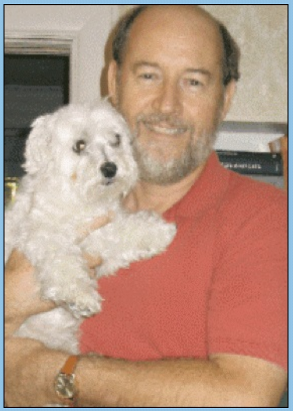
Knowledge, Truth, and Mathematics

Philosophy 405
Russell Marcus
Hamilton College, Spring 2014
Class #1: Mathematics and Philosophy

Why I Like James Robert Brown



If we taught philosophy today in a way that reflected its history, the current curriculum would be overwhelmed with the philosophy of mathematics. Think of these great philosophers and how important mathematics is to their thought: Plato, Descartes, Leibniz, Kant, Frege, Russell, Wittgenstein, Quine, Putnam, and so many others. And interest in the nature of mathematics is not confined to the so-called analytic stream of philosophy; it also looms large in the work of Husserl and Lonergin, central figures in, respectively, the continental and Thomistic philosophical traditions. Anyone sincerely interested in philosophy must be interested in the nature of mathematics... As for those who persist in thinking otherwise - let them burn in hell (Brown, xi-xii).

Philosophers Doing Mathematics

- Plato's Academy: "Let no one enter who is ignorant of geometry."
- Aristotle: "Mathematics has come to be the whole of philosophy for modern thinkers" (*Metaphysics* I.9: 992a32).
- Descartes founded analytic geometry.
- Leibniz developed the calculus.
- Frege and Russell made advances in the foundations of mathematics proper.
- Quine, Kripke, Field and many others contribute to set theory and the foundations of mathematics.

Mathematicians Doing Philosophy

- Euclid's method: proof vs truth
- Cantor's work on transfinite numbers
- Kripke and the mathematical treatment of modality
- Hilbert, Gödel, von Neumann, and Tarski

The Effects of Mathematics on Metaphysics

- Plato used the abstractness of mathematics to motivate the reality of the forms.
- Descartes cleaved thought from sensation by considering how mathematical beliefs were not ultimately sensory.
- Kant's transcendental idealism begins with the question of what the structure of our reasoning must be in order to yield mathematical certainty.
- Wittgenstein's *Remarks on the Foundations of Mathematics* contain core elements of his philosophical positions, specifically his skepticism about rule-following.

Effects of Metaphysics on Mathematics?

- Berkeley tried to debunk the calculus.
- But...
- “Philosophy may in no way interfere with the actual use of language; it can in the end only describe it. For it cannot give it any foundation either. It leaves everything as it is. It also leaves mathematics as it is, and no mathematical discovery can advance it” (Wittgenstein, *Philosophical Investigations*, §124).
- “There is no mathematical substitute for philosophy” (Kripke, “Is There a Problem About Substitutional Quantification”).

This Course

- Historical and contemporary approaches to the philosophy of mathematics
- First half: a broad survey of historical approaches to the philosophy of mathematics, from the Pre-Socratic philosophers through the early twentieth century.
  - Most of you will be familiar with the main themes of most of the central authors
  - Plato, Aristotle, Descartes, Leibniz, Locke, etc.
  - I will provide lecture notes and sometimes portions of the introductory material I am preparing for a reader and textbook on this material.
- Second half: the indispensability argument

On Indispensability

- A central problem in philosophy of mathematics is to explain how we can have knowledge of the abstract objects of mathematics.
- Rationalists claim that we have a non-sensory capacity for understanding mathematical truths.
  - That claim appears incompatible with an understanding of human beings as physical creatures whose capacities for learning are exhausted by our physical bodies.
- Logicists argue that mathematical truths are just complex logical truths.
  - But we can not reduce mathematics to logic without adding substantial portions of set theory to our logic.
- Fictionalists deny that there are any mathematical objects: mathematical sentences are vacuously true, if true at all.
- The indispensability argument attempts to justify our mathematical beliefs while avoiding any appeal to rational insight.
  - A platonist ontology with an empiricist epistemology.
- *Autonomy Platonism and the Indispensability Argument*.

The Syllabus and Website

- **readings**
- **reading précis**
  - The first ten précis are due on Friday, March 14, at 4pm.
  - The last ten précis are due on Friday, May 9, at 4pm.

    MB1. I have a clear and distinct understanding of my mind, independent of my body.

    MB2. I have a clear and distinct understanding of my body, independent of my mind.

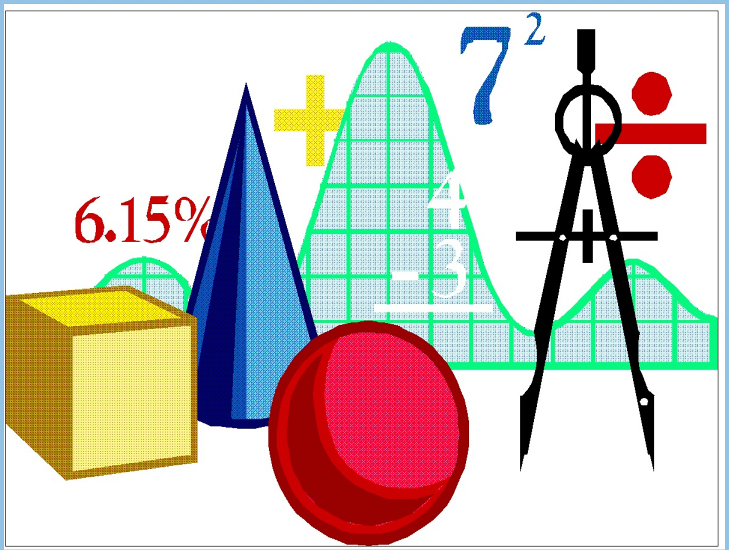
    MB3. Whatever I can clearly and distinctly conceive of as separate, can be separated by God, and so are really distinct.

    MBC. So, my mind is distinct from my body
- **seminar papers**
- **term paper**
  - Thursday, March 6: A one–to-two-paragraph abstract with a proposed bibliography
  - Thursday, April 3: Précis of your argument with an annotated bibliography
  - Tuesday, April 22: A full draft
  - Tuesday, May 6: The final draft
- **final exam**
  - **Wednesday, May 14**, from 7pm-10pm
  - Preparatory questions will be posted on the course website.

Preparing for Class

- 1. Completing the primary readings (preferably two-to-three times)
- 2. Writing a reading précis
- 3. Reading a seminar paper (if there is one) and preparing comments and questions
- 4. Completing as much of the secondary readings as you can
- For the two classes in which you will be presenting, your preparation will of course differ.
- Meet with me in advance

The Nature of Mathematics



Some Questions About Mathematics

1. Do the objects of mathematics exist? If so, how do they exist?
2. Are mathematical truths necessary?
3. Are there innate ideas?
4. Are mathematical claims truths of experience or of reason?
5. Are mathematical statements synthetic or analytic?

- The first two questions are metaphysical.
- The third and fourth are epistemological.
- The fifth is semantic.
  - I'll say more about these categories later today and through the term.

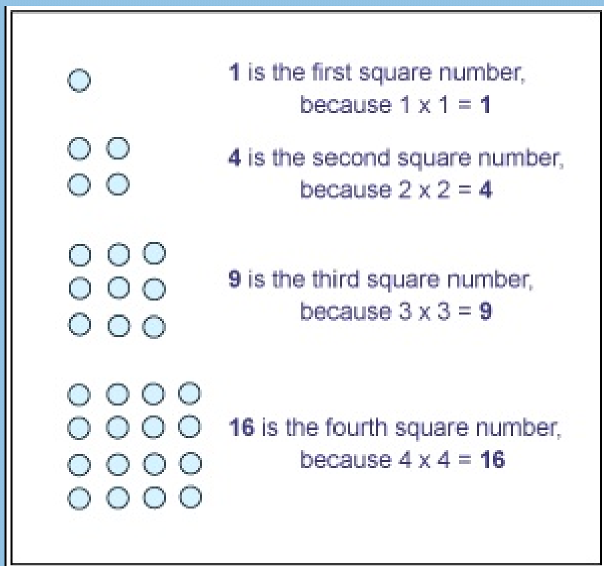
Brown's "Mathematical Image"

1. Mathematical results are certain.
2. Mathematics is objective.
3. Proofs are essential.
4. Diagrams are psychologically useful, but prove nothing.
5. Diagrams can even be misleading.
6. Mathematics is wedded to classical logic.
7. Mathematics is independent of sense experience.
8. The history of mathematics is cumulative.
9. Computer proofs are merely long and complicated regular proofs.
10. Some mathematical problems are unsolvable in principle.

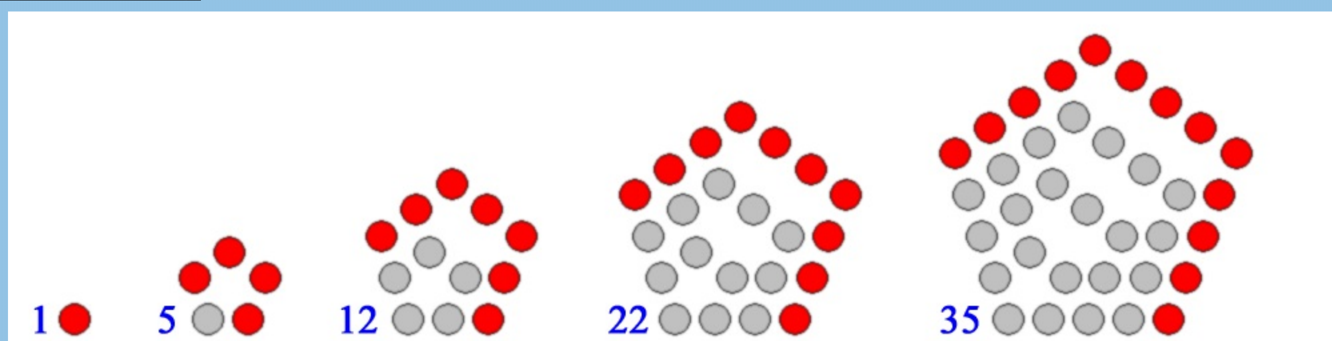
Mathematics and Proof

Brown’s #3

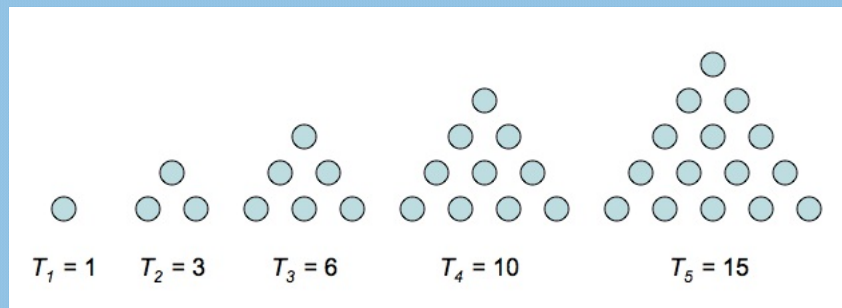
On Proof



- Greek mathematics was essentially geometric.
- The Pythagoreans were fascinated by figurate numbers: triangular numbers, square numbers, pentagonal numbers
- The triangular numbers, were especially interesting to the Pythagoreans:
  - 1, 3, 6, 10, 15, 21, 28, 36...



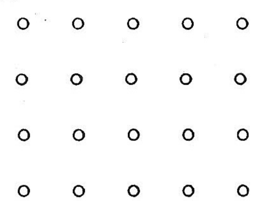
Triangular Numbers



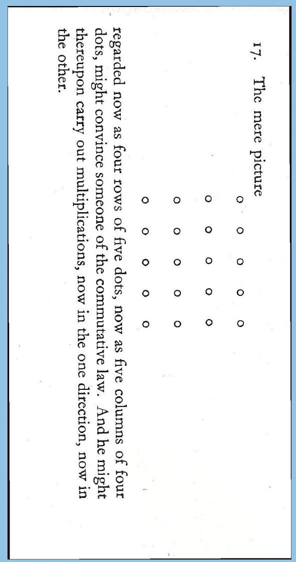
- The formula for the calculating the nth triangular number is: (n/2)(n+1)
- Theorem: The sum of two consecutive triangular numbers is a square number.
- The theorem is easily shown algebraically:
  - $(n/2)(n+1) + ((n+1)/2)((n+1)+1) =$
  - $(n^2 + n)/2 + (n+1)(n+2)/2 =$
  - $(n^2 + n)/2 + (n^2 + 3n + 2)/2 =$
  - $(2n^2 + 4n + 2)/2 =$
  - $n^2 + 2n + 1 =$
  - $(n+1)^2$
- Kline: “That the Pythagoreans could prove this general conclusion, however, is doubtful” (30).
- Is it?

Wittgenstein on Commutativity

17. The mere picture



regarded now as four rows of five dots, now as five columns of four dots, might convince someone of the commutative law. And he might thereupon carry out multiplications, now in the one direction, now in the other.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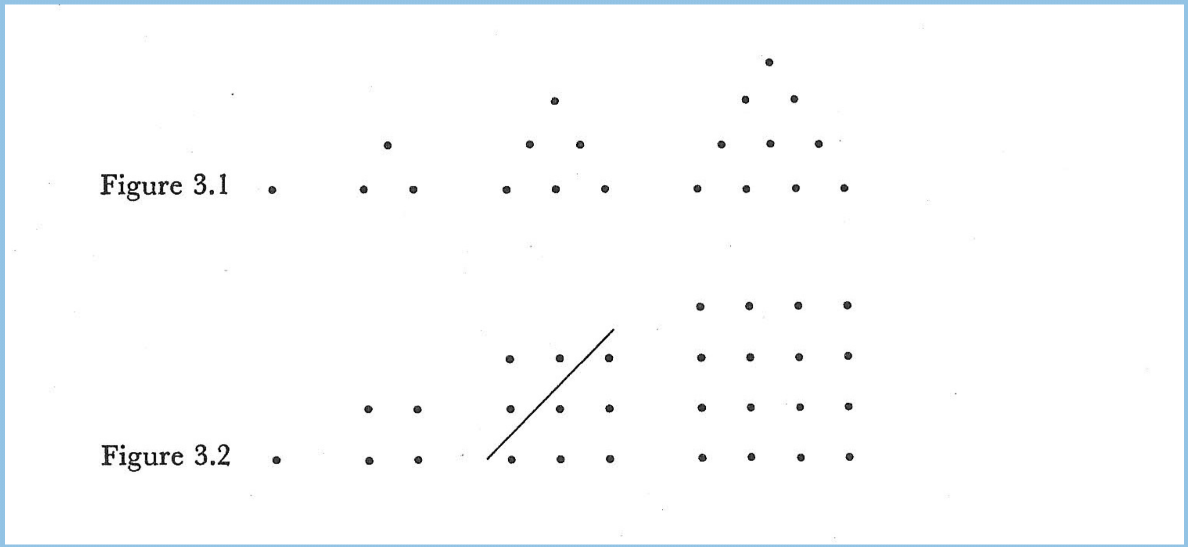
The Pythagoreans Had a Picture



Limitations of Pictures

- It is difficult to draw intuitively useful pictures of odd spaces.
- Some pictures are misleading.
- Compare the sums of two infinite series:
  - 1, 1/4, 1/9, 1/16...
  - 1, ½, 1/3, 1/4...

Graphs

1, 1/4, 1/9, 1/16...

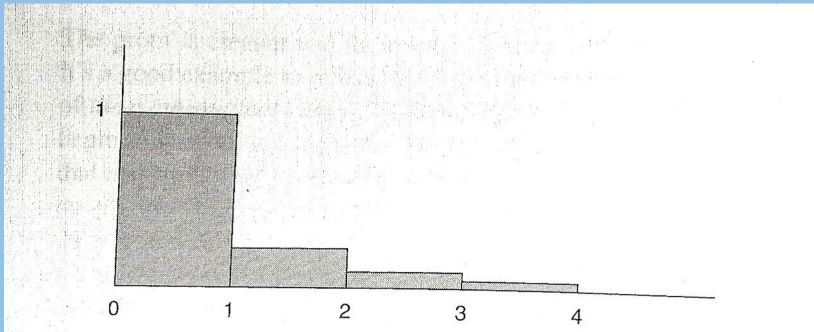


Figure 1.2 Shaded blocks correspond to terms in the series

1, ½, 1/3, 1/4...

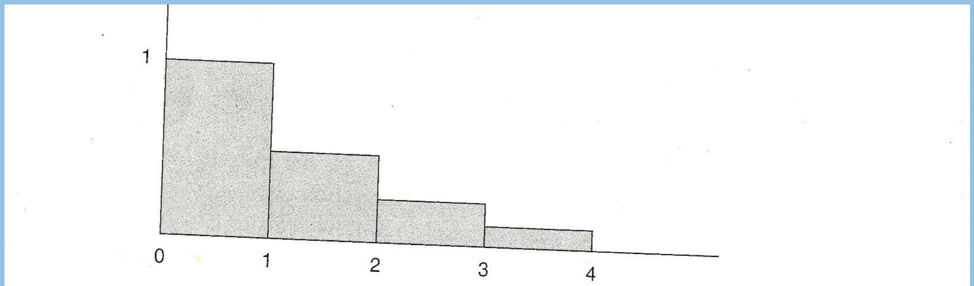
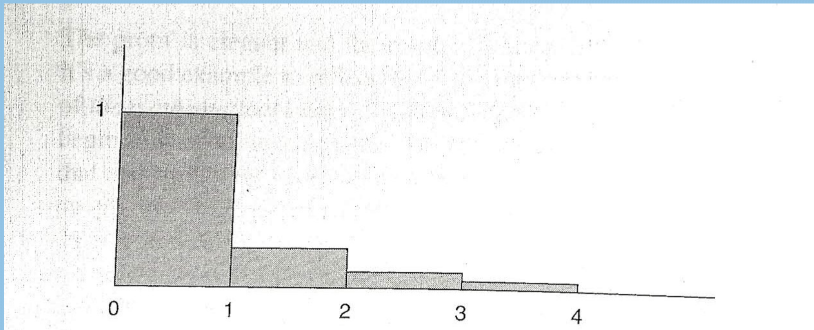


Figure 1.3

Graphs

1, 1/4, 1/9, 1/16...



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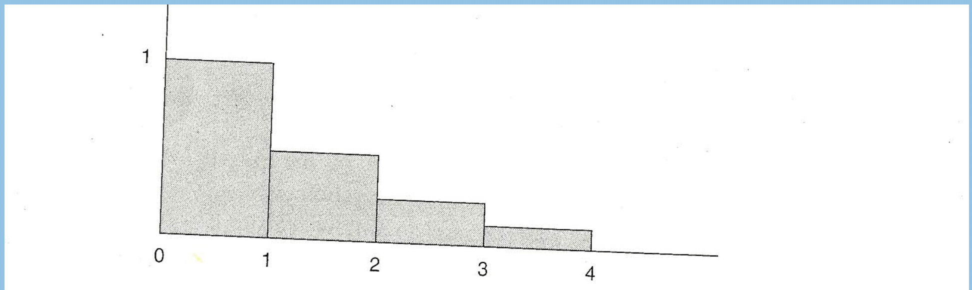

Figure 1.3

The sum of this series is a finite number, $\pi^2/6 \approx 1.64$

The sum of this series is infinite.

So much for pictures?
Is Kline right about proof?

A Priori Knowledge

Brown's #7

a priori knowledge

- The question of whether we have *a priori* knowledge is widely debated.
- A proposition is known *a priori* if the knowledge is not based on any, "Experience of the specific course of events of the actual world'"(Blackburn, in Shapiro, 22).
- The debates over the *a priori* are subtle and complex.
- But, the question of whether there is *a priori* knowledge seems easily answered in mathematics.
- We could never discover that the square root of two is irrational by experience.
  - The rationals are dense.
  - We can always find a rational which will fulfill our measurement needs.

That $\sqrt{2}$ is irrational

- Suppose that $\sqrt{2}$ is rational.
- Then, it's expressible as a/b, where a and b are integers.
- We can suppose a/b to be in lowest terms, which means that a and b have no common divisors.
- $a^2 = 2b^2$
- So, a^2 is even.
- Thus **a is even**, since only even numbers have even squares.
- So, a = 2c, for some c.
- $a^2 = 4c^2 = 2b^2$
- So, $b^2 = 2c^2$.
- Which means that **b is also even**.
- So a and b have been shown even, which contradicts our assumption that a/b is in lowest terms.
- *Tilt*

Aside on *reductio ad absurdum*

- Assume the opposite of what one wants to demonstrate, and show that it leads to a contradiction.
- Reductios assume bivalence.
- Some philosophers reject bivalence, and its object-language correlate called the law of the excluded middle:
  - Law of the excluded middle: $P \vee {\sim}P$
  - Intuitionists demand constructive proofs.

Hippasus

Apriority and Necessity

- The proof that √2 is irrational is *a priori*.
- It's also necessary that √2 is irrational.
- These aren't the same claim.
- Long confounded
  - Old view: anything believed *a priori* must be true.
- Consider Kant's claim that Euclidean space is the result of the a priori application of our concepts on the noumenal world.
  - Space is non-Euclidean
  - What seemed *a priori* turns out to be false.
  - On the old view, if a statement turns out false, it must never have been believed *a priori*.
  - Kant's entire metaphysical system depended on the application of *a priori* concepts to the noumenal world.
  - When space turned out to be non-Euclidean, Kant's system seemed to fall apart.
- Shapiro still calls apriority and necessity "twin notions" (23).
- Kripke: 'water is H2O' and 'the standard meter is one meter'.

The Fallibilist *A Priori*

- Apriority is about the acquisition and justification of our beliefs.
- Necessity is about their modal status.
- We can be wrong about a proposition, even if we hold it *a priori*.
- We can believe a proposition independently of experience, and still be wrong about that belief.
  - Cantor and Frege and the axiom of comprehension: every property determines a set
  - The set of all things that aren't woodchucks is too big.
- Had Kant held a fallibilistic *a priori*, he might have been able to salvage some of his work.
- The fallibilist can hold that statements believed on the basis of a priori reasoning are necessarily true, *if true*.
  - And if they are false, they are necessarily false.

Analyticity

- Thought by many to be an explanation of apriority
- ‘Bachelors are unmarried’
- ‘We walk with those with whom we stroll’
- Analyticity is a semantic notion, about meanings of terms.
- Apriority is an epistemic notion, about belief and knowledge.
- Necessity is a metaphysical notion, about the nature of the universe, broadly conceived.
- Certainty is an epistemic notion, masquerading as a metaphysical notion.
  - I can be certain about something non-necessary, like that I am here now.
  - I can be uncertain about something necessary, like whether Goldbach’s conjecture is true.
  - Even Brown makes mistakes: 1. Mathematical results are certain.

Thursday

Pythagoreans