

Knowledge, Truth, and Mathematics

Philosophy 405
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Class #24
The Explanatory Indispensability Argument

A Confusion of Weasels

- Weasel₁s deny that mathematical objects exist but mainly because of their rejection of metaphysical questions altogether.
 - Weasel₁s could be logical empiricists like Carnap, pragmatists like Rorty, or others of anti-metaphysical bent.
 - The question whether mathematical objects exist is ill-formed.
- Weasel₂s accept the legitimacy of the question of whether mathematical objects exist and deny that they do.
 - Melia and Leng are weasel₂s.
 - Mathematical objects play merely a representational or modeling role in science, so they enter our best theories in the wrong way to induce beliefs in those objects.
- Weasel₃s not only accept the question whether mathematical objects exist and deny that they do, but also explain the basis on which they determines their ontological commitments.
 - It is not enough baldly to deny beliefs in the existence of mathematical objects.
 - The weasel should explain *why* she denies that mathematical objects exist.
 - On what basis do we determine what objects do exist?
 - The eleatic presents one alternative.
 - Others:
 - Presence in space-time
 - Can we paint it?
 - Some other criterion

Stalemate

- Dispensabilist projects have been produced, but they have not been as satisfying as Field initially hoped.
- Colyvan, and others, have countered with stronger re-statements of the argument.
- Weasels like Melia and Leng have refused to adopt Quine's conclusions.
- A new version of the indispensability argument has recently emerged.
 - The explanatory indispensability argument

Indispensability and Goals

- Any indispensability argument has to present some goal for which mathematics is supposed to be indispensable.
- For Quine, that goal was the regimentation of our best scientific theories.
- One of the criteria for determining whether we have a good theory is whether and how much that theory explains phenomena.
- Proponents of the explanatory indispensability argument take explanation to be the central goal.
 - ▶ We should believe in the existence of mathematical objects because they are indispensable to our best scientific explanations.

Scientific Explanation

- The first robust theory was the deductive-nomological, or D-N, model.
 - Hempel
 - Requires covering laws and specific facts.
- D-N explanations are deductive
 - A specific phenomenon, the explanandum, is derived from the laws and initial conditions.
 - The covering laws are general.
- Example:
 - All swans are white. (Covering law)
 - This is a swan. (Initial conditions)
 - This swan is white. (Explanandum)



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The D-N Model and Laws

- We can use the D-N model to explain lower-level laws by higher-level, covering laws.
- The higher the level of these covering laws, the more variables are considered.
- Boyle's Law. $P_1V_1 = P_2V_2$, is a relatively low-level law.
- Similarly for Charles's Law: $V_1/T_1 = V_2/T_2$.
- The ideal law of gases is a higher-level law, combining results of the two: $PV = kT$.
- Newton's gravitational law explains Galileo's law regarding free-falling bodies.

Problems for the D-N Model

- Determining the laws
 - ▶ A piece of copper on my desk that conducts electricity confirms an hypothesis that all copper conducts electricity.
 - ▶ It does not confirm the hypothesis that anything on my desk will conduct electricity.
 - ▶ That a person in a room is first-born will confirm the hypothesis that all persons in a room are first-born.
 - ▶ But, that's not a law, even though it looks syntactically like a law.
 - ▶ Laws must support counterfactuals.
- Relevance
 - ▶ We can derive Kepler's laws of planetary motion from Newton's more general laws of motion.
 - ▶ We can also derive Kepler's laws from the conjunction of Newton's laws with, say, Mendel's laws of genetics.
- Symmetry
 - ▶ We can explain the length of a shadow by appealing to the height of, say, a flagpole.
 - ▶ We can just as easily explain the height of the flagpole by the length of the shadow.

Modifications and Alternatives

- Unification theories
- Pragmatic theories of explanation
- Causation
 - ▶ Covering laws can be distinguished from accidental generalizations by appealing to the causal connections described by those laws.
 - ▶ The asymmetry of explanations can be explained by the appeals to causal laws in the explananda.
 - ▶ The relevance condition is again an appeal to causal connections.

The Explanatory Indispensability Argument

- Mancosu:

- EI1. There are genuinely mathematical explanations of empirical phenomena.

- EI2. We ought to be committed to the theoretical posits postulated by such explanations.

- EIC. We ought to be committed to the entities postulated by the mathematics in question (Mancosu 2008: §3.2).

- Baker

- (1) We ought rationally to believe in the existence of any entity that plays an indispensable explanatory role in our best scientific theories.

- (2) Mathematical objects play an indispensable explanatory role in science.

- (3) Hence, we ought rationally to believe in the existence of mathematical objects (Baker 2009: 613).

The Explanatory Argument and the Quinean Argument

- The explanatory argument puts aside the question of whether theories can be recast in order to eliminate mathematical entities.
- Instead, the proponent of the explanatory argument wonders whether non-mathematical explanations of physical phenomena are available.
- Recent work on EI grants the availability of nominalist reformulations of standard scientific theories and continues to urge that mathematical explanations of empirical phenomena support belief in mathematical objects.

Whose Burden Is It?

- An additional demand on the platonist/option for the nominalist?
 - ▶ Even if dispensabilist constructions do not work, we should withhold commitments to mathematical objects since there are no genuinely mathematical explanations.
 - ▶ Sorin Bangu and Joseph Melia
 - ▶ The platonist has to show mathematics indispensable from both theories and explanations.
 - ▶ The nominalist needs to show that mathematics is eliminable only from explanations or theories.
- An additional option for the platonist/demand on the dispensabilist?
 - ▶ Even if the dispensabilist constructions do work, we should grant commitments to mathematical objects as long as there are genuinely mathematical explanations of physical phenomena.
 - ▶ Baker
 - ▶ The platonist needs to show that mathematics is indispensable only from explanations.
 - ▶ The nominalist must show how we can eliminate mathematics from both theories and explanations.
- Whatever

The Explanatory Argument and Ontological Commitment

- Like the weasel, the proponent of the explanatory argument has to defend a new method for determining ontological commitments.
- Baker substitutes an inference to the best explanation, or IBE.
 - ▶ “The indispensability debate only gets off the ground if both sides take IBE seriously, which suggests that *explanation* is of key importance in this debate” (Baker 225).
- But, the key element of Quine’s argument is his criterion for determining the ontological commitments of our theories, not an inference to the best explanation.
 - ▶ Baker claims that his argument is non-holistic.
 - ▶ Without Quine’s holism, it’s difficult to see how the evidence for the empirical elements of our best scientific explanations extends to the mathematical objects used in those explanations.

Baker on Colyvan

- The defender of the explanatory argument is looking for genuine mathematical explanations of physical phenomena.
- Baker believes that Colyvan's examples are insufficient for EI.
- The example of the antipodes is a predication, not an explanation.
- The relativity examples are contentious because of their reliance on geometry.
 - ▶ "Individual geometrical terms such as 'triangle' may refer either to mathematical or to physical objects, and the historical trajectory of Euclidean geometry, from descriptor of physical space to free-standing formal system, shows a similar bridging of the mathematics/physics boundary at the level of geometrical theories. This is one reason why nominalists often object that geometrical explanations are not genuinely mathematical. And it suggests that we should look elsewhere than geometry for a convincing case of mathematical explanation in science "(Baker 228).
- Colyvan's cases seem liable to weaseling responses, especially those based on an eleatic principle.

Baker's Three Conditions

on mathematical explanations of physical phenomena

- The application be external to mathematics
- The phenomenon in question must be in need of explanation
- The phenomenon must have been identified independently of the putative explanation (otherwise it is more like a prediction)

Baker's Cicadas



- That prime-numbered life-cycles minimize the intersection of cicada life-cycles with those of both predators and other species of cicadas explains why three species of cicadas of the genus *Magicicada* share a life cycle of either thirteen or seventeen years, depending on the environment.
- Baker claims that the phenomenon is explained thus:
 - ▶ CP1. Having a life-cycle period which minimizes intersection with other (nearby/lower) periods is evolutionarily disadvantageous.
 - ▶ CP2. Prime periods minimize intersection.
 - ▶ CP3. Hence organisms with periodic life-cycles are likely to evolve periods that are prime.
 - ▶ CP4. Cicadas in ecosystem-type, E, are limited by biological constraints to periods from 14 to 18 years.
 - ▶ CP5. Hence, cicadas in ecosystem-type, E, are likely to evolve 17-year periods.
- The mathematical explanans, at CP2, supports the “‘mixed’ biological/mathematical law” at CP3, which explains the empirical claim CP5.

Cicadas and the Three Criteria

- There is a biological phenomenon in question, one which was noticed before any mathematical explanation was presented.
- That phenomenon puzzled scientists, who looked for an explanation.

Mancosu's Three Examples

- Honeycombs
 - ▶ “Why do hive-bee honeycombs have a hexagonal structure?...Part of the explanation depends on evolutionary facts. Bees that use less wax and thus spend less energy have a better chance at being selected. The explanation is completed by pointing out that “any partition of the plane into regions of equal area has perimeter at least that of the regular hexagonal honeycomb tiling”. Thus, the hexagonal tiling is optimal with respect to dividing the plane into equal areas and minimizing the perimeter” (Mancosu §1).
 - ▶ The honeycomb conjecture which states that a regular hexagonal grid or honeycomb represents the best way to divide a surface into regions of equal area with the least total perimeter is a purely geometric result.
- Tennis Rackets
- The bundle of sticks
- “Such explanations...seem to be counterexamples to the claim that all explanations in the natural science [sic] must be causal” (Mancosu §1).

Two Questions

EI1. There are genuinely mathematical explanations of empirical phenomena.

EI2. We ought to be committed to the theoretical posits postulated by such explanations.

EIC. We ought to be committed to the entities postulated by the mathematics in question.

- For EI1, is the mathematics in these Colyvan/Mancosu cases really explanatory?
- For EI2, does it matter, as far as our ontological commitments are concerned?