Knowledge, Truth, and Mathematics

Philosophy 405
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Class 23: Mathematical Recreation

Naturalism and Epistemology

- “Naturalism [is the] abandonment of the goal of a first philosophy. It sees natural science as an inquiry into reality, fallible and corrigible but not answerable to any supra-scientific tribunal, and not in need of any justification beyond observation and the hypothetico-deductive method “(Quine, “Five Milestones of Empiricism” 72).
- “Naturalism does not repudiate epistemology, but assimilates it to empirical psychology. Science itself tells us that our information about the world is limited to irritations of our surfaces, and then the epistemological question is in turn a question within science: the question how we human animals can have managed to arrive at science from such limited information” (Quine, “Five Milestones of Empiricism” 72).
- There is no special, independent discipline called philosophy.
- If we want to do metaphysics, we just construct our scientific theories and interpret them.
- We look to scientists for their work.

Three Interpretations of ‘Naturalism’

- Naturalism$_1$
  - We should look to empirical science, understood holistically, as the locus of all our ontological commitments.
  - Indispensability
- Naturalism$_2$
  - Scientists isolate mathematics from the rest of their theories.
  - They do not act as if all the objects over which they quantify have the same status.
  - They distinguish between the real elements of their theories and the instrumental elements.
  - They reject Quine’s holism.
- Naturalism$_3$
  - The bare rejection of any unnatural (including mathematical) objects
  - Melia and the eleatics
- Leng’s fictionalism is the result of embracing naturalism$_2$, which respects the practice of scientists over the statements of their theories, and over holism and the double-talk argument.

Leng’s Three Characteristics

- [Insularity] 1. Mathematics is insulated from scientific discoveries, in the sense that the falsification of a scientific theory that uses some mathematics never counts as falsification of that mathematics (beyond simple cases of calculation error).
- [Euclidean Rescue] 2. In particular, a scientific observation that conflicts with some scientific theory may suggest a move to a different background mathematics, but does not suggest that mathematicians should abandon that mathematics...The success of a scientific theory does not confirm the mathematics used in that theory.
  - “It is...difficult to maintain that the empirical discoveries confirmed the truth of non-Euclidean geometry and showed the falsity of Euclidean geometry in any sense other than that one was a correct model of the physical world and the other was not. But this is not *mathematical* truth: the applicability of non-Euclidean geometry did not falsify any mathematical theorems in Euclidean geometry - the Pythagorean theorem still holds for Euclidean triangles - it merely confirmed the assumption of Gauss and others that the scope of the theorems of Euclidean geometry only covers systems that assume the parallel axiom” (Leng 402).
- [Bridging] 3. What does seem to be disconfirmed by the failure of a scientific theory that relies strongly on a background mathematics is the claim that this mathematics is applicable to the scientific phenomena that it has been used to describe (Leng 411).
  - “Catastrophe Theory became a much less popular area of research, but no one would claim that the mathematics of Catastrophe theory had been *falsified* by its magnificent scientific failures” (Leng 407).

Mathematical Recreation

- "My view of pure mathematics is oriented strictly to application in empirical science...Pure mathematics extravagantly exceeds the needs of application...but I see these excesses as a simplistic matter of rounding out...I recognize indenumerable infinites only because they are forced on me by the simplest known systematizations of more welcome matters. Magnitudes in excess of such demands, e.g., $\beth_\omega$ or inaccessible numbers, I look upon only as mathematical recreation and without ontological rights" (Quine on Mathematical Recreation).
- Colyvan follows Quine in appealing to a fictionalist interpretation of the un-applied portions of mathematics.
- The doctrine of mathematical recreation: we should be fictionalists about the portions of mathematics that are not used in science.

Leng's Proposal

- Extend the fictionalist attitude toward recreational or un-applied portions of mathematics to all of mathematics.
- From the characteristics of Insularity, Euclidean Rescue, and Bridging, Leng concludes that mathematics plays merely a modeling role in science.
- Mathematical theories lack any serious ontological rights because they are used merely as models without any presumption that they are true or refer to real objects.
- "When we use mathematics to model physical situations in this way, we never refer to mathematical objects or assume the (mathematical) truth of their relations. Rather, we interpret our mathematical stories physically and assume that our model is good enough in the relevant respects that the theorems derived in our mathematical recreations, when transcribed into physical language, will give us truths about the physical phenomena we are considering...If Colyvan is right (and I think he is) that mathematics that is not assumed by science to be true should be seen as recreational (and given some important status as such), then it follows from the modeling picture of the relationship between mathematics and science that *all* mathematics is recreational" (Leng 411-412).
- "We are not committed to belief in the existence of objects posited by our scientific theories *if their role in those theories is merely to represent configurations of physical objects*. Fictional objects can represent just as well as real objects can" (Leng 2005: 179).

A Confusion of Weasels

- Weasel$_1$s deny that mathematical objects exist but mainly because of their rejection of metaphysical questions altogether.
  - Weasel$_1$s could be logical empiricists like Carnap, pragmatists like Rorty, or others of anti-metaphysical bent.
  - The question whether mathematical objects exist is ill-formed.
- Weasel$_2$s accept the legitimacy of the question of whether mathematical objects exist and deny that they do.
  - Melia and Leng are weasel$_2$s.
  - Mathematical objects play merely a representational or modeling role in science, so they enter our best theories in the wrong way to induce beliefs in those objects.
- Weasel$_3$s not only accept the question whether mathematical objects exist and deny that they do, but also explain the basis on which they determines their ontological commitments.
  - It is not enough to deny beliefs in the existence of mathematical objects.
  - We must explain why we deny that mathematical objects exist and on what basis we determine what objects do exist.

Weasel$_3$s

- The weasel$_3$ provides an alternative method for determing the real commitments of our theories.
- The eleatic presents one alternative.
- Others are possible.
  - space-time
  - idealism
  - dualism
  - some other metaphysical attitude

Stalemate

- Dispensabilist projects have been produced, but they have not been as satisfying as Field initially hoped.
- Colyvan, and others, have countered with stronger re-statements of the argument.
- Weasels like Melia and Leng have refused to adopt Quine’s conclusions.
- A new version of the indispensability argument has recently emerged.
  - The explanatory indispensability argument