Plenitudinous Platonism (FBP)

Russell Marcus
Hamilton College

Knowledge, Truth and Mathematics

But First...

...let’s wrap up our discussion of the explanatory indispensability argument

Two Principles

Ontological Sincerity: We are committed to mathematical objects not by our casual uses of numbers, but only when we are speaking most seriously.

Explanatory Diversity: The theory we use to specify our ontological commitments may not be most useful when we want to explain facts about the world.

- When we explain in the metaphysical sense, we are ontologically serious.
- When we explain in the epistemic sense, we may not be most sincere.

§3: The explanatory indispensability argument (EI) (Baker, Colyvan)

EI1. There are genuinely mathematical explanations of empirical phenomena.

EI2. We ought to be committed to the theoretical posits postulated by such explanations.

EIC. We ought to be committed to the entities postulated by the mathematics in question.

- Colyvan’s cases
- Mancosu’s cases
- Baker’s cicadas
- Bangu’s bananas

EI Must Rely on an Epistemic Sense of 'Explanation'

- Standard (metaphysical) accounts of scientific explanation do not comfortably apply to mathematical explanation.
  - There are too many inferences available in mathematics.
  - Mathematicians distinguish between explanatory and non-explanatory proofs
- Conserving explanatory power is a standard requirement on nominalist reformulations.
  - Field's representation theorems
  - Conservativeness
- One could not successfully nominalize a scientific theory with less explanatory power, unless one is using a non-metaphysical sense of 'explanation'.

Why the Sense of ‘Explanation’ Matters

QI1. We should believe the theory which best accounts for our sense experience.
QI2. If we believe a theory, we must believe in its ontological commitments.
QI3. The ontological commitments of any theory are the objects over which that theory first-order quantifies.
QI4. The theory which best accounts for our sense experience first-order quantifies over mathematical objects.
QIC. We should believe that mathematical objects exist.

EI1. There are genuinely mathematical explanations of empirical phenomena.
EI2. We ought to be committed to the theoretical posits postulated by such explanations.
EIC. We ought to be committed to the entities postulated by the mathematics in question.

- The unavailability of a dispensabilist reformulation of a standard scientific theory is *essential* to QI.
- The availability of dispensabilist reformulations of theories are *irrelevant* to EI1.
  - A reformulation may well lose epistemic explanatory strength.

Evaluating EI

EI1. There are genuinely mathematical explanations of empirical phenomena.
EI2. We ought to be committed to the theoretical posits postulated by such explanations.
EIC. We ought to be committed to the entities postulated by the mathematics in question.

- The examples from Colyvan and Baker are sufficient for EI1.
- EI2 is the key premise.
- Once we realize that the sense of 'explanation' in question is epistemic, any force that EI2 is supposed to have is lost.
  - We have no reason to take the various examples invoked by the proponents of EI as expressing our ontological commitments.
  - Explanations which facilitate our subjective understanding may not be ones in which we reveal our ontological commitments by speaking most soberly.
- EI seems plausible, if we have a metaphysical sense of 'explanation' in mind.
  - But then it's no improvement on QI.

Toward Autonomy Platonism and the Indispensability Argument

- Benacerraf's Problem
- Quine's response
  - Responses
  - Tweaks
- Indispensabilist: Double-Talk is infelicitous at best
- Weasel/Eleatic: Invocations of mathematics are not justificatory
- There are other problems with indispensabilism.

The Essential Characteristics of Indispensability Arguments

- *EC.1: Evidentiary Naturalism*: The job of the philosopher, as of the scientist, is exclusively to understand our sensible experience of the physical world.
- *EC.2: Theory Construction*: In order to explain our sensible experience we construct a theory, or theories, of the physical world. We find our commitments exclusively in our best theory, or theories.
- *EC.3: Mathematization*: Some mathematical objects are ineliminable from our best theories.
- *EC.4: Subordination of Practice*: Mathematical practice depends for its legitimacy on empirical scientific practice.

The Unfortunate Consequences

- *UC1* *Restriction*: The indispensabilist's commitments are to only those mathematical objects required by empirical science
- UC2 Ontic Blur: The indispensabilist's mathematical objects are concrete.
- UC3 Causality: The indispensabilist's mathematical objects may have causal powers.
- UC4 Modal Uniformity: The indispensabilist's mathematical objects do not exist necessarily.
- UC5 Temporality: The indispensabilist's mathematical objects exist in time.
- UC6 Aposteriority: The Indispensabilist's mathematical objects are known a posteriori.
- UC7 Methodological Subservience: Any debate over the existence of a mathematical object will be resolved, for the indispensabilist, by the needs of empirical theory.

Toward Autonomy Platonism and the Indispensability Argument

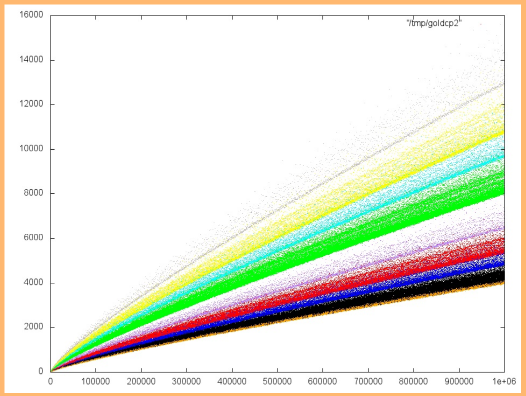
- Indispensabilist: Double-Talk is infelicitous at best
- Weasel/Eleatic: Invocations of mathematics are not justificatory
- If we had autonomous reasons to believe in mathematical claims, then we could avoid double talk and the indispensabilists's awkward inference, as well as the unfortunate consequences.
- Two Autonomy Platonisms
  - Balaguer's FBP (Today)
  - My intuition-based autonomy platonism (Thursday)

Plenitudinous Platonism

Every consistent set of mathematical axioms truly describes a universe of mathematical objects.

Open Questions

- Certain mathematical questions have no determined answer.
- Some clearly to have a correct answer.
  - Goldbach’s conjecture
- Other mathematical questions elude consensus.
  - Axiom of projective determinacy: All projective subsets of the set of functions from ω (the set of all natural numbers) to ω are determined.
    - Projective determinacy follows from some large cardinal axioms.
    - If the axiom is true, then many open questions about projective sets left undecided by ZFC are settled..
  - The elegance and strength of its consequences motivate some set theorists to accept the axiom.
  - But the axioms of ZFC are intuitively pleasing and large cardinal axioms are less obvious.
  - One might believe that at least some of the questions left unsettled by ZFC are really open.
  - Perhaps some apparently well-formed questions in mathematics are neither true nor false.
  - The size of the set theoretic universe, if there is a unique size, eludes us.

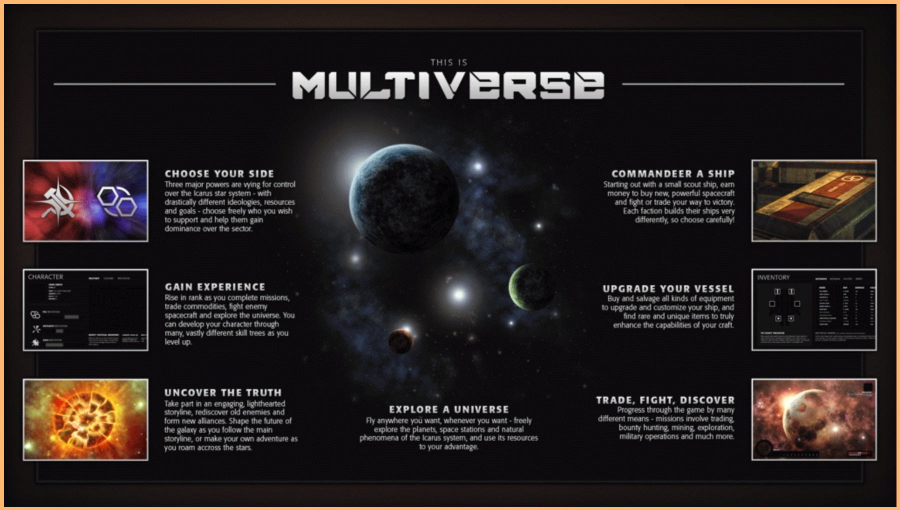


The Continuum

- Gödel famously claimed that the continuum has a unique size.
- Developments in recent decades, especially Cohen's model-theoretic proof of the independence of the continuum hypothesis from the standard axioms of set theory, have undermined Gödel's claim.
- One could adopt stronger axioms (e.g. the existence of Woodin cardinals) which settle the question univocally.
- But, it has seemed to some set theorists that no unique answer is yet warranted.

FBP and Open Questions

- FBP is largely motivated by and accommodates the view that there is no fact of the matter about certain mathematical questions.
- ZF + CH and ZF + not-CH each truly describe real set-theoretic universes.
- There is no fact of the matter whether the axiom of projective determinacy is true.



FBP and Field's Fictionalism

- Field:
  - "[M]athematicians are free to search out interesting axioms, explore their consistency and their consequences, find more beauty in some than in others, choose certain sets of axioms for certain purposes and other conflicting sets for other purposes, and so forth; and they can dismiss questions about which axiom sets are *true* as bad philosophy" (Field 1998a: 320.)
- The proponent of FBP accepts all of these claims except the last.
- Truth is no constraint on our estimation of mathematical theories.
  - Fictionalism: the mathematician is free because all mathematical theories are false or vacuous.
  - FBP: the mathematician is free since all mathematical theories are true.
- Euclidean rescues
  - There can be several equally-good geometrical theories.
  - There can be several equally-good set-theoretic universes, each with their own defining set of axioms.
  - Relative inconsistency is no problem.

Open Questions and Traditional Solutions

- Mathematicians are rarely deterred from seeking a solution to an open question by philosophical speculation that the question may be open in principle.
- The traditional platonist approaches such Euclidean rescues in mathematics warily, preferring to find a unique answer.
  - Gödel and the continuum hypothesis
- Open mathematical questions may seem unanswerable only to be later acclaimed true or false.
- Some of the questions which motivate FBP will provably require adjustments to well-entrenched and intuitive axioms.
- But axioms have been adopted and ceded before.
- The traditional platonist sees FBP as precipitously abandoning well-formed questions.

FBP as Autonomy Platonism

- FBP countenances the existence of mathematical objects and the truth of many mathematical claims.
- Only the consistency of the axioms, and not their applications in science, determines whether they are acceptable,.
  - That is, whether they truly describe a mathematical structure or universe
- It is not an indispensabilist view and does not suffer the unfortunate consequences.

Two Objections to FBP

1. Too many objects in too many true theories

- FBP portrays questions which are really open as closed.
- It provides no satisfactory account of our focus on preferred theories and interpretations.
  - E.g. on the standard model of the Peano postulates.
  - Focus on the standard model is explained by its ubiquity.
  - But mathematicians also believe it to be the correct model.
- Consistency is a minimal requirement for mathematical goodness, but it is not a sufficient condition.
- It would be nice to think that we need no capacity other than an ability to recognize contradictions to ground our estimations of various mathematical theories.
- Our mathematical practice belies this fantasy.

Two Objections to FBP

2. The denial of necessity

- For the indispensabilist, mathematical objects exist as contingently as the physical world.
  - Unfortunate Consequence
  - Pushes us toward autonomy platonism
- FBP also suffers from Modal Uniformity,
- Mathematical objects do not exist necessarily.
- Balaguer claims that nominalistic worlds are possible.
- For FBP, mathematical claims are true only in those worlds in which mathematical objects exist.

Balaguer Against Necessity

- “(a) [C]orresponding to every way that the physical world could be set up, there are two different possible worlds, one containing abstract objects and the other not; and (b) if we were “presented” with a possible world, we wouldn’t know whether it was a world containing abstract objects or a physically identical world without abstract objects, and what’s more, we wouldn’t have the foggiest idea what we could do in order to figure this out. The reason for this, if I am right, is that for any such pair of physically identical worlds, we don’t know what the difference between them really amounts to (Balaguer 1998: 166)
- “The problem here is that we just don’t have any well-motivated account of what metaphysical necessity consists in. Now, I suppose that Katz-Lewis platonists *might* be able to cook up an intuitively pleasing definition that clearly entails that the existence claims of mathematics - and, indeed, all purely mathematical truths - are metaphysically necessary. If they could do this, then their claim that mathematical truths are necessary would be innocuous after all. But (a)...the claim would still be epistemologically useless, and (b) it seems highly unlikely (to me, anyway) that Katz-Lewis platonists could really produce an adequate definition of metaphysical necessity. It just doesn't seem to me that there is any interesting sense in which ‘There exists an empty set’ is necessary but ‘There exists a purple hula hoop’ is not” (Balaguer 1998: 44-45).

On Purple Hula Hoops and Sets

- That there is a purple hula hoop depends on facts about the world over which we have some control.
  - We have interactions in events that result in the creation of hula hoops.
  - We can put together a plan for the eradication of hula hoops.
  - We can explain what contingent facts are contingent on.
- We can’t say anything about what differences could yield the existence or non-existence of mathematical objects.
  - Nothing we do or could do has any effects on the existence or non-existence of mathematical objects.
- Mathematical objects exist in all possible worlds.
  - True mathematical claims are true in all possible worlds.
  - False ones are false in all possible worlds.

The Utility of Necessity

- FBP says that every consistent mathematical theory truly describes a mathematical universe.
- This is very close to saying that the theorems of mathematics, when true, are necessarily true, and that mathematical objects exist necessarily.
- Moreover, the necessity of mathematics could help the FBPist explain why consistency entails truth.
- Also, one might merely wish to account for the commonsense belief that there is a difference between what might have been different and what could not have been different.

Explaining Necessity?

- It is difficult to see what kind of explanation for the necessity of mathematical claims one could provide.
- Causal explanations are out.
- Abstract objects are not governed by the laws of physics and so can have no explanations of the sort we provide for contingent non-existence claims.
- Purely mathematical explanations generally yield conditional claims: on the basis of certain axioms or assumptions, certain other theorems follow.
- We can explain why it is impossible for there to be a set such that its power set is the same size as itself, for example, only on the assumption of the existence of sets.
- It seems as if this is just the kind of question that doesn't have a good answer, like 'Why is there something rather than nothing?'

From FBP to Intuition-Based Autonomy Platonism

- FBP attempts to account for mathematical knowledge on the basis of merely our pre-theoretic apprehension of consistency.
- The autonomy platonist who wishes to explain our focus on the standard model will appeal to more contentious epistemic capacities.
    - mathematical intuition
- Some philosophers are skeptical of the prospects for such a view.
    - "One *might* adopt the ontological position that there are multiple 'universes of sets' and hold that nevertheless we have somehow mentally singled out one such universe of sets, even though anything we say that is true of it will be true of many others as well. But since it is totally obscure how we could have mentally singled out one such universe, I take it that this is not an option any plenitudinous platonist would want to pursue" (Field 1998b: 335.)