Philosophy 405: Knowledge, Truth and Mathematics | Hamilton College
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Class #18: The Indispensability Argument
Quine, "On What There Is"
Quine, "Existence and Quantification"
Quine on Recreation

I. A New Path to Platonism

Benacerraf's problem was that standard semantics for mathematical claims seem incommensurable with our best epistemology.
Standard readings of mathematical claims entail the existence of mathematical objects.
But our best epistemic theories seem to debar any knowledge of mathematical objects.
Thus the philosopher of mathematics faces a dilemma: either abandon standard readings of mathematical claims or give up our best epistemic theories.
Neither option is attractive.
The indispensability argument may be seen as a way out of Benacerraf's dilemma.
The indispensabilist hopes to avoid Benacerraf's dilemma by showing that our best epistemology is consistent with standard, platonistic readings of mathematical claims.

> Ordinary interpreted scientific discourse is as irredeemably committed to abstract objects - to nations, species, numbers, functions, sets - as it is to apples and other bodies. All these things figure as values of the variables in our overall system of the world. The numbers and functions contribute just as genuinely to physical theory as do hypothetical particles ("Success and the Limits of Mathematization 149-50).

Quine's route to platonism thus differs from that of the traditional rationalist.
Descartes and Leibniz defend innate ideas.
Gödel presents an argument for a platonism based on mathematical intuition, insight into the structure of the mathematical universe analogous to sense perception.
Quine's argument eschews appeal to the rationalists' innateness or intuition.
Michael Resnik, presenting an alternative version of the indispensability argument, calls it an empiricist epistemology for a platonist ontology.

Quine never presents the indispensability argument in canonical form and there are lots of different versions of the indispensability argument.
Quine's version is the original and, in my opinion, the most important.
But we must reconstruct his argument.
QI is my version.

QI
- QI1. We should believe the theory which best accounts for our sense experience.
- QI2. If we believe a theory, we must believe in its ontological commitments.
- QI3. The ontological commitments of any theory are the objects over which that theory first-order quantifies.
- QI4. The theory which best accounts for our sense experience first-order quantifies over mathematical objects.
- QIC. We should believe that mathematical objects exist.

I have more on QI in Chapters Two and Three of my book draft and in my article in the IEP.

The goal of "On What There Is" is to defend a method for determining one's commitments, or at least for expressing them or knowing what the commitments of a theory are.
This method is central to the indispensability argument and it appears in QI3.
"On What There Is" can be seen as a companion piece to "Two Dogmas of Empiricism".
In "Two Dogmas," Quine argues against the traditional empiricist's reductive method of determining ontological commitments.
In "On What There Is," Quine argues for his new method, involving the construction of a best theory of our sense experience and the modeling (or interpretation) of that theory.
The indispensability argument is a corollary of the argument for Quine's new method.

II. Meaning, Naming, and the Problem of Non-Being

In "On What There Is," Quine leads to the discussion of his positive method from the earlier part of the article in which he establishes two results.
First, there is a difference between meaning and naming.
Second, one can accept that terms and statements are meaningful without inferring that there must be objects called meanings.

This second view, called meanings skepticism or meanings nihilism, is important in the philosophy of language.
Quine rejects the abstract objects of semantics that Carnap defended (at least internally): propositions, properties, and meanings.
Quine's objections are not to their abstractness, but to the lack of proper identity conditions for their existence.
Propositions and meanings, like modal properties, are called intensions.
Sets, in contrast, are extensions.
The identity conditions for sets depend only on their members.
Two sets are identical if they have the same members and different if they have different members.
How sets are described or conceived is irrelevant to their existence.
Some terms with the same extensions have different intensions.
The objects to which 'creature with a heart' and 'creature with a kidney' refer are the same.
The terms are extensionally equivalent.
But they are intensionally distinct.
Quine rejects intensional objects.
But he is an extensionalist about abstract objects, not a nominalist.
He is happy to admit abstract objects, like sets, as long as they are extensional.
I discuss Quine's arguments against intensions the philosophy of language course, but I put them aside here.

Quine motivates the distinction between meaning and naming with a classic if overlong discussion of the problem of non-being.
The problem arises in connection with the semantics of both proper names and definite descriptions.
Names are used to pick out a particular object, often a person.
'Couper Hall' is a name, just as 'Barack Obama' is.
Definite descriptions are descriptive phrases which are supposed to pick out a unique individual, like 'the king of America', or 'the man in the corner drinking champagne', or 'the teacher of this class'.
Like names, definite descriptions are used to pick out one specific thing.

If one thinks that the meaning of a name or definite description is the thing to which it refers, it seems that one can not state N, or any equivalent, without committing oneself to the existence of Pegasus.

N There is no such thing as Pegasus.

If Pegasus does not exist, in at least some sense, then I can not deny that it exists.
I can not say something about nothing!
I am talking about a particular thing, so it has to have some sort of being.

Since there is no thing Pegasus, if we need a referent for the term we must mock-up something.
McX appeals to the idea of Pegasus as the referent of my term.
If 'Pegasus' refers to my idea of Pegasus, then N claims that my idea is not instantiated in the world.
McX's attempted solution, as Quine points out, demonstrates a basic confusion of ideas and objects.
'I am a philosophy teacher' refers to an object (me), not an idea.
'Pegasus is a winged horse' has the same grammatical structure.
'Pegasus' does not refer to an idea, on standard semantics, just as 'five' does not refer to my idea of five.
As Benacerraf argues, we need good reasons too abandon a uniform semantic account.
Such reasons are absent in this case.
'Pegasus' refers to a fictional object, not to an idea.

> McX would sooner be deceived by the crudest and most flagrant counterfeit than grant the nonbeing of Pegasus ("On What There Is" 2).

Wyman, who represents early Russell or Meinong but who also bears some resemblance to Carnap, gives Pegasus a second-class kind of existence.
He distinguishes between existence and subsistence.
All names or definite descriptions of possible objects refer to subsistent objects.
Only some of those terms refer to existent objects.
The idea of having two kinds of existence recalls Carnap's claims about internal and external questions.
We can speak as if Pegasus exists as an internal statement within the mythological framework and speak as if the question of Pegasus's existence is meaningless when wondering whether to adopt that framework.
Such double-talk is, for Quine, unacceptable.
It fails to do justice to our conventional uses of language.

> Wyman...is one of those philosophers who have united in ruining the good old word 'exist' ("On What There Is" 3).

Worse, for Wyman's account, is the claim that some well-formed phrases of English will lack meaning.
Definite descriptions of impossible objects, like the round square cupola, have to be meaningless, since they can have no sort of existence.
The claim that certain grammatical strings are meaningless rankles Quine.

> Certainly the doctrine has no intrinsic appeal... ("On What There Is" 5).

If we take 'round square' to be meaningless even though 'round' and 'square' are meaningful, we seem to be forced to abandon the compositionality of meaning, that the meanings of longer strings of our language are built out of the meanings of their component parts.

But worst, as far as Quine is concerned, is Wyman's multiplication of intensional entities.
In part, Quine's objection to possible objects is Ockhamist.
But his real argument against possible objects comes from his demand for identity conditions for any posit.

> Wyman's overpopulated universe is in many ways unlovely. It offends the aesthetic sense of us who have a taste for desert landscapes, but this is not the worst of it. Wyman's slum of possibles is a breeding ground for disorderly elements. Take, for instance, the possible fat man in that doorway; and again, the possible bald man in that doorway. Are they the same possible man, or two possible men? How do we decide? How many possible men are there in that doorway? Are there more possible thin ones than fat ones? How many of them are alike? Or would their being alike make them one? Are no *two* possible things alike? Is this the same as saying that it is impossible for two things to be alike? Or, finally, is the concept of identity simply inapplicable to unactualized possibles? ("On What There Is" 4).

Quine prefers dispensing with modal notions altogether, though.
He suggests that modalities conceived as sentential operators, rather than as modifiers of objects, are less objectionable.
Metaphysics aside, the moral of the discussion of McX and Wyman is methodological: there is a difference between meaning and naming.
Terms may be meaningful without being names or referring to things.
Quine's distinction between meaning and naming has its roots in Frege's distinction between sense and reference, though Quine denies the existence of senses, which are intensions.
Frege had argued that terms have both sense and reference.
The sense of a name is the mode of presentation of the object, the set of properties we associate with the name.
The referent of a name is the thing which the name picks out.
So, 'Russell Marcus' has a sense, though one that is different for everyone who thinks of me.
But whatever sense we associate with the name, it has the same reference.
Definite descriptions also have senses, depending on the meanings of the terms used, and references.
Empty names, like Pegasus, and non-referring definite descriptions, like 'the king of America', can lack referents and still have meaning.
So Quine and McX and Wyman can all agree that the name 'Pegasus' is meaningful and deny that it has a referent without committing themselves to the existence, in any way, of some thing called Pegasus.

The broader point is that we should not look to names to find our ontological commitments.
Names can have referents or not.
Furthermore, a result from mathematics, with which you are already familiar, also show that names can not express our ontology.
From Cantor's diagonal argument, we learned that there are more real numbers than names.
That argument does not conclude that there aren't real numbers.
It just shows, as does Quine's argument against McX and Wyman, that we should not look to names for our ontology.

The problem of non-being is just one example of the unclarity and ambiguity of natural language.
There are others.
Elsewhere Quine discusses uses of other terms which might mislead us, if we were very naive, to an errant metaphysics.

Ryle, cited in Carnap's article, complained about mistaken reification in mathematics.
Ryle rejects standard semantics to argue that our belief in sentences like 'there are two prime numbers between four and eight' should not commit us to mathematical objects.
But using mathematical examples to show that certain terms should be taken as non-referring is question-begging here, where the central question is whether to believe that there are mathematical objects.
Quine shows that there are uncontroversial cases in which we paraphrase away offending nouns.
For example, consider 'I slew a Jabberwock for Julie's sake'.
Grammatically, we are treating Julie's sake as a thing.
But we don't take ourselves to be committed to sakes.
Thus Benacerraf's counsel to prefer a standard semantics, one which follows the grammar of natural language, should not be taken as an absolute requirement on interpreting our language.
All things being equal, we should prefer standard semantics.
But if we have over-riding reasons to disavow a commitment that a standard semantics would imply, we should re-write our language.
Quine's counsel that we should rewrite natural language to avoid errant commitments is just a return to Frege's project of providing a precise language for the expression of our most sincere beliefs.
The question remains whether to believe in the existence of mathematical objects.

Russell made a significant advance in Frege's formalizing project by presenting the theory of definite descriptions to deal with failures of presupposition.
Consider the following two claims:

B The king of America is bald.
NB The king of America is not bald.

Neither B nor NB is a true proposition.
But, NB looks like the negation of B.
B looks to have the same form as DB.

DB Devendra Banhart is bald.

The falsity of DB entails that Devendra Banhart has hair.
But we don't want to conclude from the falsity of B that the king of America has hair.
The problem is that in this case we want both a proposition and its negation to be false.
If we take 'the king of America' as a name, then B is regimented as 'B_k' and NB as '$\sim B_k$'.
Then, the conjunction of the negations of B and NB is a contradiction:

$\sim Bk \bullet \sim\sim Bk$

Thus, we can never deny both a statement and its negation, as we wish to do with B and NB.

B and NB are both false because they both contain a false presupposition.
Perfectly grammatical English sentences can contain failures of presupposition.

The woman on the moon is six feet tall.
The rational square root of three is less than two.
When did you stop beating your wife?

Russell proposes that such sentences are more complex than their grammar shows.
We can unpack them.
B entails three simpler expressions:

B1	There is a king of America.	$(\exists x)Kx$
B2	There is only one king of America.	$(\forall y)(Ky \supset y=x)$
B3	That thing is bald.	Bx

So, B is false because B1 is false.
NB is also false, for a parallel reason.

Russell's claim, which Quine accepts, is that the logical form of a sentence is not the same as its grammatical form.
Our decisions about how to construct the logical form of a sentence will be guided by our desires for precision in the formal language.

Russell's theory applies to definite descriptions.
Quine adopts Russell's analysis, and urges that we can extend it to eliminate names.
We can introduce predicates which hold of exactly one person, or one object.

> We could [appeal] to the *ex hypothesi* unanalyzable, irreducible attribute of *being Pegasus*, adopting, for its expression, the verb 'is-Pegasus' or 'pegasizes'. The noun 'Pegasus' itself could then be treated as derivative, and identified after all with a description: 'the thing that is-Pegasus', 'the thing that pegasizes' ("On What There Is" 8).

For definite descriptions, we re-cast sentences in the way that Russell does.
For names, we introduce unique predicates.
Our language then contains only quantifiers, predicates, bound variables, and logical connectives.

> Whatever we say with the help of names can be said in a language which shuns names altogether ("On What There Is" 13).

Quine's arguments against taking names to indicate ontological commitments avoid the traditional debate over names.
In the traditional debate, the question is whether objects are merely collections of properties or whether they have haecceity (thisness).
Quine's arguments are methodological rather than metaphysical.
He is looking for the most elegant language in which to formulate our theories.
His claim is that theories which locate their commitments in names are less attractive than theories which rely on quantification.
Quine has other arguments for using first-order logic.
Some of them rely on technical virtues of first-order logic like completeness.
Some of them depend on the elegance of unifying a theory's referential apparatus in the quantifiers.
We will not consider those arguments here.

Here is one last argument against names, not found in Quine's work, but in his idiom nonetheless.
Given the existence of names in first-order logic, we can prove the existence of God.
Take the constant 'g' to stand for God.

	1. $\sim(\exists x)x=g$	Assumption, for indirect proof
	2. $(\forall x)x=x$	Principle of identity
	3. $(\forall x)\sim x=g$	1, Change of quantifier rule
	4. $g=g$	2, UI
	5. $\sim g=g$	3, UI
6. $(\exists x)\ x=g$		1-5, Indirect proof

QED

Quine takes two morals from the discussion of names.
The first is merely formal: we should avoid names in our best theories.
The second is metaphysical: our decisions about how to construct our formal languages should both be guided by and guide our beliefs about what exists.

III. Existence and Quantification

Quine's main argument against both Wyman's intensional profligacy and McX's conceptualist confusion consists of his positive account of how to deal with names and definite descriptions which lack referents and how to deal with debates about existence claims generally.
If I claim that electrons exist, I should be able to demonstrate how I discovered them, or how I posited them, or how their existence was revealed to me.
If you deny my claim that the tooth fairy exists, you might appeal to the fact that we have no sense experience of a tooth fairy.
To resolve such disputes about what exists, we need to agree on a method to determine what exists.

Quine urges semantic ascent, or "withdrawing to a semantical plane" ("On What There Is" 16).
We can talk about sentences in order to resolve ontological disputes rather than getting mired in confusions about whether terms must refer in order to have significance.
We can find common ground in discussing which sentences one person accepts and another denies.
I accept TF.

TF The tooth fairy exists.

You reject TF.
Our discussion of the disagreement can center around TF, and not around the tooth fairy.

Quine further argues that the least controversial and most effective way of formulating a theory is to put it in the language of first-order logic.
Then our metaphysics reduces to a process of interpreting our first-order theory.
We interpret a first-order theory by specifying a domain of discourse, a set of objects over which the quantifiers range.
We assign values to variables in order to model the theory, providing an interpretation which makes the sentences of the theory come out true.
Our metaphysics is the simple byproduct of modeling the theory.

> To be assumed as an entity is, purely and simply, to be reckoned as the value of a variable ("On What There Is" 13)

Variables in first order logic work like pronouns in natural language: this, that, something.

> We can very easily involve ourselves in ontological commitments by saying, for example, that *there is something* (bound variable) which red houses and sunsets have in common; or that *there is something* which is a prime number larger than a million. But this is, essentially, the *only* way we can involve ourselves in ontological commitments: by our use of bound variables ("On What There Is" 12).

The question of whether numbers exist, then, becomes a question about whether we quantify over them when our language is made most precise and formalized into first-order logic.
Existence questions become questions about how best to write one's best theory.

IV. Theory Construction

We must be careful not to read too much into Quine's criterion for determining what exists, his appeals to first order logic.
As Quine warns, his criterion does not turn metaphysical questions into semantic ones.

> How are we to adjudicate among rival ontologies? Certainly the answer is not provided by the semantical formula "To be is to be the value of a variable"; this formula serves rather, conversely, in testing the conformity of a given remark or doctrine to a prior ontological standard. We look to bound variables in connection with ontology not in order to know what there is, but in order to know what a given remark or doctrine, ours or someone else's, *says* there is; and this much is quite properly a problem involving language. But what there is is another question ("On What There Is" 15-16).

The question of whether mathematical objects exist is the question of how to specify the prior ontological standard.
In early work such as "On What There Is", Quine was mainly interested in establishing the proper methodology.

> We adopt, at least insofar as we are reasonable, the simplest conceptual scheme into which the disordered fragments of raw experience can be fitted and arranged. Our ontology is determined once we have fixed upon the over-all conceptual scheme which is to accommodate science in the broadest sense... ("On What There Is" 16-17).

We construct a theory of our sense experience.
Then we look at the theory and decide what values it takes for its bound variables.
The values of the bound variables are what the theory presupposes.
These are the posits, the postulated entities, of the theory.
Quine, in early work, calls them myths, as we saw in Two Dogmas.
His point in calling them myths is to emphasize that they are the result of our choice of a theory.
Our central metaphysical tasks are to construct a theory and to see what it says exists.
This is just the proper methodology and not intended to denigrate the objects posited.

> To call a posit a posit is not to patronize it (*Word and Object* 22).

We don't first commit to objects and then figure out how to make the theory say that they exist.
We choose among our theories according to their immanent virtues: simplicity, elegance, utility, explanatory strength.

Sense experience serves as the evidence for our theory.
Quine calls our experiences boundary conditions for the construction of a theory.
But sense experience under-determines any theory.
There are various theories which will fit the same experiences.
We choose among competing empirically equivalent theories according to their formal characteristics, their immanent virtues.

One consequence of the rejection of reductionism we saw in Two Dogmas is that the statements of any theory are interconnected.
When we tinker with our theory in response to new experiences, we can adjust any theory in various ways.
All evidence is evidence for the theory as a whole, not for individual statements.
The evidence is all the stimulation of our sense organs.
The posits come out all together as values for the variables.
All posits, including mathematical ones, come out of the same best theory.

> The considerations which determine a reasonable construction of any part of that conceptual scheme, for example, the biological or the physical part, are not different in kind from the considerations which determine a reasonable construction of the whole ("On What There Is" 17).

Quine's method serves as a rejection not only of the reductionism of the logical empiricists, but of their willingness to speak equivocally about mathematics and empirical science.
Quine agrees with Carnap's claim that metaphysical claims, from an internal perspective, are trivial.

> One's ontology is basic to the conceptual scheme by which he interprets all experiences, even the most commonplace ones. Judged within some particular conceptual scheme - and how else is judgment possible? - an ontological statement goes without saying, standing in need of no separate justification at all ("On What There Is" 10).

Further, Quine agrees with Carnap that we can choose among various theories or conceptual schemes.
He disagrees with Carnap's characterizations of the choices among conceptual schemes.
If, with Carnap, we say that numbers exist (internally) while denying, at the same time, that "numbers exist" is meaningful, we are contradicting ourselves.
Carnap's physicist uses mathematics in his work but denies the existence of mathematical objects.
Unlike Carnap, who dismisses the problem, Quine takes the physicist's guilty conscience seriously.
He accuses such a view of intellectual dishonesty.

> For us common men who believe in bodies and prime numbers, the statements "There is a rabbit in the yard" and "There are prime numbers between 10 and 20" are free from double-talk. Quantification does them justice ("Existence and Quantification" 99).

Quine's early focus on his method of theory construction and positing is clear in his agnosticism between

physicalism and phenomenalism.
According to physicalists, the ultimate constituents of the world are physical objects.
That is, the values of the bound variables of the theory which best explains our sense experience are things like electrons and quarks and photons.
According to phenomenalists, the ultimate constituents of the world are sense data.
The bound variables of the best phenomenalist theory, as Carnap tried to construct in the *Aufbau*, are reports of sense experience themselves.

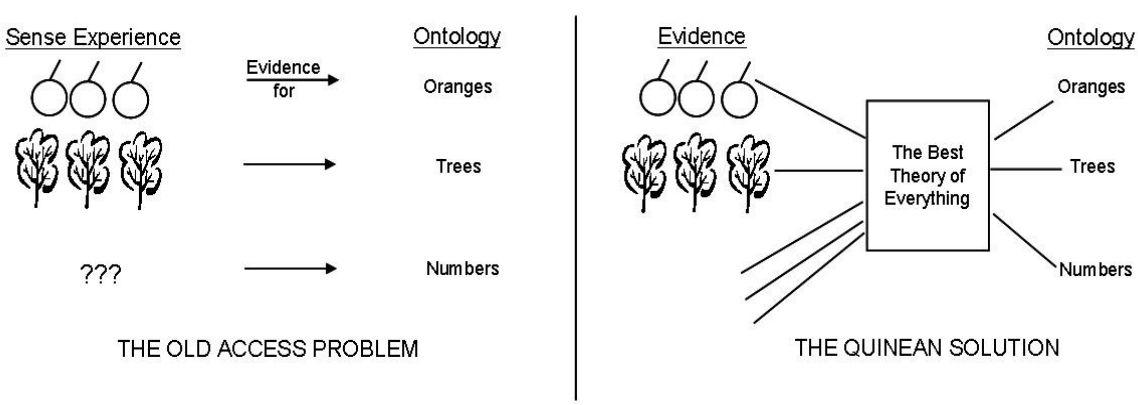
> Here we have two competing conceptual schemes, a phenomenalistic one and a physicalistic one. Which should prevail? Each has its advantages; each has its special simplicity in its own way. Each, I suggest, deserves to be developed. Each may be said, indeed, to be the more fundamental, though in different senses: the one is epistemology, the other physically, fundamental ("On What There Is" 17).

Later, as in "Success and the Limits of Mathematization", Quine was clear that he believes that the best theory is physical theory.
Physical science requires mathematics as well as other objects which we can not directly sense.

The indispensability argument is thus a combination of Quine's commitment to a single, best theory, that of physical science, and his method for determining the commitments of that theory, which involves regimenting that theory into quantificational logic and looking at the values of the bound variables.

V. Indispensability and Access

Quine, like his empiricist predecessors, sought the best theories for explaining our sense experience.
Unlike traditional empiricists, he does not reduce all claims of existence directly to sense experiences.
Traditional empiricists are burdened with an access problem: how can we justify beliefs in objects unavailable to our senses?
The access problem is the source of the epistemological branch of Benacerraf's problem.
Benacerraf argued that knowledge of mathematical objects seemed debarred by our limited epistemic capacities: we have no causal (or otherwise reliable) access to abstract objects.
Quine's method avoids the access problem by denying the possibilities of reduction.
Our best epistemology is just figuring out how best to construct and interpret scientific theory.



Thus, according to Quine, there is nothing preventing us from having both a standard semantics and our best epistemology.
The question is whether the indispensability argument is successful in the goals Quine has for it.

VI. Indispensability's Minor Premise

Most discussion of the indispensability argument has centered not on its revolutionary major premise, that if mathematical objects are required for the construction of scientific theory then we should believe that they exist.
Instead, the debate following Quine's work has focused on its minor premise, that we do indeed need mathematical objects to form our best scientific theory.
In his earliest work, Quine believed that we did not need mathematics, or at least that scientific theories could be constructed without referring to mathematical objects.
We could have competing scientific theories regarding the uses of mathematics, just as phenomenalistic and physical theories were portrayed as competing in "On What There Is."

> Let us see how, or to what degree, natural science may be rendered independent of platonistic mathematics; but let us also pursue mathematics and delve into its platonistic foundations ("On What There Is" 19).

In 1947, he wrote "Steps Toward a Constructive Nominalism" with Nelson Goodman, describing how one could eliminate mathematical objects from our best discourse.
But he abandoned that project for the stronger defense of platonism we see in his later work.
Next week we will look at the most prominent response to Quine's indispensability argument, Hartry Field's dispensabilist project in *Science without Numbers*.

VII. A Few Worries, Very Briefly

I'll mention three concerns I have about the indispensability argument.
First, central to Quine's argument is his argument against the logical empiricists' reductionism.
In response, Quine presents a view, called confirmation holism in which all the statements of a theory are confirmed and disconfirmed together.
As Quine puts it in "Two Dogmas," the unit of empirical significance is the theory as a whole.
On confirmation holism, when we test an empirical theory, all the elements of the theory are tested.
If we find that there is a problem in the theory, if it, say, predicts something that doesn't happen, we have to revise the theory.
On holism, we have free choice about which elements to revise.
If we find that five pairs of socks yield nine socks, we can always give up the mathematics rather than our observational claims.

Quine's point, as a logical matter, is correct.
We can adjust either our mathematics or our observational claims to restore consistency when we lose it.
But this point may be misleading.
In practice, we always protect mathematical statements from revision on empirical grounds.
That protection is the point of Elliot Sober's foxes and chickens example.
Quine argues that our protection of mathematics is a decision, not an indication that holism is false.

He presents a metaphor of a web of belief.
Our best theory is like a web of interconnected statements.
Certain beliefs, like the logical and mathematical ones, are central to the web.
Giving them up forces us to revise too many other strands.
In contrast, particular observational beliefs lie on the periphery of the web.
We can cede them without changing other strands.
So our choices of which beliefs to ceded will be guided by pragmatics of theory construction.
In practice we don't give up mathematical beliefs.
But that practice, Quine says, is no indication that the mathematical beliefs are any different in kind from our observations.

Second, Quine's indispensability argument makes the justification of mathematical beliefs subordinate to the justification of empirical scientific beliefs.
Only a small portion of mathematics is actually used in science.
Beliefs in the applied portions of mathematics may be justified by the indispensability argument.
But beliefs in those portions, like almost all of transfinite set theory, that are not used in science seem unjustifiable by indispensability considerations.

> My view of pure mathematics is oriented strictly to application in empirical science. Parsons has remarked, against this attitude, that pure mathematics extravagantly exceeds the needs of application. It does indeed, but I see these excesses as a simplistic matter of rounding out...I recognize indenumerable infinites only because they are forced on me by the simplest known systematizations of more welcome matters. Magnitudes in excess of such demands, e.g., $\beth_\omega$ or inaccessible numbers, I look upon only as mathematical recreation and without ontological rights (Quine on Mathematical Recreation).

Quine's distinction between legitimate and recreational mathematics might bring to mind Hilbert's distinction between real and ideal elements of mathematical theories, though the lines are drawn in different places.
Still, from the point of view of the mathematician there is no difference between applied and unapplied portions of mathematics.
Mathematical methodology is consistent across mathematical theories, no matter what the scientists do with the mathematical results.

Last, there are alternatives to Quine's definition of ontological commitment, even accepting holism.
We might be instrumentalist about some of our quantifications.
Consider a center of mass.
Our best theory might analyze a physical system in terms of its center of mass.
We know that the center of mass, for some systems, does not even lie on the object.
Consider a system of two springs tied together and tossed in the air.
As the springs move, the center of mass shifts around the object.
To describe the motion of the system, we refer to the center of mass even if it is not on the object.
We accept some idealizations and some instrumentalist posits, for the purposes of theoretical or pragmatic simplicity.
So, our theory may be committed, in the formal sense, to objects to which we are not committed.
Among those objects could be the mathematical objects.

Check out this quiz on "On What There Is."