Explanation and Indispensability

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Knowledge, Truth and Mathematics

Stuff

- Dinner tonight
- Papers due Tuesday

Outline

- A rough distinction between epistemic explanations and metaphysical explanations
- The old-fashioned (Quinean) indispensability argument: a stalemate
- The explanatory indispensability argument: an attempted tiebreaker
- The explanatory argument fails since it relies on an epistemic sense of ‘explanation
  - Colyvan
  - Baker
  - Bangu

§1: Epistemic and Metaphysical Explanations

Casual Explanation and Serious Theory

Q: Should the following inference convince someone that there are numbers?

- IM

  I have two mangoes.

  Andrés has three different mangoes.

  So, together we have five mangoes.

A: No, simple uses of arithmetic are easily eliminated.

- IN

  $(\exists x)(\exists y)(Mx \bullet My \bullet Hix \bullet Hiy \bullet x \neq y)$

  $(\exists x)(\exists y)(\exists z)(Mx \bullet My \bullet Mz \bullet Hax \bullet Hay \bullet Haz \bullet x \neq y \bullet x \neq z \bullet y \neq z) \bullet (\forall x)[(Mx \bullet Hax) \supset {\sim}Hmx]$

  $\therefore (\exists x)(\exists y)(\exists z)(\exists w)(\exists v)[Mx \bullet My \bullet Mz \bullet Mw \bullet Mv \bullet x \neq y \bullet x \neq z \bullet x \neq w \bullet x \neq v \bullet y \neq z \bullet y \neq w \bullet y \neq v \bullet z \neq w \bullet z \neq v \bullet w \neq v \bullet (Hix \vee Hax) \bullet (Hiy \vee Hay) \bullet (Hiz \vee Haz) \bullet (Hiw \vee Haw) \bullet (Hiv \vee Hav)]$

Why are there five mangoes here?

- IM provides a perspicuous, easily-understood, and satisfying explanation.
- The fact is only awkwardly demonstrated, if explained at all, by IN.
- But the conclusion of IN follows from the premises in standard first-order logic.
- IM is explanatory, but not to be taken literally.

IM

I have two mangoes.
Andrés has three different mangoes.
So, together we have five mangoes.

IN

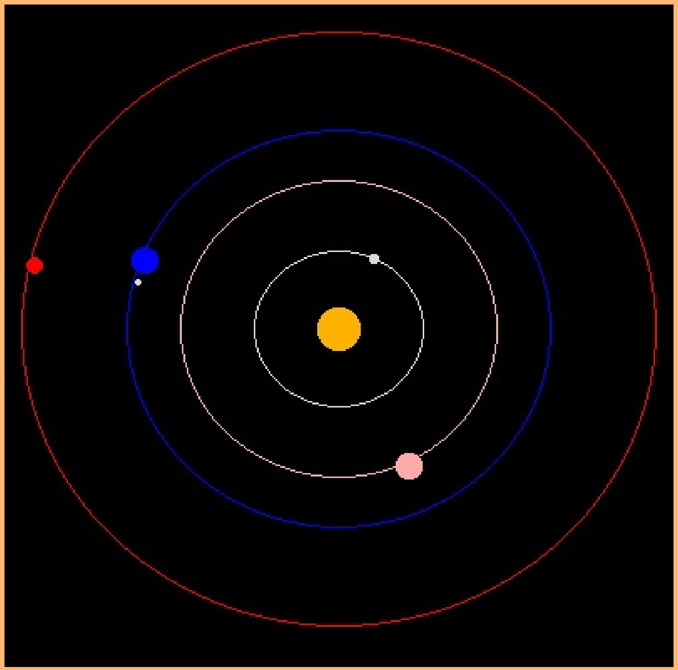
$(\exists x)(\exists y)(Mx \bullet My \bullet Bxm \bullet Bym \bullet x \neq y)$

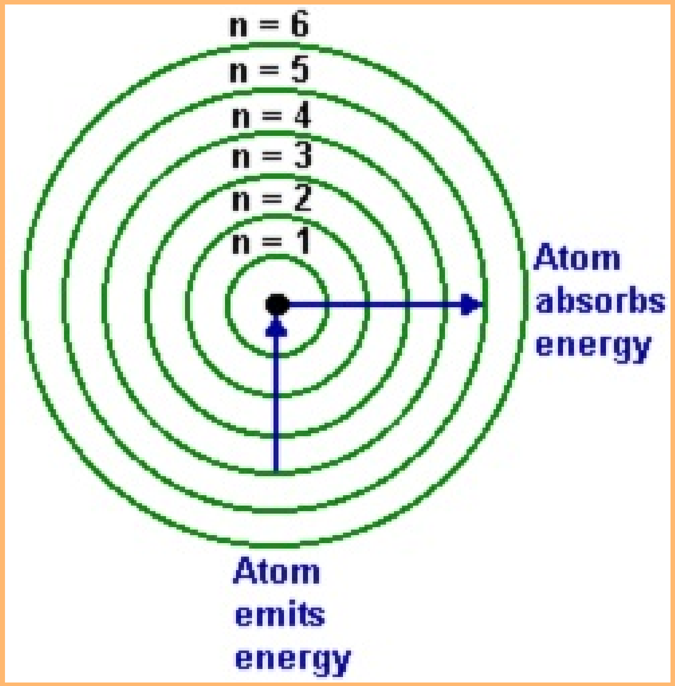
$(\exists x)(\exists y)(\exists z)(Mx \bullet My \bullet Mz \bullet Bxa \bullet Bya \bullet Bza \bullet x \neq y \bullet x \neq z \bullet y \neq z)$

$(x)[(Mx \bullet Bxa) \supset {\sim}Bxm]$

$\therefore (\exists x)(\exists y)(\exists z)(\exists w)(\exists v)(Mx \bullet My \bullet Mz \bullet Mw \bullet Mv \bullet x \neq y \bullet x \neq z \bullet x \neq w \bullet x \neq v \bullet y \neq z \bullet y \neq w \bullet y \neq v \bullet z \neq w \bullet z \neq v \bullet w \neq v)$

The Bohr Model





Like IM, the Bohr model of the atom can be explanatory without being taken literally.

Two distinct senses of ‘explanation’

- Physicists generally seek explanations in terms of laws and unifying principles.
- These *metaphysical* explanations may be taken as aiming at the most fundamental characteristics of the world.
- Sometimes we prefer explanations in less fundamental terms.
- These *epistemic* explanations are aimed at increasing our subjective understanding.
- The casual inference IM and the Bohr model exemplify epistemic explanation.
  - Van Fraassen’s pragmatic account
- The formal inference IN exemplifies metaphysical explanation.
  - D-N model
  - Unificationist model (Friedman, Kitcher)
- Metaphysical explanations increase subjective understanding in the most learned persons.
- Both senses of ‘explanation’ are useful.
  - It seems to me obvious that the only rational approach ...would be the following: We should reconcile ourselves with the fact that we are confronted, not with one concept, but with several different concepts which are denoted by one word; we should try to make these concepts as clear as possible; to avoid further confusions, we should agree to use different terms for different concepts; and then we may proceed to a quiet and systematic study of all concepts involved, which will exhibit their main properties and mutual relations (Tarski on truth).
- The explanatory indispensability argument depends on an equivocation between the two.

Two morals

Moral 1: We are committed to mathematical objects not by our casual uses of numbers, but only when we are speaking most seriously.

Moral 2: The theory we use to specify our ontological commitments may not be most useful when we want to explain facts about the world, at least in the epistemic sense of ‘explain’.

§2: Quine's indispensability argument (QI)

QI1. We should believe the theory which best accounts for our sense experience.
QI2. If we believe a theory, we must believe in its ontological commitments.
QI3. The ontological commitments of any theory are the objects over which that theory first-order quantifies.
QI4. The theory which best accounts for our sense experience first-order quantifies over mathematical objects.
QIC. We should believe that mathematical objects exist.

Note that the argument is formulated in terms of theories, not explanations.

- But: you could re-cast it in terms of metaphysical explanations, because they rely on our most austere theories

QI derives its strength, in large part, from Quine's connection between ontology and the construction of formal scientific theory.

"The quest of a simplest, clearest overall pattern of canonical notation is not to be distinguished from a quest of ultimate categories, a limning of the most general traits of reality."

Stalemate

Dispensabilists deny QI4

- Field's attempt to rewrite Newtonian Gravitational Theory quantifying over space-time regions rather than real numbers
- John Burgess's later improvements
- Mark Balaguer's sketch of a dispensabilist project for quantum mechanics
- Burgess and Rosen's argument that the lack of dispensabilist projects currently available is weak evidence for their eventual non-existence
- The dispensabilist has reasonable hope of finding moderately attractive reformulations of large swaths of scientific theory.

Indispensabilists affirm QI4

- Curved space-time (general relativity)
- Statistical theories (QM, many special sciences)
- No neat, first-order theory which eschews all mathematical axioms will suffice for all of current and future science.

QI4. The theory which best accounts for our sense experience first-order quantifies over mathematical objects.

§3: The explanatory indispensability argument (EI) (Baker, Colyvan)

EI1. There are genuinely mathematical explanations of empirical phenomena.

EI2. We ought to be committed to the theoretical posits postulated by such explanations.

EIC. We ought to be committed to the entities postulated by the mathematics in question.

- "Even if nominalisation via [a dispensabilist construction] is possible, the resulting theory is likely to be less explanatory; there is explanatory power in phase-space formulations of theories, and this explanatory power does not seem recoverable in alternative formulations" (Lyon and Colyvan 2008: 242).

EI must rely on an epistemic sense of ‘explanation’

- Standard (metaphysical) accounts of scientific explanation do not comfortably apply to mathematical explanation.
  - There are too many inferences available in mathematics.
  - Mathematicians distinguish between explanatory and non-explanatory proofs
- Unlike standard scientific theories, dispensabilist reformulations will be imperspicuous, and not useful to working scientists
  - Dispensabilists generally do not suggest that scientists adopt the reformulations.
  - The dispensabilist grants that standard theories are more explanatory in the epistemic sense.
- Conserving explanatory power is a standard requirement on nominalist reformulations.
  - Field’s representation theorems
  - Conservativeness
- One could not successfully nominalize a scientific theory with less explanatory power, unless one is using a non-metaphysical sense of ‘explanation’.

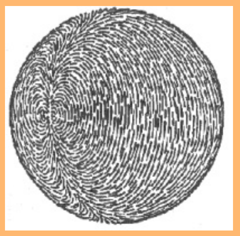
Why the sense of 'explanation' matters

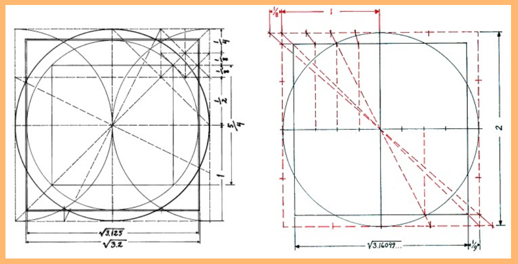
QI1. We should believe the theory which best accounts for our sense experience.
QI2. If we believe a theory, we must believe in its ontological commitments.
QI3. The ontological commitments of any theory are the objects over which that theory first-order quantifies.
QI4. The theory which best accounts for our sense experience first-order quantifies over mathematical objects.
QIC. We should believe that mathematical objects exist.

EI1. There are genuinely mathematical explanations of empirical phenomena.
EI2. We ought to be committed to the theoretical posits postulated by such explanations.
EIC. We ought to be committed to the entities postulated by the mathematics in question.

- The availability of a dispensabilist reformulation of a standard scientific theory is *essential* to QI.
- The availability of a dispensabilist reformulation of a standard scientific theory is *irrelevant* to EI.
  - A reformulation may well lose explanatory strength
- Debate over EI has focused on its first premise.

Colyvan’s support for EI1





- The Borsuk-Ulam topological theorem explains the existence of two pressure/temperature antipodes in the Earth’s atmosphere.
- That π is transcendental explains why we can not square the circle.
- Simpson’s paradox may help explain the persistence of maladaptive traits like altruism.
- Such examples provide decisive, if unsurprising, evidence for EI1, since ‘explanation’ should be taken in the epistemic sense.
- We are free to invoke fiction and metaphor.
- The question of whether such examples can be reformulated to eliminate mathematical objects is precisely what EI is designed to avoid.

Baker's Cicadas

- That prime-numbered life-cycles minimize the intersection of cicada life-cycles with those of both predators and other species of cicadas explains why three species of cicadas of the genus Magicicada share a life cycle of either thirteen or seventeen years, depending on the environment.
- Baker claims that the phenomenon is explained thus:

  CP1. Having a life-cycle period which minimizes intersection with other (nearby/lower) periods is evolutionarily disadvantageous.

  CP2. Prime periods minimize intersection.

  CP3. Hence organisms with periodic life-cycles are likely to evolve periods that are prime.

  CP4. Cicadas in ecosystem-type, E, are limited by biological constraints to periods from 14 to 18 years.

  CP5. Hence, cicadas in ecosystem-type, E, are likely to evolve 17-year periods.
- The mathematical explanans, at CP2, supports the "'mixed' biological/ mathematical law" at CP3, which explains the empirical claim CP5.

Bangu's criticism

CP5. Hence, cicadas in ecosystem-type, E, are likely to evolve 17-year periods.

- The explanandum in question at CP5 is, like CP3, a mixed statement, composed of both mathematical and physical facts.
  - a physical phenomenon (the time interval between successive occurrences of cicadas)
  - the concept of a life-cycle period
  - the number seventeen
  - the mathematical property of primeness
- The mathematical explanation only explains the mathematical portions of the explanandum.
- "If the explanandum is the relevance of the primeness of a certain number, since primeness is a mathematical property, it is not surprising that we have to advance a mathematical explanation of its relevance, in terms of specific theorems about prime numbers."

Smuggling-in the metaphysical sense of 'explanation'

- Bangu's criticism of EI depends on whether we can reformulate CP5, eliminating the mathematical portion.
- If the mathematical elements of CP5 were inseparable, then we could conclude, with the indispensabilist, that there are essentially mathematical elements of our descriptions of physical phenomena.
- Conversely, if we can eliminate the mathematical elements of CP, then we can deny that it provides support to EI.
- Bangu's criticism thus replays the dialectic between the indispensabilist and the dispensabilist over QI4.
- But the whole point of introducing EI was to avoid precisely this dispute.
- As it stands, CP is an (epistemic) explanation of a biological fact which refers to mathematical objects.
- Re-casting it, to avoid mathematical objects, would reduce its epistemic explanatory force in order to speak most seriously.

Evaluating EI

EI1. There are genuinely mathematical explanations of empirical phenomena.
EI2. We ought to be committed to the theoretical posits postulated by such explanations.
EIC. We ought to be committed to the entities postulated by the mathematics in question.

- The examples from Colyvan and Baker are sufficient for EI1.
- EI2 is the key premise.
- The real problem with CP (the cicada example) is not that it doesn't support EI1.
- The real problem is that we have no reason to take CP as expressing our ontological commitments.

Why the explanatory argument fails

EI1. There are genuinely mathematical explanations of empirical phenomena.
EI2. We ought to be committed to the theoretical posits postulated by such explanations.
EIC. We ought to be committed to the entities postulated by the mathematics in question.

- Once we realize that the sense of 'explanation' in question is epistemic, any force that EI2 is supposed to have is lost.
- EI seems plausible, if we have a metaphysical sense of 'explanation' in mind.
  - But then it's no improvement on QI.
- There is little reason to believe that the explanations which facilitate our subjective understanding are ones in which we reveal our ontological commitments by speaking most soberly.

Four final comments

1. I remain agnostic about whether the Quinean argument is successful.
2. My argument against EI is based on ascribing an epistemic notion of explanation.
   Perhaps there is an independent sense of 'explanation' on which EI might be based, and on which it would be more successful.
3. I presented EI as an additional option for the platonist, and thus an additional demand on the nominalist.
   Alternatively, we could see the argument as an additional demand on the platonist.
   My criticisms of EI are neutral, I believe, between the two views.
4. Two distinct attitudes toward mathematical explanations of empirical phenomena
   A. Instrumentalist/Representationalist: Melia, Leng, Azzouni
   B. Demanding an account of application (platonist or fictionalist)