions, it is doubtful that we could discover this fact.
- This is a skeptical worry, like the impossibility of proving the consistency of our mathematical theories.
- Let's look at the broader view more carefully now.

From Mathematical Intuition to Mathematical Theory

- The intuition-based epistemology that I am proposing is simple and unsurprising.
- Start with a thin, fallibilist version of mathematical intuition.
- Intuition yields some mathematical beliefs.
- But we need some method to improve our beliefs.
  - to systematize
  - to check
- Two components of the process
  - An apprehension of some elementary mathematical truths
  - An apprehension of the connection of those truths with other truths
    - axiomatic systems
    - some logical apparatus
    - We compare different systematizations for their mathematical and more-general theoretical virtues.
- The processes of refining and extending the mathematical beliefs generated by intuition are just the natural and well-refined methods of mathematics.

The Method of Seeking Reflective Equilibrium in Mathematics

- We balance our intuitive apprehension of basic mathematical facts with our evaluations of the systematizations of our mathematical knowledge.
- The intuitions are constraints on the system-building.
- The systems are constraints on the intuitions.
  - “When pure mathematics is organized as a deductive system - i.e. as the set of all those propositions that can be deduced from an assigned set of premises - it becomes obvious that, if we are to believe in the truth of pure mathematics, it cannot be solely because we believe in the truth of the set of premises. Some of the premises are much less obvious that some of their consequences and are believed chiefly because of their consequences. This will be found to be always the case when a science is arranged as a deductive system. It is not the logically simplest propositions of the system that are the most obvious, or that provide the chief part of our reasons for believing in the system” (Russell 1924: 325).



Relevant Tools

- Two main cognitive tools:
  - 1. Our ability to recognize consistency
  - 2. Our mathematical intuition
- The former guides the mathematician categorically.
  - rigorous formal systems
  - FBP
- The latter guides the mathematician (fallibly) where the former is silent.

The Fallibility of Intuition

Two ways in which intuition may steer us wrong.

1. We might give up some basic claims which appear intuitive.
   - Unrestricted comprehension axiom
   - Euclid's parallel postulate
   - Leibniz's work with infinitesimals and Newton's work with fluxions seemed intuitively false to many mathematicians.
     - How could the sum of an infinite number of infinitely small quantities result in a finite quantity?
2. We might find that certain systematizations better organize mathematical phenomena than others.
   - Our intuitive judgments about forrmal systems may vary and err.
   - Mere apprehension of consistency can not guide our choices among provably equivalent models or axiomatizations.
   - Arithmetic with Peano axioms, by those axioms modeled within set theory, by those axioms modeled within category theory, or within second-order logic?
   - Benecerraf WNCNB

The Axiom of Choice

- In some formulations, the axiom seems obviously and intuitively true.
  - For any set of sets, there is a choice set, one which consists of exactly one member of each of the sets.
- Other formulations are far less intuitive.
  - Equivalent to the well-ordering theorem (with ZF in background): every set can be well-ordered
- On pain of consistency, the intuitions can not all be correct.
- Moreover both the axiom of choice and its denial are consistent with the axioms of ZF, if ZF is consistent.
- In such cases, our intuitions are of limited direct use.

When Intuitions Fail

- We look at the ways in which it facilitates inferences and at what theorems can be proved with it and without it.
- We evaluate both the broader systems and the further theorems by their intuitiveness, balancing a range of factors.
  - strength
  - elegance
- Whether or not we should believe the axiom of choice will thus be guided by our intuitions about both axiomatizations of set theory and particular further theorems.
- Some of our intuitions are misleading.
- We seek reflective equilibrium between our particular intuitions and our systematizations, again guided by consistency and our intuitions about theoretical virtues.
- Standard mathematical practice.

Naturalism and Intuition

- Intuition seems to some folks to be a mysterious psychic ability.
  - "The naturalism driving contemporary epistemology and cognitive psychology demands that we not settle for an account of mathematical knowledge based on processes, such as a priori intuition, that do not seem to be capable of scientific investigation or explanation" (Resnik 1997: 3-4.)
- Naturalism: Everyone wants to avoid both mysticism and an unsatisfying empiricism.
- On the empiricism side:
  - Failures of logicism and logical empiricism
  - No one wants a pure Millian account of mathematics.
- On the mysticism side:
  - Plato, Descartes, and Gödel.
  - "The trouble with [Gödel's] Platonism is that it seems flatly incompatible with the simple fact that we think with our brains, and not with immaterial souls. Gödel would reject this 'simple fact', as I just described it, as a mere naturalistic prejudice on my part; but this seems to me to be rank medievalism on *his* part" (Putnam 1994: 503.)
- If naturalism is to do any work at all, it must forestall an autonomous mathematical epistemology as long as autonomy platonism is seen as entailing mysticism.
- Also, every one wants to avoid Kantian psychologism, which is like mysticism in positing substantial mental structures without empirical evidence.
- But neither motivation weighs at all against an account like the one I am presenting.

Can Intuition be Investigated and Explained?

- Resnik's claim that *a priori* intuition is incapable of scientific investigation reveals a lack of scientific ingenuity.
- We can ask people about their intuitions, judge their consistency and reliability.
- We can use standard neuroscientific tools like fMRIs to see what their brains are doing when they have them.
- Mathematical intuition must be compatible with a mature psychology.
- But an epistemology which includes intuition is plausible.
- We don't need a dedicated brain structure for mathematical perception.
  - "We cannot envisage *any* kind of neural process that could even correspond to the 'perception of a mathematical object'" (Putnam 1994: 503).
  - Appeals to intuition are, "[U]nhelpful as epistemology and unpersuasive as science. What neural process, after all, can be described as the perception of a mathematical object? Why of one mathematical object rather than another?" (Putnam 1980: 10).
  - The claim that there are neural processes which provide mathematical perception would be part of an empirical account of mathematics, not an apriorist one.

Challenges to Intuition-Based Autonomy Platonism

Reliability

- To show that our intuitions are generally reliable, I need to show that our best mathematical theories, the results of the process of attempting to achieve equilibrium between our intuitions and our theories, are reliable.
- It is a brute fact about our abilities to analyze concepts *a priori* that we often get such analyses right.
  - We consider concepts, look at their inferential relations, and derive further concepts from the ones we hold.
  - We recognize consistency and can distinguish interesting mathematical questions from more pedestrian or dull ones.
  - Our ability to systematize, model, and connect different mathematical theories provides a constraint on our reasoning.
- We all not are born with good and reliable mathematical intuitions.
  - We require training to hone our skills.
  - Similarly, our early perceptual and small motor skill are lousy at first; we mature.
- From a phenomenological point of view, at least some of us have experiences which can be called intuitions about mathematics.
  - immediate, non-inferential graspings of mathematical concepts and relations
- Those experiences presumably have some neural correlates and so are not un-natural or spooky.
- The question is whether they can play a legitimate role in mathematical epistemology.

Challenges to Intuition-Based Autonomy Platonism (IBAP?)

Circularity

- I may have done justice to the ways in which mathematics is practiced.
- Still, one may wonder whether I have actually justified any beliefs.
- The method of reflective equilibrium on which my account is founded seems liable to be untethered to mathematical truth.
  - We justify the beliefs based on our mathematical intuitions by appealing to the constructions of mathematical theories.
  - We justify the mathematical theories at least partly by their consistency with our intuitions.
- This method threatens to undermine the justificatory role that an epistemology for mathematics is supposed to provide.
- I believe that the circularity is not problematic and does not debar my account from justifying our mathematical beliefs.
- But that argument will have to go elsewhere.

The End