Knowledge, Truth, and Mathematics

Philosophy 405
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Class #19: Field's Project

Three Worries About Indispensability

- 1. Holism is false
  - Sober’s foxes and chickens
  - Basic beliefs
- 2. Quine’s indispensability argument makes the justification of mathematical beliefs subordinate to the justification of empirical scientific beliefs.
  - “My view of pure mathematics is oriented strictly to application in empirical science. Parsons has remarked, against this attitude, that pure mathematics extravagantly exceeds the needs of application. It does indeed, but I see these excesses as a simplistic matter of rounding out...I recognize indenumerable infinites only because they are forced on me by the simplest known systematizations of more welcome matters. Magnitudes in excess of such demands, e.g., $\beth_{\omega}$ or inaccessible numbers, I look upon only as mathematical recreation and without ontological rights” (Quine on Mathematical Recreation).
  - Mathematical methodology is consistent across mathematical theories, no matter what the scientists do with the mathematical results.
- 3. Instrumentalism
  - Our theory may be mention objects to which we are not committed.
  - Center of mass
  - Asymptotic reasoning
  - Among those objects could be the mathematical objects.
- These worries are about the first three premises of QI.

QI.1: We should believe the theory which best accounts for our sense experience.
QI.2: If we believe a theory, we must believe in its ontological commitments.
QI.3: The ontological commitments of any theory are the objects over which that theory first-order quantifies.
QI.4: The theory which best accounts for our sense experience first-order quantifies over mathematical objects.
QI.C: We should believe that mathematical objects exist.

Field's Project

- Field's work is aimed at the fourth premise of QI.
- He denies QI4.
  - The theory which best accounts for our experience need not quantify over mathematical objects.
- If Field is right that QI4 is false, Quine's indispensability argument fails, and Benacerraf's dilemma re-emerges.
- At the heart of Field's project is his proposed nominalistic reformulation of Newtonian Gravitational Theory (NGT).
  - Replaces quantification over mathematical objects in NGT with quantification over space-time points or regions.

Formal Axiomatic Physical Theories

- Standard Theories
  - 1. A logical system, used for inference
  - 2. Mathematical axioms
  - 3. Scientific axioms
- Bridge functions relate the theorems of mathematics to the theorems of science.
  - measurements of quantities like mass and velocity
  - The speed of light $c = 3 \times 10^8$ cm/s^2
  - $G = 8 \pi T$, where G is the gravitational tensor and T is the stress-energy tensor
- Field produces representation theorems to show how the space-time points and regions can do the work that mathematical objects do in standard theories.

Fictionalism

- Mathematical terms are empty names.
- Mathematical sentences are either false, for existence claims, or vacuously true, for purely mathematical entailments.
- Field aims at the indispensability argument:
  - "The only non-question-begging arguments that I have ever heard for the view that mathematics is a body of truths all rest ultimately on the applicability of mathematics to the physical world" (Field viii).
  - "Nothing in this monograph purports to be a positive argument for nominalism. My goal rather is to try to counter the most compelling arguments that have been offered against the nominalist position" (Field 4).
- An Argument
  - FF1. We should take mathematical sentences at face value.
  - FF2. If we take (some of them) to be non-vacuously true, then we have to explain our access to them.
  - FF3. The only good account of access is the indispensability argument.
  - FF4. But, the indispensability argument fails.
  - FFC. So, we should take the non-vacuous ones to be false.

Reformulating Mathematical Theories

- Some nominalists try to re-interpret the mathematical axioms.
  - Benacerraf and the combinatorial views
  - Hilbert and inscriptions
  - possible inscriptions
- Such strategies give up standard semantics for mathematical propositions.
- Field’s work eliminates, rather than re-interprets, the mathematical axioms.
- He interprets the mathematical axioms standardly, and then claims that mathematical propositions are false.

Two Parts to Field's Project

- First, he develops a nominalist counterpart to a standard scientific theory
  - Remove the math.
  - "If one *just* advocates fictionalism about a portion of mathematics, without showing how that part of mathematics is dispensable in applications, then one is engaging in intellectual doublethink: one is merely taking back in one's philosophical moments what one asserts in doing science..." (Field 2).
  - Choose some arbitrary regions of space-time to serve as the bases for measurement in place of the real numbers.
  - Impose a structure, including measurement (greater and less) and order (betweenness), on space-time regions.
- Second, he argues that mathematics applies conservatively to nominalist theories, to assure us that nominalist counterparts are adequate substitutes.
  - Show that we don't need it.
  - He proves representation theorems to show that any uses of the real numbers can be replaced by the structure he describes for space-time.
  - The representation theorems are supposed to map space-time points onto R^4.
  - Field's representation theorems explain how mathematics can be useful, given a nominalist theory, by showing that statements which use mathematics are convenient shorthand for nominalistically acceptable sentences about space-time points.

Ground Rules One and Two

- GR.1: *Adequacy*: A reformulation must not omit empirical results of the standard theory.
- GR.2: *Logical Neutrality*: A reformulation must not reduce ontology merely by extending logic, or ideology.
  - First-order logic makes no commitments
  - Second-order logic is “set theory in sheep’s clothing” (Quine).
  - Modal logics make commitments to possible worlds.
    - Possible inscriptions
    - Possible arrangements of physical objects
  - “Avoidance of modalities is as strong a reason for an abstract ontology as I can well imagine” (Quine, “Reply to Charles Parsons” 397).
  - “[I]t can be seen that there is something dubious about the practice of just helping oneself to whatever logical apparatus one pleases for purposes of nominalistic reconstruction while ignoring any customary definitions that would make the apparatus nominalistically unpalatable: for by doing so, one can make the task of nominalistic reconstruction absolutely trivial – and so absolutely uninteresting” (Burgess and Rosen, *A Subject with No Object* 175).

Ground Rule Three

- GR.3: *Conservativeness*: The addition of mathematics to the reformulated theory should license no additional nominalist conclusions.
  - Deductive conservativeness: mathematics does not allow new theorems to be derived from the nominalistic theory.
  - Semantic conservativeness: no additional statements come out true in any model of the theory which includes mathematics.
- The definition of conservativeness
  - Let A be any nominalistically statable assertion, N any body of such assertions, and S any mathematical theory.
  - Take 'Mx' to mean that x is a mathematical object.
  - Let A* and N* be restatements of A and N with a restriction of the quantifiers to non-mathematical objects.
  - S is conservative over N* if A* is not a consequence of N*+S+'∃x~Mx' unless A is a consequence of N.
- "Standard mathematics *might* turn out not to be conservative...for it might conceivably turn out to be inconsistent, and if it is inconsistent it certainly isn't conservative. We would however regard a proof that standard mathematics was inconsistent as extremely surprising, and as showing that standard mathematics needed revision. Equally, it would be extremely surprising if it were to be discovered that standard mathematics implied that there are at least 10^6 non-mathematical objects in the universe, or that the Paris Commune was defeated; and were such a discovery to be made, all but the most unregenerate rationalist would take this as showing that standard mathematics needed revision. *Good* mathematics *is* conservative; a discovery that accepted mathematics isn't conservative would be a discovery that it isn't good "(Field 13).

The Importance of Conservativeness

- First, it serves as a check on the adequacy of the nominalist reformulation.
  - If mathematics does not apply conservatively to NGT*, then the standard theory will yield more consequences.
  - The nominalist theory will be shown to omit some theorems of the standard theory.
- Second, conservativeness provides an account of the applicability of mathematics to science.
  - Why is mathematics useful?
- Field's project is especially alluring.
  - He does not merely eliminate mathematics from scientific theory.
  - He attempts to show that our ordinary uses of mathematics are consistent with nominalist principles.

Attractiveness: A Fourth Ground Rule

- GR.4: *Attractiveness*: The dispensabilist must show, "[T]hat one can always reaxiomatize scientific theories so that there is no reference to or quantification over mathematical entities in the reaxiomatization (*and one can do this in such a way that the resulting axiomatization is fairly simple and attractive*)." (Field viii, emphasis added)
  - Subjective?
  - Few axioms
  - Elegant proofs
  - Hilbert and Birkhoff
- A Bad Theory: All and only the nominalistic consequences of standard science
  - will not reduce diverse experiences to a few, simple axioms
- "If no attractiveness requirement is imposed, nominalization is trivial... Obviously, such ways of obtaining nominalistic theories are of no interest" (Field 41).

Purpose of the Reformulation

- Useful theory?
  - But, the standard theory regimented in first-order logic is similarly unacceptable.
  - Conservativeness
  - GR.4, in this sense, is too strong a requirement.
- Explanatory Strength?
  - "The elimination of numbers [from science], unlike the elimination of electrons, helps us to further a plausible methodological principle: the principle that *underlying every good extrinsic explanation there is an intrinsic explanation*. If this principle is correct, then real numbers (unlike electrons) have *got* to be eliminable from physical explanations, and the only question is how precisely this is to be done" (Field 44).
  - No theory regimented into first-order logic can be explanatory, since it can not be perspicuous.
  - The original un-regimented theory is the one which does all the explanatory work.
- Reduce the laws to a neat and tidy few axioms.
- Translate axioms of standard science directly into nominalist language.

Worries About Field's Project I: Space-Time Points

- There are two ways in which mathematics might be said to remain in Field's nominalist theory.
- First, we might claim that space-time points are really mathematical objects.
  - Hartry Field has claimed...that one can do physics without reference to abstract entities. But his construction requires that we accept *absolute space-time points* and arbitrary sets of space-time points as 'concrete'; most philosophers (including myself) would regard this as 'cheating' (Putnam 1981: 175).
  - Owing to the richness of Field's physical ontology, philosophers in Quine's tradition might object that Field has just hidden his mathematical objects in physical disguises (Resnik 1985b: 192).
  - [In space-time, we have] everything essential to the real numbers (Maddy 1990b: 201).

Against Worry I

- We use space-time points in field formulations of physical theories.
- They are empirical posits, as opposed to *a priori* ones.
  - "Perhaps it is a bit odd to use the phrase 'physical entity' to apply to space-time points. But however this may be, space-time points are not abstract entities in any normal sense. After all, from a typical platonist perspective, our knowledge of mathematical structures of abstract entities (e.g. the mathematical structure of real numbers) is *a priori*; but the structure of physical space is an empirical matter" (Field 31).
- Field even argues that we have sensory access to space-time points.
  - "For there are quite unproblematic physical relations, viz., spatial relations, between ourselves and space-time regions, and this gives us epistemological access to space-time regions. For instance, because of their spatial relations to us, certain space-time regions can fall within our field of vision" (Field 1982a: 68).
  - Sensory access to empty regions is implausible.
  - Worries about access to space-time points are moot.

Worries About Field’s Project II: Ideology

- A second about what remains in the reformulated theory: the ideology that goes with both mathematics and Field’s nominalist theory is what really matters to mathematics.
- Both Field’s space-time ontology and the standard mathematical ontology contain continuous objects.
- Both kinds of objects satisfy the mathematical properties required by science.
- But:
  - “Postulating physical space isn’t like postulating real numbers...the ideology that goes with the postulate of points of space is less rich than that which goes with the postulate of the real numbers” (Field 32).

Worries About Field's Project III: Are Reformulations Available?

- Classical Hamiltonian mechanics, a phase-space theory
- Quantum mechanics
- General relativity
  - There is no available suitable version of the curved space-time geometry.
- Statistical inference
  - Psychology and economics
  - Physics may be easier to nominalize, just because it is so mathematized.
- Speculation
  - We do not know the nature of our ideal physical theory.
  - And we do not know what kinds of dispensabilist techniques may be developed.
  - "As a consequence of nominalism's being mainly a philosopher's concern, this open research problem is...one that has so far been investigated only by amateurs - philosophers and logicians -not professionals - geometers and physicists; and the failure of amateurs to surmount the obstacles is no strong grounds for pessimism about what could be achieved by professionals" (Burgess and Rosen 1997: 118).

Fictionalism and Mathematical Knowledge

Field accounts for what is ordinarily seen as mathematical knowledge in three ways.

1. Much of what we take to be mathematical knowledge is really empirical knowledge.
   - Of space-time
   - Of which axioms are generally accepted by the mathematical community
2. For limited results, there is no knowledge to be had.
   - no physical facts
3. We have logical knowledge of which theorems follow from which axioms.
   - “[Y]ou don’t need to make mathematics actually be about anything for it to be possible to objectively assess the logical relations between mathematical premises and mathematical conclusions” (Field 1998a: 317).