VAN Heijenoort, Jean
From Frege to Godel: A Sourcebook in Mathematical Logic, 1879-1931. Cambridge, Harvard UP 1967

Letter to Frege

BERTRAND RUSSELL

(*1902*)

Bertrand Russell discovered what became known as the Russell paradox in June 1901 (see *1944*, p. 13). In the letter below, written more than a year later and hitherto unpublished, he communicates the paradox to Frege. The paradox shook the logicians' world, and the rumbles are still felt today.

The Burali-Forti paradox, discovered a few years earlier, involves the notion of ordinal number; it seemed to be intimately connected with Cantor's set theory, hence to be the mathematicians' concern rather than the logicians'. Russell's paradox, which makes use of the bare notions of set and element, falls squarely in the field of logic. The paradox was first published by Russell in *The principles of mathematics* (*1903*) and is discussed there in great detail (see especially pp. 101–107). After various attempts, Russell considered the paradox solved by the theory of types (*1908a*). Zermelo (below, p. 191, footnote 9) states that he had discovered the paradox independently of Russell and communicated it to Hilbert, among others, prior to its publication by Russell.

In addition to the statement of the paradox, the letter offers a vivid picture of Russell's attitude toward Frege and his work at the time.

The formula in Peano's notation at the end of the letter can be read more easily if one compares it with formula 450 in *Peano 1898a*, p. VII (or *1897*, p. 15).

Russell wrote the letter in German, and it was translated by Beverly Woodward. Lord Russell read the translation and gave permission to print it here.

Friday's Hill, Haslemere, 16 June 1902

Dear colleague,

For a year and a half I have been acquainted with your *Grundgesetze der Arithmetik*, but it is only now that I have been able to find the time for the thorough study I intended to make of your work. I find myself in complete agreement with you in all essentials, particularly when you reject any psychological element [[Moment]] in logic and when you place a high value upon an ideography [[Begriffsschrift]] for the foundations of mathematics and of formal logic, which, incidentally, can hardly be distinguished. With regard to many particular questions, I find in your work discussions, distinctions, and definitions that one seeks in vain in the works of other logicians. Especially so far as function is concerned (§ 9 of your *Begriffsschrift*), I have been led on my own to views that are the same even in the details. There is just one point where I have encountered a difficulty. You state (p. 17 [[p. 23 above]]) that a function,

too, can act as the indeterminate element. This I formerly believed, but now this view seems doubtful to me because of the following contradiction. Let w be the predicate: to be a predicate that cannot be predicated of itself. Can w be predicated of itself? From each answer its opposite follows. Therefore we must conclude that w is not a predicate. Likewise there is no class (as a totality) of those classes which, each taken as a totality, do not belong to themselves. From this I conclude that under certain circumstances a definable collection [[Menge]] does not form a totality.

I am on the point of finishing a book on the principles of mathematics and in it I should like to discuss your work very thoroughly.[1] I already have your books or shall buy them soon, but I would be very grateful to you if you could send me reprints of your articles in various periodicals. In case this should be impossible, however, I will obtain them from a library.

The exact treatment of logic in fundamental questions, where symbols fail, has remained very much behind; in your works I find the best I know of our time, and therefore I have permitted myself to express my deep respect to you. It is very regrettable that you have not come to publish the second volume of your *Grundgesetze*; I hope that this will still be done.

Very respectfully yours,

BERTRAND RUSSELL

The above contradiction, when expressed in Peano's ideography, reads as follows:

$$w = \mathrm{cls} \cap x \ni (x \sim\varepsilon\, x) . \supset : w\, \varepsilon\, w \,.=.\, w \sim\varepsilon\, w.$$

I have written to Peano about this, but he still owes me an answer.

[1] [[This was done in *Russell 1903*, Appendix A, "The logical and arithmetical doctrines of Frege".]]