Philosophy 405: Knowledge, Truth and Mathematics
Fall 2014

Hamilton College
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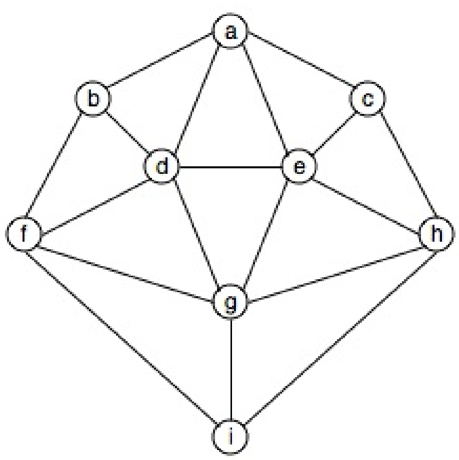
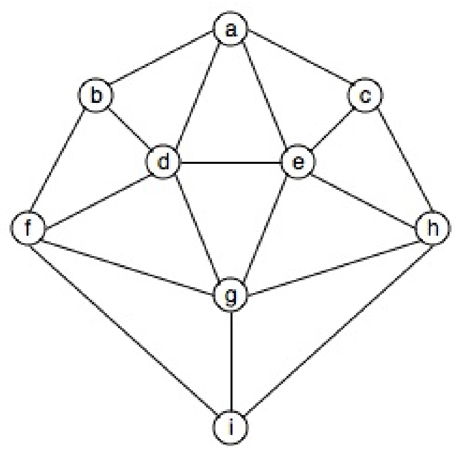
Constructive and Non-Constructive Proofs

<u>A Constructive Proof</u>:

Definition: A coloring of a graph is an assignment of a color to each node of the graph.
Definition: A graph is 3-colorable if any coloring which uses only three colors does not assign the same color to any two nodes which share a branch.
Definition: A graph is 4-colorable if any coloring which uses only four colors does not assign the same color to any two nodes which share a branch.

Theorem: There are graphs which are 4-colorable but which are not 3-colorable.
Proof: In two stages. First, present a graph which is not 3-colorable. Second, show that it is 4-colorable.





<u>A Non-Constructive Proof</u>

Claim: There exist irrational numbers x and y such that x^y is rational.

Proof:

Let $z = \sqrt{2}^{\sqrt{2}}$.
Either z is rational or z is irrational, though we do not know which.
If z is rational then z is our desired number with $x = y = \sqrt{2}$.
If z is irrational, then let x = z and $y = \sqrt{2}$.

$$x^y = \sqrt{2}^{\sqrt{2} \cdot \sqrt{2}} \quad \sqrt{2}^{\sqrt{2} \bullet \sqrt{2}} \quad \sqrt{2}^{2} \quad = \quad = 2.$$

On these different assignments of irrational values to x and y, x^y is again rational.
Whether z is rational or irrational, there exist irrational numbers x and y such that x^y is rational.

QED