Philosophy 405: Knowledge, Truth and Mathematics
Spring 2014
Tuesdays and Thursdays: 1pm - 2:15pm
Philosophy Seminar Room

Hamilton College
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Syllabus

Course Description and Overview

This course is divided into two parts. The first part, covering roughly the first eight weeks of the term, is an historical survey of philosophical questions about mathematics. Do we have a priori knowledge of necessary truths? Is our knowledge of mathematics empirical? Do we have mathematical knowledge at all? The readings in the first part of the course, covered mainly chronologically, range from ancient philosophy through the twentieth century. We will devote the second part of the course, the last six weeks of the term, to recent work on the indispensability argument.

Mathematics has a long and prominent place in philosophy. Plato's students were implored to excel in mathematics; a sign over the door to his Academy said, "Let no one enter who is ignorant of geometry." Aristotle wrote, "Mathematics has come to be the whole of philosophy for modern thinkers" (*Metaphysics* I.9: 992a32).

Some prominent philosophers in the early modern period were also important mathematicians, including Descartes, who developed analytic geometry, and Leibniz, who developed the calculus. In the late nineteenth and early twentieth centuries, philosophers including Frege and Russell made advances in the foundations of mathematics proper. In recent years, many philosophers have made contributions to set theory and mathematical logic, independently of their philosophical work.

In the other direction, mathematicians from Euclid forward have contributed to philosophy. Cantor's work on transfinite numbers transformed the philosopher's concept of infinity, which had played a central role in philosophical debate about God and the origins of the universe for millennia. Other philosophical topics like necessity and contingency have received mathematical treatment which has changed the way philosophers argue about these concepts. Indeed some mathematicians, like Hilbert, Gödel, von Neumann, and Tarski, are central philosophical figures.

Even philosophers who have not contributed to mathematics have made mathematical insights central to their work. Berkeley tried to debunk the calculus on philosophical grounds. Kant's transcendental idealism begins with the question of what the structure of our reasoning must be in order to yield mathematical certainty. Wittgenstein's *Remarks on the Foundations of Mathematics* contain core elements of his philosophical positions.

Still, some philosophers who spend time with mathematics deny that the relationship of mathematics to philosophy is particularly close. Wittgenstein wrote that philosophy, "Leaves mathematics as it is, and no mathematical discovery can advance it." (*Philosophical Investigations*, §124) Kripke implored that, "There is no mathematical substitute for philosophy."

The second half of the course focuses on a topic of central importance in contemporary philosophy of mathematics: the indispenability argument. The central problem in the philosophy of mathematics is to explain how we can have knowledge of the abstract objects of mathematics. Rationalists claim that we have a non-sensory capacity for understanding mathematical truths, but that claim appears incompatible with an understanding of human beings as physical creatures whose capacities for learning are exhausted by our physical bodies. Logicists argue that mathematical truths are just complex logical truths, but we can not reduce mathematics to logic without adding substantial portions of set theory to our logic. Fictionalists deny that there are any mathematical objects.

The indispensability argument attempts to justify our mathematical beliefs while avoiding any appeal to rational insight, to provide a platonist ontology with an empiricist epistemology. We will read a variety of recent work, including excerpts from my manuscript in progress, *Autonomy Platonism and the Indispensability Argument.*

Texts

Most primary readings are available on the course website. In addition, I am assigning optional but vaulable secondary readings from two excellent sources:

James Robert Brown, *Philosophy of Mathematics: An Introduction to the World of Proofs and Pictures*, 2nd edition, Routledge, 2008.

Stewart Shapiro, *Thinking About Mathematics: The Philosophy of Mathematics*, New York: Oxford, 2000.

On-Line Resources

The website for this course is:

http://www.thatmarcusfamily.org/philosophy/Course_Websites/Math_S14/Course_Home.html

The course website includes an html syllabus, readings, assignments, class notes, handouts, and other links. Limited material will be available on Blackboard.

Assignments and Grading

Your responsibilities for this course include the following, with their contributions to your grade calculation in parentheses:

1. All the primary readings listed below, including seminar papers.
2. Twenty reading précis (10%)
3. Two seminar papers/presentations (2-4 pages) (30%; 15% each)
4. Term paper (8-12 pages) (40%)
5. Final exam (20%)

Readings are to be completed before the class indicated. The Primary Readings are required; the secondary readings are optional. Some secondary readings, notably the readings from the Brown and Shapiro texts, are introductory elucidations of the primary readings. Some secondary readings are further scholarly articles on a given topic, critical commentaries on the primary readings, or extended studies of a point we will study only briefly. All of the readings on the syllabus that are not from either the Brown or Shapiro texts will be accessible from the course website. The course bibliography includes further readings, many of which are also accessible from the course website.

Reading précis are 100- to 150- word distillations of some argument in an assigned reading. A précis is a skeletal version of an argument, the argument in its most efficient form. In preparing for most classes, you should write one prècis before class. You may choose to write about an entire reading or to focus on a portion of the reading. If there is more than one reading, choose one on which to focus. You need not complete précis for the two classes in which you are presenting a seminar paper. In lieu of up to five précis, you can write a list of 6-8 detailed questions on the reading. The first ten précis are due on Friday, March 14, at 4pm. The last ten précis are due on Friday, May 9, at 4pm. You will mainly be graded on the completion of twenty précis, rather than their quality. I expect that the précis will be useful to you in preparing both for classes and for the final exam.

Many classes will run as extended discussions of a 750- to 1500-word **seminar paper**. Seminar papers should assimilate the assigned readings and summarize the main arguments. I also encourage you to include some critical analysis. It is good practice to end a seminar paper with a few questions you believe will be useful for the class to consider. Each seminar paper is due at noon by email to all seminar participants the day before the class in which it will be discussed (i.e. Monday or Wendesday). This

deadline is necessary for all participants in the seminar to be able to read the paper and prepare comments and questions for class.

You will lead the class on the day we discuss your seminar paper. You may be creative with your presentation. You may focus on the content of your paper. You may also discuss any particular difficulties in the material, or topics that you were unable to cover in the paper. Your grade for the seminar paper will depend on both the paper and your presentation of it. Each student in the course will write and present two seminar papers.

Your **term papers** will be completed in four stages. A one–to-two-paragraph abstract of your paper with a proposed bibliography is due on Thursday, March 6. A précis of your argument with an annotated bibliography is due on Thursday, April 3. A full draft of your term paper is due on Tuesday, April 22. The final draft is due on Tuesday, May 6. See the Paper Assignment handout for various options for paper topics. I will be happy to meet with you to discuss your topic, in advance. Failure to hand in a draft, or handing in an insufficient draft, will reduce your final paper grade by two steps (e.g. from B+ to B-).

The **final exam** will be on Wednesday, May 14, from 7pm to 10pm. Preparatory questions will be posted on the course website.

Grades on assignments will be posted on Blackboard, along with a running total, which I call your grade calculation. Your grade calculation is a guide for me to use in assigning you a final grade. There are no rules binding how I translate your grade calculation into a letter grade. The Hamilton College key for converting letter grades into percentages is not a tool for calculating your final grade. I welcome discussion of the purposes and methods of grading, as well as my own grading policies.

Both the **Writing Center** and the **Oral Communications Center** have an astoundingly wonderful set of resources to help you write and speak more effectively.

The Hamilton College Honor Code will be enforced.

Preparing for Class

Preparation for most classes will consist of the following tasks

1. Completing the primary readings (preferably two-to-three times)
2. Writing a reading précis
3. Reading a seminar paper (if there is one) and preparing comments and questions
4. Completing as much of the secondary readings as you can

For the two classes in which you will be presenting, your preparation will of course differ. I encourage you to find time to meet with me in advance.

Contacting Me

My office hours for the Spring 2014 term are 10:30am - noon, Tuesdays and Thursdays. My office is 202 College Hill Road, Room 210. My email address is rmarcus1@hamilton.edu.

Schedule:

	Date	Topic	Primary Readings	Secondary Readings
1	Tuesday, January 21	Mathematics and the Philosophy of Mathematics?	Brown, Chapter 1 Shapiro, pp 21-29	
2	Thursday, January 23	Pythagoras and the Pythagoreans	Kline, "The Creation of Classical Greek Mathematics" Kline, "The Greek Rationalization of Nature"	
3	Tuesday, January 28	Plato's Platonism	Selections from Plato on Mathematics Aristotle, *Metaphysics* I.9	Shapiro, pp 49-63 Brown, Chapter 2
4	Thursday, January 30	Aristotle	Aristotle, *Metaphysics* XIII.1-3 Aristotle, *Physics* II.2 Lear, "Aristotle's Philosophy of Mathematics"	Shapiro, pp 63-71 Aristotle, *Metaphysics* XIII- XIV
5	Tuesday, February 4	Modern Rationalism I	Descartes, Third and Fifth Meditations Descartes, Synthetic Presentation from Second Replies Leibniz, "Meditations on Knowledge, Truth, and Ideas"	Kline, "Coordinate Geometry" Kline, "The Mathematization of Science"
6	Thursday, February 6	Modern Rationalism II	Locke, *Essay*, Bk 1, Ch. 1 Leibniz, Selections from *New Essays*	Kline, "The Creation of the Calculus"
7	Tuesday, February 11	Modern Empiricism	Locke, *Selections on Mathematics* Selections from Berkeley's *Principles* Selections from Hume on Mathematics	
8	Thursday, February 13	The Synthetic *A Priori* I	Kant, *Prolegomena*, §§1-2 Selections from Kant's *Critique*	Shapiro, pp 76-91
9	Tuesday, February 18	The Synthetic *A Priori* II		Kitcher, "Kant and the Foundations of Mathemtics"
10	Thursday, February 20	Radical Empiricism	Mill, *System of Logic*, Book II, §V and §VI Frege, from *The Foundations of Arithmetic*, I	Shapiro, pp 91-102
11	Tuesday, February 25	Cantor's Paradise	Tiles, "Cantor's Transfinite Paradise"	Tiles, "Numbering the Continuum"
12	Thursday, February 27	Logicism	Frege, from *The Foundations of Arithmetic*, II Russell, "Letter to Frege" Frege, "Letter to Russell"	Shapiro, pp 107-115 Russell, "On Our Knowledge of General Principles" Russell, "How *A Priori* Knowledge is Possible"
13	Tuesday, March 4	Formalism and Incompleteness	Hilbert, "On the Infinite" Johann von Neumann, "The Formalist Foundations of Mathematics"	Brown, Chapter 5 Shapiro, pp 140-168 Smullyan, "The General Idea Behind Gödel's Proof"

	Date	Topic	Primary Readings	Secondary Readings
14	Thursday, March 6 **Term Paper Abstracts Due**	Intuitionism	Brouwer, "Intuitionism and Formalism" Heyting, "Disputation"	Brown, Chapter 8 Shapiro, pp 172-189 Brouwer, "Consciousness, Philosophy, and Mathematics"
15	Tuesday, March 11	Conventionalism	Carnap, "Empiricism, Semantics and Ontology" Ayer, "The A Priori"	Shapiro, pp 124-133 Brown, Chapter 9
16	Thursday, March 13	Two Dogmas of Empiricism	Quine, "Two Dogmas of Empiricism"	Shapiro, pp 212-220 Grice and Strawson, "In Defense of a Dogma"
	Friday March 14	**First 10 Reading Précis are due**		
		Spring	**Break**	
17	Tuesday, April 1	The Problem	Benacerraf, "Mathematical Truth" Field, "Knowledge of Mathematical Entities"	Shapiro, pp 29-39
18	Thursday, April 3 **Term Paper Précis due**	The Indispensability Argument	Quine, "Existence and Quantification" Quine, "On What There Is" Quine on Recreation	Azzouni, "On 'On What There Is'" Marcus, ""Chapter 2: The Quinean Indispensability Argument"
19	Tuesday, April 8	Dispensabilism I	Field, from *Science without Numbers*	Shapiro, pp 226-237 Brown, Chapter 4
20	Thursday, April 10	Dispensabilism II	Field, "Introduction: Fictionalism, Epistemology, and Modality"	MacBride, "Listening to Fictions: A Study of Fieldian Nominalism" Melia, "Field's Programme: Some Interference"
21	Tuesday, April 15	The Weasel	Melia, "Weaseling Away the Indispensability Argument"	Colyvan, "Mathematics and Aesthetic Considerations in Science? Melia, "Response to Colyvan"
22	Thursday, April 17	Eleatics and Indispensabilists	Colyvan, "The Quinean Backdrop" Colyvan, "The Eleatic Principle"	Marcus, "The Eleatic and the Indispensabilist"
23	Tuesday, April 22 **Term Paper Draft Due**	Mathematical Recreation	Leng, "What's Wrong with Indispensability? (Or, the Case for Recreational Mathematics)"	Maddy, "Indispensability and Practice" Sober, "Mathematics and Indispensability"

	Date	Topic	Primary Readings	Secondary Readings
24	Thursday, April 24	The Explanatory Argument	Baker, "Are There Genuine Mathematical Explanations of Physical Phenomena?" Mancosu, "Mathematical Explanation: Problems and Prospects," §3	Lyon and Colyvan, "The Explanatory Power of Phase Spaces"
25	Tuesday, April 29	The Nominalist Against the Explanatory Argument	Bangu, "Inference to the Best Explanation and Mathematical Realism"	
26	Thursday, May 1	The Platonist Against the Explanatory Argument	Marcus, "How Not to Enhance the Indispensability Argument"	Marcus, "Chapter 5: The Unfortunate Consequences" Brown, Chapter 3
27	Tuesday, May 6 **Term Paper Due**	Autonomy Platonism I: Plenitudinous Platonism	Balaguer, "On Plenitudinous Platonism" Balaguer, "A New Platonist Epistemology"	Marcus, "Chapter 9: Two Versions of Autonomy Platonism, §1-§2"
28	Thursday, May 8	Autonomy Platonism II	Marcus, "Chapter 9: Two Versions of Autonomy Platonism, §3-§6" Marcus, "Chapter 10: Circles"	Shapiro, pp 201-212

Deadline Summary

Thursday, March 6: Term paper abstracts and bibliography
Friday, March 14, 4pm: First ten reading précis
Thursday, April 3: Term paper précis and annotated bibliography
Tuesday, April 22: First draft of term paper
Tuesday, May 6: Final draft of term paper
Friday, May 9, 4pm: Final ten Précis due
Wednesday, May 14, 7-10pm: Final Exam

Seminar papers are due, by email to me and the rest of the class by noon on the day before your presentation (i.e. Monday or Wednesday). All other deadlines are at classtime, except where noted.