Knowledge, Truth, and Mathematics

Philosophy 405
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Class #15: Conventionalism

A Little (Summery?) Summary

Philosopher	Is Mathematics Analytic or Synthetic?	Math Knowledge	Are Mathematical Truths Necessary?	What are Mathematical Objects?	The Infinite
Frege's Logicism	Arithmetic is analytic; geometry is synthetic.	All mathematical knowledge is *a priori* logic.	Yes	Platonic entities, but logical constructions	Accepts the actual infinite
Brouwer's Intuitionism	Synthetic	Synthetic *a priori*, constructed out of our intuition of time	Yes; disproving mathematical truths is unthinkable	Mental constructions	Accepts only a potential infinite; nothing beyond ω
Hilbert's Formalism	Analytic for ideal elements; synthetic for real elements	Proofs are known *a priori* (logic); elementary truths are known by intuition.	Yes; and all mathematical truths are provable	Finite mathematics is about stroke symbol sequences; Infinite mathematics is about nothing	Accepts the infinite as an ideal, meaningless element, useful for deriving further finitary mathematical results

A Fourth School

Conventionalism

- Held by logical empiricists (logical positivists).

- Wittgenstein's *Tractatus Logico-Philosophicus*

- Vienna Circle
  - ‣ Rudolph Carnap
  - ‣ Otto Neurath
  - ‣ Moritz Schlick
  - ‣ Herbert Feigl

- Berlin Circle
  - ‣ Hans Reichenbach

- A.J. Ayer, from England

Logical Empiricism

- British empiricism (more Hume than Locke) plus logic

- Seeking a systematic foundation for our scientific beliefs that relied on sense experience.

- Hume and Locke imagined that all knowledge could be traced to sense experience.

- The logical empiricists tried actually to trace construction of science from sense data.

- Carnap's 1928 *The Logical Structure of the World* or *Aufbau*
  - ‣ Student of Frege's in Jena
  - ‣ Attempts to develop scientific theory out of sense-data, using the tools of Fregean logic.

- Wittgenstein and the logical empiricists were responding in large part to Hegelian idealism and speculative metaphysics generally.

- Like Hume, they were intent on ridding philosophy of what they deemed to be pseudo-problems, pseudo-questions, meaningless language, and controversial epistemology.

The Verification Theory of Meaning

- Hume: for a term to be meaningful, it has to stand for an idea in one's mind that can be traced back (in some sense) to an initial sense impression.
- Verification theory
  - ‣ For a sentence to be meaningful, it must be verifiable on the basis of observation.
- All our justifiable claims are traceable to a core set of claims which refer only to things or events that we can experience.
- All of science (and philosophy) can be founded on the basis of observation statements in conjunction with the logical and mathematical principles used to regiment and derive those observations.
- Claims evidence for which is not observable may be derived from the axiomatic observations or introduced by definition.
- All and only meaningful statements will be of one of three types
  - ‣ analytic
  - ‣ observable
  - ‣ derivable (using logic) from observable axioms

Getting Rid of Metaphysics

Using the Principle of Verification

- Any sentence which is unverifiable is meaningless.
  - A. The meaning of life
  - B. The existence (or non-existence) of God
  - C. Whether the world was created, say, five minutes ago
  - D. Why there is something rather than nothing
  - E. Emergent evolutionary theory and the elan vital
  - F. Freudian psychology
  - G. Marxist theories of history

- In particular, questions about the reality of the external world were deemed pseudo-questions.

  "The Circle rejected both the thesis of the reality of the external world and the thesis of its irreality as pseudo-statements; the same was the case for both the thesis of the reality of universals (abstract entities, in our present terminology) and the nominalistic thesis that they are not real and that their alleged names are not names of anything..." (Carnap 215).

The Challenge

- The challenge for the logical empiricists was to clarify what it means to verify a sentence.
  - ‣ more difficult than it seemed
  - ‣ led to the development of the philosophy of science
- We'll return to this challenge.
- Now, let's look at the logical empiricist's views about mathematics

The *Tractatus* and the Picture Theory

- The culmination of the enterprise of logical analysis begun by Frege

- Both the world and our language consist of independent atomic elements which are combined according to strictly logical principles.

- The structure both of language and of the world is governed by strict logical rules.

- The world is a collection of independent states of affairs.

- Language consists of atomic statements of those facts, connected (into more complex statements) by logical principles.

- Language provides a picture of the world, and mirrors the world by providing logical structure which is isomorphic to the structure of the world.

- Getting straight about the world and about language are concomitant projects.

- To understand logic and mathematics, we have to understand logical and mathematical language.

Deflating Logic and Mathematics

- We might (with Descartes, say) characterize logical truths as necessary.

- Wittgenstein and the logical empiricists believe that characterizing logical truths as necessary imbues them with too much importance.

- Wittgenstein calls them nonsense.
  - ▸ "Propositions show what they say; tautologies and contradictions show that they say nothing. A tautology has no truth conditions, since it is unconditionally true; and a contradiction is true on no condition. Tautologies and contradictions are not pictures of reality. They do not represent any possible situations. For the former admit *all* possible situations, and the latter *none*" (Wittgenstein, *Tractatus* §4.461-4.462).

- Logical truths are unknowable because they are too thin to be objects of knowledge.

- They don't picture any fact.
  - ▸ "It is to be noticed that the proposition "Either some ants are parasitic or none are" provides no information whatsoever about the behavior of ants, or, indeed, about any matter of fact. And this applies to all analytic propositions. They none of them provide any information about any matter of fact. In other words, they are entirely devoid of factual content" (Ayer 79).

Ayer's Deflationism

- Ayer agrees with the rationalists and the logicists that mathematics consists of necessary truths.
  - No experience could refute them.
  - Five pairs of socks yielding only nine socks
    - We would give up our claim to having five pairs.
    - We would look for a missing sock.
    - We would *not* give up the claim that five times two is ten.
  - "One would say that I was wrong in supposing that there were five pairs of objects to start with, or that one of the objects had been taken away while I was counting, or that two of them had coalesced, or that I had counted wrongly. One would adopt as an explanation whatever empirical hypothesis fitted in best with the accredited facts. The one explanation which would in no circumstances be adopted is that ten is not always the product of two and five" (Ayer 75-6).

- There is no way to verify, empirically, the existence of numbers or circles.

- Mathematical theorems tell us about the ways in which we use language.

- They do not tell us about the way the world is.

Carnap's Conventionalism

- Carnap, like Ayer, maintains the logical empiricist's view that logic and mathematics are analytic, and thus devoid of empirical content.

- Mathematical objects are used in science, which the logical empiricists esteemed most highly.

- But Carnap does not believe that our uses of mathematical terms in science commit us to the existence of abstract entities.

- They are artifacts of the conventions of language.

- We choose languages, or linguistic frameworks, on the basis of pragmatic considerations.

- Once we have chosen a language, certain truths follow within the language.

- The question of the correct language is merely conventional.
  - Mathematical theorems are necessary, once we have adopted mathematical language.
  - The choice of whether to adopt mathematical language does not reflect any transcendent necessity.

External Questions
and Linguistic Frameworks

- External questions regard whether or not to adopt a linguistic framework.

- A linguistic framework consists of a general term, like 'number' or 'proposition', and variables which range over those objects.
    - F1. Thing language
    - F2. Mathematical languages
    - F3. The language of propositions
    - F4. Property language
    - F5. Systems of space-time points

Choosing a Framework

- The choice of whether to adopt a linguistic framework is a practical decision.
  - ▸ We can talk about things, concrete objects.
  - ▸ Equivalently, we can talk about our sense data.
  - ▸ There is no fact of the matter about whether things or sense data actually exist.

- "The decision of accepting the thing language, although itself not of a cognitive nature, will nevertheless usually be influenced by theoretical knowledge, just like any other deliberate decision concerning the acceptance of linguistic or other rules. The purposes for which the language is intended to be used, for instance, the purpose of communicating factual knowledge, will determine which factors are relevant for the decision. The efficiency, fruitfulness, and simplicity of the use of the thing language may be among the decisive factors. And the questions concerning these qualities are indeed of a theoretical nature. But these questions cannot be identified with the question of realism" (Carnap 208).

- "It can only be judged as being more or less expedient, fruitful, conducive to the aim for which the language is intended" (Carnap 214).

Internal Questions

- Internal questions arise once a framework has been adopted.

- The easy argument for the existence of abstract objects
  - EA1. There are two prime number between 4 and 8.
  - EA2. So, there are (at least) two prime numbers.
  - EAC. So, there are numbers.

- "Nobody who meant the question "Are there numbers?" in the internal sense would either assert or even seriously consider a negative answer" (Carnap 209).

Are There Mathematical Objects?

- The question, in itself, is ambiguous.
  - ‣ If it is internal, it is obviously the case.
  - ‣ If it is meant as an external question, it is nonsensical.
- Philosophers who ask whether there are numbers, as an external question, are posing an ill-formed question.
- "Unfortunately, these philosophers have so far not given a formulation of their question in terms of the common scientific language. Therefore our judgment must be that they have not succeeded in giving to the external question and to the possible answers any cognitive content. Unless and until they supply a clear cognitive interpretation, we are justified in our suspicion that their question is a pseudo-question..." (Carnap 209).

Verification and Linguistic Frameworks

- Whether a question is internal or external does not admit of degrees.

- A framework may be more or less useful depending on context.

- Either we have a method of verification or we do not.

- If there is a method to verify an answer, then the question has content, and can not be merely external.

- If there is no method, then we have a pseudo-question.

The Empiricist's Bind

- Statements of mathematics are both impossible to verify empirically and essential to the construction of empirical science.

- "The truths of mathematics appear to everyone to be necessary and certain. But if empiricism is correct no proposition which has a factual content can be necessary or certain. Accordingly the empiricist must deal with the truths of logic and mathematics in one of the two following ways: he must say either that they are not necessary truths, in which case he must account for the universal conviction that they are; or he must say that they have no factual content, and then he must explain how a proposition which is empty of all factual content can be true and useful and surprising" (Ayer 72-3).

- A deflationary necessity: convention
  - ‣ "The principles of logic and mathematics are true universally simply because we never allow them to be anything else. And the reason for this is that we cannot abandon them without contradicting ourselves, without sinning against the rules which govern the use of language..." (Ayer 77).

Surprise!

- Ayer's remaining challenge is to explain how mathematical sentences, mere empty tautologies, can nevertheless appear surprising.

- Logical truths are supposed to be obvious.

- But mathematical sentences often appear to be full of interesting, unexpected content.

- Ayer appeals to psychological factors to explain how some tautologies can be surprising.
  - ‣ Tautologies tell us about how we use language.
  - ‣ We have limited intellects, and some language use is complicated.
  - ‣ "The power of logic and mathematics to surprise us depends, like their usefulness, on the limitations of our reason. A being whose intellect was infinitely powerful would take no interest in logic and mathematics. For he would be able to see at a glance everything that his definitions implied, and, accordingly, could never learn anything from logical inference which he was not fully conscious of already" (Ayer 85-6).
  - ‣ "The fact that most of us need the help of an example to make us aware of those consequences does not show that the relation between them and the axioms is not a purely logical relation. It shows merely that our intellects are unequal to the task of carrying out very abstract processes of reasoning without the assistance of intuition. In other words, it has no bearing on the nature of geometrical propositions, but is simply an empirical fact about ourselves" (Ayer 83).

Double-Talk and
the Guilty Conscience

- "A physicist who is suspicious of abstract entities may perhaps try to declare a certain part of the language of physics as uninterpreted and uninterpretable, that part which refers to real numbers as space-time coordinates or as values of physical magnitudes, to functions, limits, etc. More probably he will just speak about all these things like anybody else but with an uneasy conscience, like a man who in his everyday life does with qualms many things which are not in accord with the high moral principles he professes on Sundays" (Carnap 205).

- If we claim there are numbers (in science) at the same time as thinking that there are no numbers (in philosophy), we seem to be contradicting ourselves.
  - ‣ We use real numbers ubiquitously for measurement and for constants, for functions and formulas and space-time coordinates.
  - ‣ It seems illicit to deny in one context what we affirm in another.

- Carnap's internal/external distinction is an attempt to legitimate these two different ways of speaking.
  - ‣ When we are doing science, and using mathematics, we are admitting mathematical objects as an internal matter.
  - ‣ When we step out of the language of science, we can deny that we mean anything by such talk.
  - ‣ "Some contemporary nominalists label the admission of variables of abstract types as "Platonism". This is, to say the least, an extremely misleading terminology. It leads to the absurd consequence, that the position of everybody who accepts the language of physics with its real number variables (as a language of communication, not merely as a calculus) would be called Platonistic, even if he is a strict empiricist who rejects Platonic metaphysics" (Carnap 215).

Carnap's Distinction
and the Verification Theory

- Internal questions are meaningful, since we have a way of verifying them.

- External questions are meaningless, since we lack a way of verifying them.
  - "I cannot think of any possible evidence that would be regarded as relevant by both philosophers, and therefore, if actually found, would decide the controversy or at least make one of the opposite these more probably than the other...Therefore I feel compelled to regard the external question as a pseudo-question, until both parties to the controversy offer a common interpretation of the question as a cognitive question; this would involve an indication of possible evidence regarded as relevant by both sides" (Carnap 219).

- So, it is critical for the logical empiricist to defend the verifiability theory of meaning.

Problems with Verification

- The theory claims that a proposition is meaningless unless it is verifiable.

- But, to know whether the statement is verifiable, we need to know what it means.

- Kichwa chake kikubwa.
  - ‣ The meaning of life is 42?
    - – Not verifiable
  - ‣ His head is big.
    - – Verifiable

- We need to know what a proposition means before we can verify it.
  - ‣ But then verificationism is not doing any semantic work.

- There is a difference between nonsense and metaphysical claims.

Quine Against Conventionalism for Logic

- Quine argues that for logic to be conventional, in Carnap's sense, we would have to adopt a framework including it.

- But the adoption of a framework is itself guided by logical laws.

- So some logic has to be presupposed.

- Similar claims might be made for mathematics.

Logical Empiricism and Basic Truths

- The logical empiricists derided as pseudo-statements all claim that could not be verified.

- It is thus central to the logical empiricist's project that some statements be taken as basic truths.
  - ▸ "There is a class of empirical propositions of which it is permissible to say that they can be verified conclusively. It is characteristic of these propositions, which I have elsewhere called "basic propositions," that they refer solely to the content of a single experience, and what may be said to verify them conclusively is the occurrence of the experience to which they uniquely refer... Propositions of this kind are "incorrigible,"...[in that] it is impossible to be mistaken about them except in a verbal sense" (Ayer, *Language, Truth and Logic*, 10).

- Among these basic, incorrigible principles are the logical and mathematical principles, which are analytic.

- These analytic claims are atomic facts.
  - ▸ "Every logical proposition is valid in its own right. Its validity does not depend on its being incorporated in a system" (Ayer 81).