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Class #25: Bangu Against the Explanatory Argument
Bangu, "Inference to the Best Explanation and Mathematical Realism"
Bangu, "Indispensability and Explanation"

I. The Explanatory Argument, Redux

In our last class we saw Baker's explanatory indispensability argument.
Here, again, is Mancosu's version of the argument, EI.

> EI EI1. There are genuinely mathematical explanations of empirical phenomena.
> EI2. We ought to be committed to the theoretical posits postulated by such explanations.
> EIC. We ought to be committed to the entities postulated by the mathematics in question.

The central question about EI1 is whether the mathematics in an alleged case is really explanatory.
The central question about EI2 is whether E1 matters, as far as our ontological commitments are concerned.
To evaluate EI, we will focus on Baker's cicada example.
Today, we will look at Sorin Bangu's objections to that example.
Then, we will look at my new paper on the argument which focuses on EI2.

For Bangu's first paper, I won't spend much time thinking about what I take to be a mis-interpretation of the indispensability argument as an explanatory indispensability argument.

> Field noted that even if, contrary to what he argued in his (1980), mathematical posits turn out to be indispensable to scientific theorizing, they still can't be granted ontological rights until they are shown to be indispensable in a stronger, more specific sense; in particular, the realists should be able to show that mathematical posits are indispensable for scientific explanations (Field, 1989, pp. 14-20) (Bangu 2008: 13-4).

As I mentioned in my previous notes, I think this is a poor interpretation of Field's argument.
Field does refer to a special indispensability argument for scientific explanations.
But that argument is supposed to contrast with the one for metalogic, not with the one for scientific theories.
Field adopts a straight Quinean argument, like QI, using explanatory strength as one of the criteria for good explanations.

> What we must do is make a bet on how best to achieve a satisfactory overall view of the place of mathematics in the world... My tentative bet is that we would do better to try to show that the explanatory role of mathematical entities is not what is superficially appears to be; and the most convincing way to do that would be to show that there are some fairly general strategies that can be employed *to purge theories of all reference to mathematical entities* (Field 1989: 18, emphasis added; see also fn 15 on p 20).

Whatever we think of the Baker/Bangu interpretation of Field's argument, we can evaluate the explanatory indispensability argument on its own merits.

II. Externality

Baker provided three criteria which any example of a mathematical explanation of a physical phenomenon would have to meet in order to be considered genuine.

B1 The application must be external to mathematics.
B2 The phenomenon in question must be in need of explanation.
B3 The phenomenon must have been identified independently of the putative explanation.

Baker used these conditions to motivate his cicada example by showing that Colyvan's purported examples each violate one or more of B1-B3.
If the application for which mathematics is purportedly required is mathematical, in violation of B1, then the indispensability argument does not apply as its proponents allege.
Indeed, it seems circular to argue that we should be committed to mathematical objects in order to explain our knowledge of mathematics.
Of course we do![1]
Such an argument is not convincing to the indispensabilist, who accepts the premise that we are committed to mathematical objects only if we need them for science.

Let's distinguish between inter-theoretic and intra-theoretic indispensability arguments.
An indispensability argument transfers evidence for one set of claims to another.
If the transfer crosses disciplinary lines, we can call the argument an inter-theoretic indispensability argument.
If evidence is transferred within a theory, we can call the argument an intra-theoretic indispensability argument.
The indispensability argument in the philosophy of mathematics transfers evidence from natural science to mathematics.
Thus, this argument is an inter-theoretic indispensability argument.

As an example of an intra-theoretic indispensability argument, consider an argument for the belief in the existence of atoms.
Atomic theory makes accurate predictions which extend to the observable world.
It has led to a deeper understanding of the world, as well as further successful research.
Despite our lacking direct perception of atoms, they play an indispensable role in atomic theory.
According to atomic theory, atoms exist.
Thus, according to an intra-theoretic indispensability argument, we should believe that atoms exist.

As an example of an intra-theoretic indispensability argument within mathematics, consider Church's Thesis.
Church's Thesis claims that our intuitive notion of an algorithm is equivalent to the technical notion of a recursive function.
Church's Thesis is not provable, in the ordinary sense.
But, it might be defended by using an intra-theoretic indispensability argument: Church's Thesis is fruitful, and, arguably, indispensable to our understanding of mathematics.

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[1] OK, that's too quick. One can be a sentence realist without being an object realists, as many structuralists are. But, the charge of circularity still holds.

Intra-theoretic indispensability arguments are little-researched, and controversial.
Even if some intra-theoretic indispensability arguments are acceptable, the claim that we need mathematical objects in order to do mathematics is not convincing, especially to the Quinean, or any related indispensabilist with Ockhamist tendencies.
It begs the question.

Bangu, following Mary Leng, argues that B1, Baker's externality criterion, is properly interpreted as a requirement that the explanandum be true.
Here's Leng:

> Genuine explanations must have a true explanandum, and when the explanandum is mathematical, its truth will also be in question (Leng 2005: 174).

Here's Bangu:

> The explanandum can't be a mathematical statement. Suppose it were; because we also had to assume the explanandum were true (in order to make sense to advance an explanation of it), the entities it features must exist. But this is just to assume that realism is correct, i.e. to beg the question against the nominalist (Bangu 2008: 17)

Bangu's criticism of Baker's cicada example, then, is that Baker violates his own requirement of externality.

III. Bangu on Baker's Cicadas

Bangu presents four desiderata of examples used to support EI1.
First, perhaps obviously, they must invoke indispensable uses of mathematics.
Second, the explanations must be genuinely mathematical.
Some of Baker's concern about Colyvan's examples focus on this second criterion.
Third, they should be fairly simple.

> The examples presented to the nominalist must be non-elementary—hard—precisely because it must be hard to see how the explanatory power of the mathematized theories can be reproduced without mathematics. The idea is that the nominalist must be overwhelmed by the complexity of the example and declare that nominalization manoeuvres...aren't in sight, and thus mathematics is indispensable to formulating the explanation (Bangu 2013: 260).

Since rewriting theories to avoid quantification over mathematical objects is mainly a philosopher's project, not of compelling interest to many scientists or mathematicians, relevant techniques for eliminating mathematics may not yet be developed.
The indispensabilist should avoid resting the case on a lack of nominalist strategies which is due only to the difficulty of the task.

Fourth, and most relevant here, proponents of EI should not beg the question by presenting examples in which the explanandum contains ineliminable uses of mathematics.
Baker's cicada explanation proceeds as follows.

CP
- CP1. Having a life-cycle period which minimizes intersection with other (nearby/lower) periods is evolutionarily advantageous.
- CP2. Prime periods minimize intersection.
- CP3. Hence organisms with periodic life-cycles are likely to evolve periods that are prime.
- CP4. Cicadas in ecosystem-type, E, are limited by biological constraints to periods from 14 to 18 years.
- CP5. Hence, cicadas in ecosystem-type, E, are likely to evolve 17-year periods (Baker 2005: 233).

Bangu notes, with Baker, that CP3 is a mixed biological/mathematical law.
Similarly, Bangu argues, the explanandum in question at CP5 is, like CP3, a mixed statement, composed of both mathematical and physical facts.
CP5 contains a physical phenomenon, the time interval between successive occurrences of cicadas.
It contains the concept of a life-cycle period, expressed in years.
It refers to the number 17.
And it contains the mathematical property of primeness.
Bangu claims that once we separate these portions of CP5, we realize that the mathematical explanation only explains the mathematical portions of the explanandum.

> If the explanandum is the relevance of the primeness of a certain number, since primeness is a mathematical property, it is not surprising that we have to advance a mathematical explanation of its relevance, in terms of specific theorems about prime numbers (Bangu 2008: 18).

In other words, if we decompose CP5, we can see that the mathematical portions may be explained by mathematical theorems, without accepting that the non-mathematical portions are explained by the mathematical theorems.
To apply the mathematical theorem, we need bridge laws to assurance that number theory applies to the cicadas' cycles.
The pure number-theoretic premise refers to numbers and says nothing about life-cycles and their intersections.
If we accept that the whole of CP5 is true, we have already admitted the truth of the mathematical portions of CP5.
Thus, Baker is really only providing a question-begging intra-theoretic indispensability argument.

> Baker assumes realism before he argues for it (Bangu 2008: 18).

IV. Bangu's Bananas

Bangu's argument against Baker undermines Baker's support for EI1.
But Bangu does not intend to undermine the inference EI.
Indeed, Bangu supports EI by proposing an alternative case of mathematical explanation, the banana game, derived from work in economic theory.
With this example, Bangu hopes to avoid what he sees as Baker's violation of the fourth desideratum.

In the banana game, two players compete to collect bananas by choosing among crates filled with unknown numbers of bananas.

By adjusting the probabilities of choosing some crates over others, the game can be constructed so as to ensure the victory of one side over the other, even when the losing side has more bananas to choose from.
The explananda invoke mathematics along the way.
As long as the probability of choosing the crates with large numbers of bananas is sufficiently low, the relevant phenomenon will appear.
The explananda thus include mathematics in the forms of probabilities and expected values.

Bangu argues that the nominalist lacks resources for an explanation which is as satisfying as the one which appeals to probabilities.

> A correct and complete formulation of [a non-mathematical or qualitative explanation] (hence a rigorous proof of it) seems to be beyond the nominalist's conceptual resources (Bangu 2013: 270-1).

So the explanation invoked for the banana game satisfies his first criterion that explanations must invoke indispensable uses of mathematics.
The explanations to which Bangu appeals are genuinely mathematical.

> The crucial point is that the result in the first Game (one crate winning almost always) tends to occur because of an inequality of expectation values: the value corresponding to crate X is higher than the one corresponding to Y. Essentially, the same reasoning can be transferred to the other game, Game*... Hence, if one wants to know what is common to both games, and thus what accounts for the explanandum, the realist offers this: in both games, we have an inequality of expectation values. A common feature of the games was identified, and this is what explains why the two games evolve the same way in the long run. This feature has been shown, in a rigorous fashion, to be responsible for the observed unidirectionality, that is, the explanandum. This explanation is given in terms of a simple mathematical notion ('expectation value'), so we are entitled to count this explanation as a mathematical one (Bangu 2013: 268).

So, they fulfil Bangu's second criterion.
Moreover, the banana case is as simple as Baker's cicada case, so the third (simplicity) criterion is satisfied.
And the explanandum concerns the victory of one player over another.
It does not contain ineliminable uses of mathematics.
Thus, we can not consider any explananda of the phenomenon problematically circular.
Bangu's fourth criterion for mathematical explanations of scientific phenomena, the one which he invokes against Baker's case, is also satisfied.

Despite Bangu's refinements, I believe that the explanatory argument rests on a fundamental error about the nature of the indispensability argument.
In our next class, we'll look at my argument (which was just published over spring break!).