

# Knowledge, Truth, and Mathematics

Philosophy 405  
Russell Marcus

Class #25: Bangu on the Explanatory Argument

# Business

- Dinner Thursday
  - Food restrictions?
- The Final/Author Meets Critics Session
  - Tuesday, May 6: Papers to me
  - Thursday, May 8: Papers to whole class
  - Sunday May 11: First Critics send their comments to author, second critic, and me
  - Tuesday, May 13, noon: Second Critics sent their comments to author, first critic, and me
  - Wednesday , May 14, 7-10pm: Our Session

# Quick Quiz

## On the Indispensability Argument

QI1. We should believe the theory which best accounts for our sense experience.

QI2. If we believe a theory, we must believe in its ontological commitments.

QI3. The ontological commitments of any theory are the objects over which that theory first-order quantifies.

QI4. The theory which best accounts for our sense experience first-order quantifies over mathematical objects.

QIC. We should believe that mathematical objects exist.

- What does Field say about it?
- What does Melia's weasel say about it?
- What does Leng's recreational argument say about it?
- How does the proponent of EI alter the argument?

# The Explanatory Indispensability Argument

E1. There are genuinely mathematical explanations of empirical phenomena.

E2. We ought to be committed to the theoretical posits postulated by such explanations.

E3. We ought to be committed to the entities postulated by the mathematics in question (Mancosu 2008: §3.2).

## Three Conditions on E1

- ▶ B1. The application be external to mathematics
- ▶ B2. The phenomenon in question must be in need of explanation
- ▶ B3. The phenomenon must have been identified independently of the putative explanation.

B1-B3 eliminate Colyvan's examples.

# Four Better Examples

- Three from Mancosu
  - ▶ Honeycombs
  - ▶ Tennis Rackets
  - ▶ Sticks
- Baker's Cicadas
  - ▶ That prime-numbered life-cycles minimize the intersection of cicada life-cycles with those of both predators and other species of cicadas explains why three species of cicadas of the genus *Magicicada* share a life cycle of either thirteen or seventeen years, depending on the environment.

# Two Questions

EI1. There are genuinely mathematical explanations of empirical phenomena.

EI2. We ought to be committed to the theoretical posits postulated by such explanations.

EIC. We ought to be committed to the entities postulated by the mathematics in question.

- For EI1, is the mathematics in these Colyvan/Mancosu cases really explanatory?
- For EI2, does it matter, as far as our ontological commitments are concerned?
  - E1 for today

# A Note on Bangu's Misinterpretation

- “Field noted that even if, contrary to what he argued in his (1980), mathematical posits turn out to be indispensable to scientific theorizing, they still can't be granted ontological rights until they are shown to be indispensable in a stronger, more specific sense; in particular, the realists should be able to show that mathematical posits are indispensable for scientific explanations” (Field, 1989, pp. 14-20) (Bangu 13-4).
- “Hartry Field, one of the more influential recent nominalists, writes that the key issue in the platonism-nominalism debate is ‘one special kind of indispensability argument: one involving indispensability for explanations’ (Field 1989, p. 14)” (Baker 225).
- “What we must do is make a bet on how best to achieve a satisfactory overall view of the place of mathematics in the world... My tentative bet is that we would do better to try to show that the explanatory role of mathematical entities is not what is superficially appears to be; and the most convincing way to do that would be to show that there are some fairly general strategies that can be employed *to purge theories of all reference to mathematical entities*” (Field 1989: 18, emphasis added; see also fn 15 on p 20).
- An explanation is, “A relatively simple non-*ad hoc* body of principles from which [the phenomena] follow” (Field 1989: 15).
- The difference between QI and EI is important.
  - More on Thursday

# Bangu and B1

B1. The application must be external to mathematics.

- If the application for which mathematics is purportedly required is mathematical, then the indispensability argument does not apply as its proponents allege.
- Circularity
- Mathematical explanations of mathematical phenomena are not convincing to the indispensabilist
  - Leveraging argument



# Inter- and Intra-Theoretic Indispensability arguments

- An indispensability argument transfers evidence for one set of claims to another.
  - If the transfer crosses disciplinary lines, we can call the argument an inter-theoretic indispensability argument.
  - If evidence is transferred within a theory, we can call the argument an intra-theoretic indispensability argument.
- The indispensability argument in the philosophy of mathematics transfers evidence from natural science to mathematics.
  - inter-theoretic
- An intra-theoretic indispensability argument in science
  - Atomic theory makes accurate predictions which extend to the observable world.
  - Thus we should believe that atoms exist.
- An intra-theoretic indispensability argument in mathematics
  - Church's Thesis claims that our intuitive notion of an algorithm is equivalent to the technical notion of a recursive function.
  - It is fruitful, and, arguably, indispensable to our understanding of mathematics.
- Even if some intra-theoretic indispensability arguments are acceptable, the claim that we need mathematical objects in order to do mathematics is not convincing, especially to the Quinean, or any related indispensabilist with Ockhamist tendencies.

# Bangu and Leng on Externality

The explanandum must be true.

- “Genuine explanations must have a true explanandum, and when the explanandum is mathematical, its truth will also be in question” (Leng 2005: 174).
- “The explanandum can’t be a mathematical statement. Suppose it were; because we also had to assume the explanandum were true (in order to make sense to advance an explanation of it), the entities it features must exist. But this is just to assume that realism is correct, i.e. to beg the question against the nominalist” (Bangu 17).

# Bangu's Four Criteria for Mathematical Explanations

1. **Indispensability:** They must invoke indispensable uses of mathematics.
2. **Genuine Mathematicality:** The explanations must be genuinely mathematical.
  - An explanation in terms of space-time wouldn't count.
3. **Simplicity:** They should be fairly simple.
  - No argument by overwhelming complexity!
4. **Clean Explanandum:** Proponents of EI should not beg the question by presenting examples in which the explanandum contains ineliminable uses of mathematics.
  - The Circularity/Externality Objection

# CP

CP1. Having a life-cycle period which minimizes intersection with other (nearby/lower) periods is evolutionarily advantageous.

CP2. Prime periods minimize intersection.

CP3. Hence organisms with periodic life-cycles are likely to evolve periods that are prime.

CP4. Cicadas in ecosystem-type, E, are limited by biological constraints to periods from 14 to 18 years.

CP5. Hence, cicadas in ecosystem-type, E, are likely to evolve 17-year periods (Baker 2005: 233).

- ▶ CP3 is a mixed biological/mathematical law.
- ▶ Bangu says that CP5 is mixed, as well.

# CP5 is a Mixed Claim

CP5. Hence, cicadas in ecosystem-type,  $E$ , are likely to evolve 17-year periods.

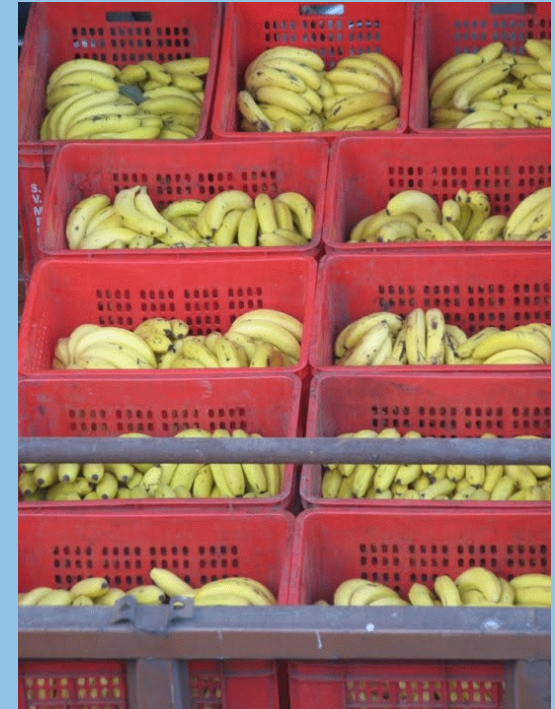
- CP5
  - ▶ It contains a physical phenomenon, the time interval between successive occurrences of cicadas.
  - ▶ It contains the concept of a life-cycle period, expressed in years.
  - ▶ It refers to the number 17.
  - ▶ And it contains the mathematical property of primeness.
- If we decompose CP5, we can see that the mathematical portions may be explained by mathematical theorems, without accepting that the non-mathematical portions are explained by the mathematical theorems.

# Bangu on Baker

- Bangu claims that Baker violates his own requirement of externality.
- If we accept that the whole of CP5 is true, we have already admitted the truth of the mathematical portions of CP5.
- Thus, Baker is really only providing a question-begging intra-theoretic indispensability argument.
  - ▶ “If the explanandum is the relevance of the primeness of a certain number, since primeness is a mathematical property, it is not surprising that we have to advance a mathematical explanation of its relevance, in terms of specific theorems about prime numbers “(Bangu 2008: 18).
  - ▶ “Baker assumes realism before he argues for it” (Bangu 18).

# Bangu's Bananas

- Bangu supports EI by proposing an alternative case of mathematical explanation.
- The banana game is derived from work in economic theory.
  - Bangu hopes to avoid what he sees as Baker's violation of the fourth desideratum.
- Two players compete to collect bananas by choosing among crates filled with unknown numbers of bananas.
- By adjusting the probabilities of choosing some crates over others, the game can be constructed so as to ensure the victory of one side over the other, even when the losing side has more bananas to choose from.
- The explananda invoke mathematics along the way.
  - probabilities and expected values
- The nominalist lacks resources for an explanation which is as satisfying as the one which appeals to probabilities.
  - "A correct and complete formulation of [a non-mathematical or qualitative explanation] (hence a rigorous proof of it) seems to be beyond the nominalist's conceptual resources" (Bangu 2013: 270-1).



# The Banana Game and Bangu's Criteria

1. **Indispensability**
2. **Genuine Mathematicality**
3. **Simplicity**
4. **Clean Explanandum**

- The banana game seems to satisfy the first criterion.
- Second, the explanations to which Bangu appeals are genuinely mathematical.
  - ▶ The crucial point is that the result in the first Game (one crate winning almost always) tends to occur because of an inequality of expectation values: the value corresponding to crate X is higher than the one corresponding to Y. Essentially, the same reasoning can be transferred to the other game, Game\*... Hence, if one wants to know what is common to both games, and thus what accounts for the explanandum, the realist offers this: in both games, we have an inequality of expectation values. A common feature of the games was identified, and this is what explains why the two games evolve the same way in the long run. This feature has been shown, in a rigorous fashion, to be responsible for the observed unidirectionality, that is, the explanandum. This explanation is given in terms of a simple mathematical notion ('expectation value'), so we are entitled to count this explanation as a mathematical one (Bangu 2013: 268).



# The Banana Game and Bangu's Criteria

1. **Indispensability**
2. **Genuine Mathematicality**
3. **Simplicity**
4. **Clean Explanandum**

- Third, the banana case is as simple as Baker's cicada case.
- Fourth, the explanandum concerns the victory of one player over another.
  - It does not contain ineliminable uses of mathematics.
  - Thus, we can not consider any explananda of the phenomenon problematically circular.



# Two Questions (Redux)

- For EI1, is the mathematics in these Colyvan/Mancosu cases really explanatory?
  - Bangu: Not in the Mancosu case
  - How about the other cases?
- For EI2, does it matter, as far as our ontological commitments are concerned?
  - That's for Thursday.