Philosophy 405: Knowledge, Truth and Mathematics Hamilton College
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Reading Guide #10 - Formalism
David Hilbert, "On the Infinite"
John Von Neumann, "The Formalist Foundations of Mathematics"

Hilbert

1. What was Weierstrass's project? How did he accomplish it? What questions regarding analysis remain?
2. What is Hilbert's goal?
3. Describe two criteria for mathematical legitimacy. How are they related?
4. What is the relation between nature and infinite divisibility? What is the relation between nature and the infinitely large?
5. How does the concept of infinity apply in number theory?
6. How do the uses of ideal elements introduce infinity into geometry?
7. What problems arise for theories of transfinite numbers? How does Hilbert want to solve them?
8. How does Hilbert criticize Frege's project? How does he defend Kant?
9. What is the subject matter of mathematics? How does this claim mitigate worries about the infinite?
10. How can an existential statement lapse into meaninglessness?
11. How does Hilbert adopt a finitary view?
12. Why do we introduce ideal statements (e.g. into Aristotelian logic)?
13. "We therefore conclude that a, b, =, +, as well as the whole formula $a + b = b + a$ mean nothing in themselves..." (196). Explain.
14. To what kinds of formulas does the law of the excluded middle (*tertium non datur*) apply?
15. Are logical theorems meaningful? Explain.
16. What is a consistency proof? Why is it important for Hilbert's program? Why is it important more broadly?
17. Is every mathematical problem solvable? What contribution to the answer to this question does Hilbert hope to provide?

Von Neumann

18. What is Hilbert's theory of proof?
19. Why does Hilbert want to investigate methods of proof?
20. What are the four requirements for a Hilbert-style system? Be especially clear on the fourth requirement.