Knowledge, Truth, and Mathematics

Philosophy 405
Russell Marcus
Hamilton College, Spring 2014

Class #2: Pythagoreans

Sign-Up for Seminar Papers/Presentations

Brown's "Mathematical Image"

1. Mathematical results are certain.
2. Mathematics is objective.
3. Proofs are essential.
4. Diagrams are psychologically useful, but prove nothing.
5. Diagrams can even be misleading.
6. Mathematics is wedded to classical logic.
7. **Mathematics is independent of sense experience.**
8. The history of mathematics is cumulative.
9. Computer proofs are merely long and complicated regular proofs.
10. Some mathematical problems are unsolvable in principle.

A Priori Knowledge

Brown's #7

a priori knowledge

- The question of whether we have *a priori* knowledge is widely debated.
- A proposition is known *a priori* if the knowledge is not based on any, “Experience of the specific course of events of the actual world’”(Blackburn, in Shapiro, 22).
- The debates over the *a priori* are subtle and complex.
- But, the question of whether there is *a priori* knowledge seems easily answered in mathematics.
- We could never discover that the square root of two is irrational by experience.
  - The rationals are dense.
  - We can always find a rational which will fulfill our measurement needs.

That $\sqrt{2}$ is irrational

- Suppose that $\sqrt{2}$ is rational.
- Then, it's expressible as a/b, where a and b are integers.
- We can suppose a/b to be in lowest terms, which means that a and b have no common divisors.
- $a^2 = 2b^2$
- So, a^2 is even.
- Thus **a is even**, since only even numbers have even squares.
- So, a = 2c, for some c.
- $a^2 = 4c^2 = 2b^2$
- So, $b^2 = 2c^2$.
- Which means that **b is also even**.
- So a and b have been shown even, which contradicts our assumption that a/b is in lowest terms.
- *Tilt*

Aside on *reductio ad absurdum*

- Assume the opposite of what one wants to demonstrate, and show that it leads to a contradiction.
- Reductios assume bivalence.
- Some philosophers reject bivalence, and its object-language correlate called the law of the excluded middle:
  - Law of the excluded middle: $P \vee \sim P$
  - Intuitionists demand constructive proofs.

Hippasus

Metaphysics, Epistemology, Semantics

Metaphysics of Mathematics

- Metaphysics:
  - What is there?
  - What is it like?
- Realism: numbers exist, objectively
  - Plato, Descartes, Frege, Gödel, Quine
  - Sentence Realists vs. Object Realists
- Idealism: numbers are mental constructs
  - Kant, Brouwer, maybe Locke
- Nominalism: numbers do not exist
  - Denies that there are any types corresponding to number tokens, inscriptions.
  - Berkeley, Field
- I've focused here on numbers, but the same goes for sets, geometric or topological spaces, knots and graphs, i.e. all mathematical objects.

Necessity and Apriority

- The proof that $\sqrt{2}$ is irrational is *a priori*.
- It's also necessary that $\sqrt{2}$ is irrational.
- These aren't the same claim.
  - Apriority is about the acquisition and justification of our beliefs (i.e. epistemology).
  - Necessity is about their modal status (i.e. metaphysical).
- Nevertheless, they are often confounded.
  - Old view: anything believed *a priori* must be true.
- Consider Kant's claim that Euclidean space is the result of the a priori application of our concepts on the noumenal world.
  - Space is non-Euclidean
  - What seemed *a priori* turns out to be false.
  - On the old view, if a statement turns out false, it must never have been believed *a priori*.
  - Kant's entire metaphysical system depended on the application of *a priori* concepts to the noumenal world.
  - When space turned out to be non-Euclidean, Kant's system seemed to fall apart.
- Shapiro still calls apriority and necessity "twin notions" (23).
- Kripke: 'water is H2O' and 'the standard meter is one meter'.

The Fallibilist *A Priori*

- We can be wrong about a proposition, even if we hold it *a priori*.
- We can believe a proposition independently of experience, and still be wrong about that belief.
  - Cantor and Frege and the axiom of comprehension: every property determines a set
  - The set of all things that aren't woodchucks is too big.
- Had Kant held a fallibilistic *a priori*, he might have been able to salvage some of his work.
- The fallibilist can hold that statements believed on the basis of a priori reasoning are necessarily true, *if true*.
  - And if they are false, they are necessarily false.

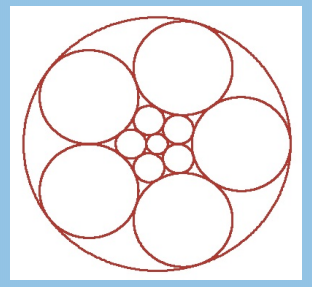
Analyticity

- Analyticity, a semantic notion, has been thought to explain apriority.
  - ‘Bachelors are unmarried’
  - ‘We walk with those with whom we stroll’
- Analyticity is about meanings of terms.
- Apriority is an epistemic notion, about belief and knowledge.
- Necessity is a metaphysical notion, about the nature of the universe, broadly conceived.
- Certainty is an epistemic notion, masquerading as a metaphysical notion.
  - I can be certain about something non-necessary, like that I am here now.
  - I can be uncertain about something necessary, like whether Goldbach’s conjecture is true.
  - Even Brown makes mistakes: 1. Mathematical results are certain.

Forward to the Past

The Greek Achievement in Mathematics

- Earlier or contemporaneous civilizations, like the Babylonians or the Egyptians, took mathematics to be all applied.
- Circles were pizzas and frisbees (and fields of wheat, say)
- The Pythagoreans recognized that realism about mathematics entails believing in an unseen world.
- They demanded proofs of mathematical theorems, rather than mere practical utility.



Kline's Chaos Claim:

The civilizations that preceded the Greek or were contemporaneous with it regarded nature as chaotic, mysterious, capricious, and terrifying (Kline, 146).

The Kline View

- The world, as we perceive it, is orderly and predictable.
  - The odd surprise (a tsunami, a terrorist attack) is always explicable, in hindsight.
- Kline implies that this order is unnatural.
- His claim is that the Greeks, through a variety of methods, tamed the chaos.
- The Ionians argued that all diversity is the result of different combinations of a few familiar substances, or even a single substance.
- Thales thought that everything is water.
- Democritus favored unseen atoms.
- Empedocles posited four elements: earth, air, fire, and water.
- The Pythagoreans: all things are numbers.
  - Numbers are to the Pythagoreans what atoms are to us.

The Pythagoreans Cult

1. To abstain from beans.
2. Not to pick up what has fallen.
3. Not to touch a white cock.
4. Not to break bread.
5. Not to step over a crossbar.
6. Not to stir the fire with iron.
7. Not to eat from a whole loaf.
8. Not to pluck a garland.
9. Not to sit on a quart measure.
10. Not to eat the heart.
11. Not to walk on highways.
12. Not to let swallows share one's roof.
13. When the pot is taken off the fire, not to leave the mark of it in the ashes, but to stir them together.
14. Do not look in a mirror beside a light.
15. When you rise from the bedclothes, roll them together and smooth out the impress of the body (Bertrand Russell, *A History of Western Philosophy* 31-2).

Concerns about Kline's Chaos Claim

- It's hard to see what it would mean to regard the world as chaotic, mysterious, capricious, and terrifying.
- Kant: we impose order on a noumenal world using our concepts.
- Perception is ordering.

Ontogeny, Phylogeny, and Developmental Analogies

- Kline's claim is that in seeing the world as mathematical, the Pythagoreans were able to turn a chaotic world into a tame one.
- Let's push on this analogy a little bit.
- The individual begins life by perceiving a chaotic world, and learns to order and organize that world around him/her.
- Perhaps, the baby is given a chaotic world.

The Four Piagetian Stages

1. Sensorimotor stage (birth - age 2): The child builds concepts about the external world and how it works, correlating sense experiences with external objects. The child lacks, and learns, object permanence.

2. Pre-operational stage (ages 2 - 7): The child is not able to think abstractly. The child lacks and learns conservation of quantity.

3. Concrete operations (ages 7 - 11): The child starts to reason logically about concrete events. Some limited abstract problem-solving is possible, but only applied to concrete phenomena.

4. Formal operations (ages 11 - 15): The child develops abstract reasoning.

Is the Metaphor Successful?

- Kline: the Greeks provided a rational view of nature.
- The mythology of earlier cultures makes life and death the whims of the gods.
- But, the gods are presumably rational, too.
  - Their reasons are just potentially hidden from our view.
- The Greeks provided scientific reasons, as opposed to mythological reasons.
  - Natural explanations in place of human rational ones.
  - Moving away from the anthropomorphicization of nature.
- They were not giving order to chaos.
- They were providing natural explanations where only mythic ones were available.

The Limits of Natural Explanation

- If we want to know why A fell off of a cliff, it is useful to know that B pushed her.
- It raises the question of *why* B pushed her.
  - We might find out that C pushed B.
  - And that D pushed C.
  - *Et cetera*.

Ultimate Causes

- In natural explanations, ultimate causes get pushed back, but are not disappeared.
- We do get an order to some portion of the universe, which may be Kline's point.
- But we don't have ultimate explanations, ones which trace back all the way.
- It is for that reason that some philosophers (and theologians) posit an uncaused cause, God.
- The cosmological argument for the existence of God says that natural explanations for events are ultimately incomplete, that we need to have some sort of rational explanation for the world.
- Some who favor natural explanations see the presumption of rational order underlying the cosmological argument as merely mythological.

What's the Point of Natural Explanations?

- What does it mean to have a sufficient (or complete) explanation?
- Scientific explanations tend to replace a visible world with a less-visible world.
  - Electrons
  - Quarks
  - Strings
  - Numbers
- The Pythagorean claim was that the less-visible world is mathematical in nature.
- Is there any sense that we can make of the claim that the world is mathematical?

Galileo's Pythagoreanism

Philosophy is written in this grand book of the universe, which stands continually open to our gaze. But the book cannot be understood unless one first learns to comprehend the language and to read the alphabet in which it is composed. It is written in the language of mathematics, and its characters are triangles, circles and other geometric figures, without which it is humanly impossible to understand a single word of it; without these, one wanders in a dark labyrinth *(The Assayer*, 1623).

Coulomb's Law

$$F = k\,|q_1 q_2|/\,r^2$$

Contemporary Pythagoreanism

arises out of Quine's indispensability argument

- We start thinking of bodies as physical objects, but these have vague boundaries, and puzzling identity conditions over time.
  - Am I the same person I was when I was younger, or the same person I will be later?
  - Is the cragged old tree the same as the small sapling?
  - We can take bodies to be composed of smaller particles.
  - We can think of the world as composed of four-dimensional aggregates of these atomic elements.
- Problems with atomism
  - Electrons have weird identity conditions.
  - It is arbitrary, at times, to say whether two point events are moments in the career of one electron, or two different ones.
- Field theory of distributions of states over space-time
  - electromagnetic fields, gravitational fields, etc.
  - The world as space-time points and their states
  - The objects are the space-time regions and their properties.
- We can identify space-time regions with Cartesian coordinates.

Vanishing Matter

Predicates that formerly attributed states to points or regions will now apply rather to quadruples of numbers, or to sets of quadruples... I seem to have ended up with this as my ontology: pure sets (Quine, “Whither Physical Objects”, 501-2).