Hamilton College
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Class #23: Mathematical Recreation
Leng, "What's Wrong with Indispensability? (Or, the Case for Recreational Mathematics)"

I. Naturalism and Epistemology

Leng defends a weaseling version of mathematical fictionalism against the indispensability argument.
The first part of Leng's paper concerns a tension in Quine's view between his epistemology and the indispensability argument.
Leng believes that she has found a way to reject the conclusion of the indispensability argument while maintaining a broadly Quinean (i.e. empiricist) framework by exploiting this tension.
This tension was explored earlier, by Penelope Maddy, who describes the problem as rooted in different understandings of Quine's naturalism.
It will be worth a moment to try to be clearer about the 'naturalist' label and Quine's use of it.

While Quine, in places, calls himself an empiricist, and the indispensability argument is clearly an empiricist's argument, Quine prefers to use the term 'naturalism' to describe his approach to philosophy.
He characterizes naturalism as the rejection of the view of philosophy as independent of, and as a way of evaluating, science.

> Naturalism [is the] abandonment of the goal of a first philosophy. It sees natural science as an inquiry into reality, fallible and corrigible but not answerable to any supra-scientific tribunal, and not in need of any justification beyond observation and the hypothetico-deductive method (Quine, "Five Milestones of Empiricism" 72).

As with any distillation of a variety of complex views to a simple term, there are different interpretations of 'naturalism'.
One version of naturalism, Quine's at his most radical, is the relegation of epistemology to empirical psychology.
If there is no supra-scientific tribunal, then there is no first philosophy (metaphysics) and no epistemology, no way to establish what exists independently of our best scientific theories.
Quine's holism ensures that claims in philosophy, physics, biology, economics, and mathematics are all inter-related.
No claim is prior to or independent of any other.
Epistemology, our theory of knowledge, is continuous with neuroscience, our theories of the brain, and cognitive psychology, our theories of our thought, and biology, our theories of sensation, and information processing.

> Naturalism does not repudiate epistemology, but assimilates it to empirical psychology. Science itself tells us that our information about the world is limited to irritations of our surfaces, and then the epistemological question is in turn a question within science: the question how we human animals can have managed to arrive at science from such limited information (Quine, "Five Milestones of Empiricism" 72).

Concomitantly, Quine views philosophy as in the service of (some would say 'as a handmaiden to') the sciences.
Philosophers can make progress in mathematics, say, as Quine did in set theory, or in empirical science, by contributing to the understanding of empirical research.

But there is no special, independent discipline called philosophy.
If we want to do metaphysics, we just construct our scientific theories and interpret them.
We look to scientists for their work.

As Leng reviews in the first part of her paper, Maddy argues that various interpretations of Quine's naturalism are in tension with one another.
One interpretation, call it naturalism$_1$, is closest to the indispensability argument.
On naturalism$_1$, we should look to empirical science, understood holistically, as the locus of all our ontological commitments.
We should take scientists' words seriously and note that they use mathematics ubiquitously.
So the naturalist$_1$ should believe in the existence of mathematical objects.

On naturalism$_2$, we notice that scientists isolate mathematics from the rest of their theories.
They do not act as if all the objects over which they quantify have the same status.
Scientists make a distinction between the real elements of their theories and the instrumental elements.
In particular, they seem to rely on something closer to an eleatic principle, when evaluating the commitments of their theories.
In practice, they reject Quine's holism.
The naturalist seems to have to decide between accepting Quine's holism, with naturalism$_1$, or rejecting it in favor of a practical instrumentalism, with naturalism$_2$.

A third version of the term, call it naturalism$_3$, is just the bare rejection of any unnatural (including mathematical) objects.
Melia and the eleatics might be seen as naturalist$_3$s.
Naturalist$_3$s and naturalist$_2$s welcome instrumentalism.

Quine could respond that instrumentalism, which we saw in the work of Carnap and Melia, employs double-talk about ontological commitment.
But according to Leng and Maddy, such a response privileges philosophy over scientific practice.
The holistic response elevates philosophical reflection over scientific practice, and thus conflicts with a proper understanding of naturalism, e.g. naturalism$_2$.
A consistent naturalism, Maddy and Leng argue, would not reject instrumentalism if that view is a part of scientific practice.
And, Maddy argues, instrumentalism is an accepted principle of scientific practice.
As Sober argues, scientists never actually test mathematical hypotheses.
Instead, they hold them fixed in the background.
Experiments only confirm the empirical portions of our hypotheses.
Counterexamples only hold against the empirical portions, too.
Remember the foxes and chickens!

Maddy thus concludes, and Leng follows her, that the indispensability argument fails because the ontological commitments of a theory are not to be found exclusively in the quantifications of the theory.
Instead, naturalism entails that epistemology is to be assimilated to empirical psychology and that metaphysics is to be assimilated to empirical science.
The ontological commitments of a theory should be discovered by looking at the actual practice of scientists.
This is the approach which Maddy adopts most recently and which she calls Second Philosophy.

Leng's fictionalism, then, is the result of embracing naturalism$_2$, which respects the practice of scientists over the statements of their theories, and over holism and the double-talk argument.
Let's look at her evaluation of the relation between mathematics and science.

II. Mathematics and Science

Relying on the work of Maddy and Sober describing the relationship of scientific practice and attitudes toward mathematics, Leng presents three characteristics of mathematics with regard to science, which I will call insularity, Euclidean rescue, and bridging.

> [Insularity] 1. Mathematics is insulated from scientific discoveries, in the sense that the falsification of a scientific theory that uses some mathematics never counts as falsification of that mathematics (beyond simple cases of calculation error).
>
> [Euclidean Rescue] 2. In particular, a scientific observation that conflicts with some scientific theory may suggest a move to a different background mathematics, but does not suggest that mathematicians should abandon that mathematics...The success of a scientific theory does not confirm the mathematics used in that theory.
>
> [Bridging] 3. What does seem to be disconfirmed by the failure of a scientific theory that relies strongly on a background mathematics is the claim that this mathematics is applicable to the scientific phenomena that it has been used to describe (Leng 411).

Quine argues that when we are faced with a contradiction in our theory, as when an hypothesis is contradicted by an experiment, we have the choice to revise an empirical hypothesis in our grand, holistic theory, or a mathematical hypothesis, or a logical one.
Since the entire theory is interconnected in a single, holistic web, we can restore consistency in lots of distinct ways.
As a logical matter, Quine's confirmation holism is indisputable.
Sober's argument against the Quinean view derives from the observation that in practice we never choose to give up the logical or mathematical principles.
Again, the practical fact of insularity is not contentious.
The question between Sober and Quine, here, is whether insularity is the result of our practical choices (Quine) or whether it derives from a discontinuity between mathematics and science which shows holism to be false (Sober).
Quine explains insularity as the consequence of a methodological principle of theory choice: the maxim of minimum mutilation.
In accordance with the maxim of minimum mutilation, we hold logical and mathematical principles fixed in order to do as little damage as possible to other portions of our theory.
We can adjust logic or mathematics if we want to do so, but we always choose not to.
Sober and Leng take the fact that we never give up our mathematics in light of recalcitrant data to indicate that mathematics is insulated from science, in principle.

The term 'Euclidean rescue' comes from the work of Michael Resnik.
Before the nineteenth century, people generally thought that there were only one geometry.
When it became clear that there were consistent non-Euclidean spaces, people tended to think of them as somehow lesser geometries.

Some people thought of non-Euclidean geometries as consistent but false or uninterpreted.
When it became clear that physical space is non-Euclidean, some people inferred that hyperbolic geometry is true, and Euclidean geometry is false, or uninterpreted.
We perform a Euclidean rescue when, in contrast to these kinds of attitudes, we accept all three geometries that result from the three different parallel postulates as equally true.
In this paradigmatic case, the Euclidean rescue entails that the discovery of which geometry is actually applied in our space is irrelevant to the truth of the mathematical theory.

> It is...difficult to maintain that the empirical discoveries confirmed the truth of non-Euclidean geometry and showed the falsity of Euclidean geometry in any sense other than that one was a correct model of the physical world and the other was not. But this is not *mathematical* truth: the applicability of non-Euclidean geometry did not falsify any mathematical theorems in Euclidean geometry - the Pythagorean theorem still holds for Euclidean triangles - it merely confirmed the assumption of Gauss and others that the scope of the theorems of Euclidean geometry only covers systems that assume the parallel axiom (Leng 402).

Euclidean rescues are not limited to this one case.
They are available in all cases in which a mathematical theory is shown inapplicable in science.
Leng cites the case of catastrophe theory.
Initially, the mathematical theory called catastrophe theory was thought to have profound implications for physical science.
Later it was seen not to apply as broadly as was initially thought.
Still, the mathematics was not impugned.
Euclidean rescues are related to insularity in that if one thinks that mathematics is insular, then one is predisposed to perform Euclidean rescues.
One might perform Euclidean rescues because one thinks that mathematics is and should be held insular.

Similarly, Euclidean rescues are supported by Bridging.
For a mathematical theory to be used in a physical theory, there must be bridge principles which map some mathematical claims into some physical claims.
When one finds an inconsistency in one's physical theory, one can always restore consistency without falsifying one's mathematical claims by denying just the bridge principles and not the actual mathematical theorems.
By denying only the bridge principles, we perform a Euclidean rescue, holding mathematics to be insulated from any falsification of an empirical theory.
In Leng's case of catastrophe theory, only the bridge principles, the claims of its applicability to physics, were denied.

> Catastrophe Theory became a much less popular area of research, but no one would claim that the mathematics of Catastrophe theory had been *falsified* by its magnificent scientific failures (Leng 407).

Leng's fictionalist proposal rests both on her exploitation of the tension in Quinean naturalism and these three characteristics of mathematical practice and its relation to science.
It also relies on Quine's attitude toward the mathematics which is not used in empirical science.

III. Mathematical Recreation

We saw Quine's view concerning the un-applied portions of mathematics when we first looked at the indispensability argument.

> My view of pure mathematics is oriented strictly to application in empirical science...Pure mathematics extravagantly exceeds the needs of application...but I see these excesses as a simplistic matter of rounding out...I recognize indenumerable infinites only because they are forced on me by the simplest known systematizations of more welcome matters. Magnitudes in excess of such demands, e.g., $\beth_\omega$ or inaccessible numbers, I look upon only as mathematical recreation and without ontological rights (Quine on Mathematical Recreation).

Strictly speaking, the indispensabilist has no justification for beliefs in mathematics that are not used in physical theory.
There is some room for extending our beliefs in the disjoint pieces of mathematics that are applied in science to full mathematical theories.
There is no room for beliefs in portions of mathematics which have no use in our empirical science.
Quine agrees with the fictionalist that unused portions of mathematical theories are false or merely vacuously true.
Colyvan follows Quine in appealing to a fictionalist interpretation of the un-applied portions of mathematics.
This is what Leng calls the doctrine of mathematical recreation: we should be fictionalists about the portions of mathematics that are not used in science.

There seem to be two different kinds of accounts of our knowledge of fictional objects.
Moby Dick and other fictional stories seem not to be constructed in the way that non-fictional works are created.
We just make up fictional stories, unconstrained by the nature of the real world.
But non-fictional narratives must be researched and discovered.
The methods of fiction and non-fiction are distinct.
In contrast, mathematical methodology is the same for portions which are used in science and portions which are not used in science.
So-called recreational results are continuous with applied mathematical results.
They are reducible to the same foundational theories, like set theory.
They are written in the same, formal language.
There are no mathematical differences among the two groups of results.

Leng's proposal, then, is simply to extend the fictionalist attitude toward recreational or un-applied portions of mathematics to all of mathematics.
From the characteristics of Insularity, Euclidean Rescue, and Bridging, Leng concludes that mathematics plays merely a modeling role in science.
Mathematical theories lack any serious ontological rights because they are used merely as models without any presumption that they are true or refer to real objects.

> When we use mathematics to model physical situations in this way, we never refer to mathematical objects or assume the (mathematical) truth of their relations. Rather, we interpret our mathematical stories physically and assume that our model is good enough in the relevant respects that the theorems derived in our mathematical recreations, when transcribed into

> physical language, will give us truths about the physical phenomena we are considering...If Colyvan is right (and I think he is) that mathematics that is not assumed by science to be true should be seen as recreational (and given some important status as such), then it follows from the modeling picture of the relationship between mathematics and science that *all* mathematics is recreational (Leng 411-412).

In later work, Leng discusses the modeling aspect of mathematical objects in terms of their ability to represent physical objects.

> We are not committed to belief in the existence of objects posited by our scientific theories *if their role in those theories is merely to represent configurations of physical objects*. Fictional objects can represent just as well as real objects can (Leng 2005: 179).

From Quine's naturalism, Leng concludes that there is no mathematical reason to distinguish between applied and un-applied results in mathematics.
She also infers that the work of mathematicians (and scientists) need not entail our beliefs in the truth of mathematical claims.
Where Quine claims that un-applied portions of mathematics could be considered recreational, Leng argues that all of mathematics should have that status.

IV. A Real Platonist Option

For those impressed with the argument concerning the continuity of applied and un-applied results, there is a non-fictionalist option.
I agree with Maddy and Leng that Quine's naturalism is in tension with itself.
The indispensabilist's view of mathematics is in tension with the actual practice of mathematics.

The continuum hypothesis provides a clear example of how mathematical practice conflicts with the indispensabilist's philosophy of mathematics.
According to the indispensabilist, mathematical questions are to be answered by examining our best scientific theory.
Our current mathematical axioms do not settle the question of the size of the continuum and provably so.
Gödel suggested that we adopt new axioms to settle the question.
The question arises about how to decide on which axioms to adopt.
Do we look to physics?
Or do we look directly to mathematics?
The indispensabilist insists that the answers have to be found in the needs of our best scientific theory.
The mathematician looks to purely mathematical criteria.
Thus the methods of the indispensabilist conflict with the methods of the mathematician.

If we are looking to the practice of science as having consequences for what we should believe exists, we might look more broadly.
We can reject Quine's view that there is no supra-scientific tribunal and develop criteria for determining which so-called scientific practices are legitimate.
If we do so, we find that mathematics itself is good science according to the same principles by which we esteem natural science.

So the lesson we take from the continuity claim is either to accept all of mathematics or to reject it all.
Leng argues that a good Quinean, with her preference for desert landscapes, should reject it all.
Another option would be to accept all of mathematics on the basis of the practice of mathematicians and the legitimacy of mathematics as a science.
This is not a naturalistic view on any of the three versions we considered above, even naturalism$_1$, the one which underlies the indispensability argument.
We might, though, find a version of naturalism, say naturalism$_4$, with which it is compatible.
Naturalism$_4$ would accept any pursuit which meets the criterion for legitimacy for scientific practice and is pursued by fully natural persons, physical objects rather than soul-like objects.
Instead of calling all of mathematics recreational, the naturalist$_4$ calls it all true!

V. A Confusion of Weasels

We have been looking at the Quinean indispensability argument, and some of the most important responses to it: the dispensabilist and the weasel.
Most dispensabilists and weasels are fictionalists.
Most people who oppose the indispensability argument write from from an anti-platonist point of view.
My work, which we will read at the end of the term, opposes the indispensability argument from the platonist side and is sympathetic to weaseling.
For now, let's proceed on the assumption that dispensabilists and weasels are fictionalists.
Further, let's not distinguish between fictionalists and nominalists.
So, we are looking at various anti-platonist responses to the indispensability argument.
We have seen three different kinds of weasels.
Let's look at them in order of increasing, say, dignity.

The most basic weasel, weasel$_1$, denies that mathematical objects exist but mainly because of her rejection of metaphysical questions altogether.
Weasel$_1$s could be logical empiricists like Carnap, pragmatists like Rorty, or others of anti-metaphysical bent.
For the weasel$_1$, the very question whether mathematical objects exist is ill-formed.
Similarly, the weasel$_1$ denies the legitimacy of the question of the existence of the external world.

A slightly more dignified weasel, weasel$_2$, accepts the legitimacy of the question of whether mathematical objects exist and denies that they do.
Weasel$_2$s generally believe in the existence of the external world and, unlike weasel$_1$s, will accept and assert sentences that affirm the existence of the external world.
They deny the existence of mathematical objects regardless of whether they refer to them in their most serious sentences about the world.
Melia and Leng are weasel$_2$s.
They argue that mathematical objects play merely a representational or modeling role in science, so they enter our best theories in the wrong way to induce beliefs in those objects.
Despite our uses of mathematics in science, and despite our beliefs in the external world, we need not extend such beliefs to the mathematical objects used in science.

Lastly, the most dignified of the weasels, weasel$_3$, not only accepts the question whether mathematical objects exist, and denies that they do, but also explains the basis on which she determines her ontological commitments.

The most common of the weasel$_3$s is the eleatic, who denies the existence of mathematical objects because she believes that all and only existing objects are located within the causal realm.
Since mathematical objects are non-causal, they do not exist.
I take David Armstrong to be a weasel$_3$,
We did not read Armstrong's work, but we did see Colyvan's response to some of Armstrong's eleatic view.
Weasel$_3$s recognize that it is not enough to deny beliefs in the existence of mathematical objects.
They must also explain why they deny that mathematical objects exist and on what basis they determine what objects do exist.

If we rely on a weaseling approach to the quantifications of our best theories, then, the most dignified position will present an alternative method for determining the real commitments of our theories.
The eleatic presents one alternative.
Others are possible.
We could say that all and only objects in space-time exist, for example.
Or we might be idealists, or dualists, or have some other metaphysical attitude.

I have argued that the Quinean argument is most resilient to weaseling responses because of its reliance on independent arguments that we find our ontology in the domain of quantification of our best theory.
Whether we think that the weaseling response is satisfactory depends on how strong we take Quine's double-talk argument to be.
If double-talk is really inadmissible, then no weaseling will be allowable.
If our uses of mathematics are really just representational or for modeling, as Melia and Leng argue, then weaseling may be defensible.