QUINE

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Existence and Quantification

The question whether there are numbers, or qualities, or classes, is a metaphysical question, such as the logical positivists have regarded as meaningless. On the other hand the question whether there are rabbits, or unicorns, is as meaningful as can be. A conspicuous difference is that bodies can be perceived. Still, this is not all that matters; for we can evidently say also, meaningfully and without metaphysics, that there are prime numbers between 10 and 20.

What typifies the metaphysical cases is rather, according to an early doctrine of Carnap's,[1] the use of category words, or *Allwörter*. It is meaningful to ask whether there are prime numbers between 10 and 20, but meaningless to ask in general whether there are numbers; and likewise it is meaningful to ask whether there are rabbits, or unicorns, but meaningless to ask in general whether there are bodies.

But this ruling is unsatisfactory in two ways. The first difficulty is that there is no evident standard of what to count as a category, or category word. Typically, in terms of formalized

[1] Carnap, *Logical Syntax of Language,* p. 292.

quantification theory, each category comprises the range of some distinctive style of variables. But the style of variable is an arbitrary matter, and surely of no help in distinguishing between meaningful questions of existence and metaphysical questions of existence. For there are no external constraints on styles of variables; we can use distinctive styles for different sorts of number, or a single style for all sorts of numbers and everything else as well. Notations with one style of variables and notations with many are intertranslatable.

There is another idea of category that may superficially seem more profound. It is the idea of semantic category, as Leśniewski called it,[2] or what linguists call a substitution class. Expressions belong to the same substitution class if, whenever you put one for the other in a meaningful sentence, you get a meaningful sentence. The question whether numbers constitute a category gives way, in these terms, to a question of the meaningfulness of the sentences that we obtain by supplanting number words by other words. However, what to count as meaningful is not at all clear. The empirical linguist manages the point after a fashion by considering what sentences could be elicited by reasonable means from naive native speakers. But such a criterion is of little value to a philosopher with a reform program. In fact, the question what existence sentences to count as meaningless was where we came in.

Existence questions were ruled meaningless by Carnap when they turned on category words. This was, I said, an unsatisfactory ruling in two respects. We have seen one of the respects: the tenuousness of the idea of category word. Now the other respect is that anyway sense needs to be made of categorial existence questions, however you choose your categories.

[2] Stanisław Leśniewski, "Grundzüge eines neuen Systems der Grundlagen der Mathematik," §§ 1–11, *Fundamenta Mathematicae* 14 (1929), pp. 1–81.

For it can happen in the austerest circles that some one will try to rework a mathematical system in such a way as to avoid assuming certain sorts of objects. He may try to get by with the assumption of just numbers and not sets of numbers; or he may try to get by with classes to the exclusion of properties; or he may try, like Whitehead, to avoid points and make do with extended regions and sets of regions. Clearly the system-maker in such cases is trying for something, and there is some distinction to be drawn between his getting it and not.

When we want to check on existence, bodies have it over other objects on the score of their perceptibility. But we have moved now to the question of checking not on existence, but on imputations of existence: on what a theory says exists. The question is when to maintain that a theory assumes a given object, or objects of a given sort—numbers, say, or sets of numbers, or properties, of points. To show that a theory assumes a given object, or objects of a given class, we have to show that the theory would be false if that object did not exist, or if that class were empty; hence that the theory requires that object, or members of that class, in order to be true. How are such requirements revealed?

Perhaps we find proper names of the objects. Still, this is no evidence that the objects are required, except as we can show that these proper names of the objects are used in the theory *as* proper names of the objects. The word "dog" may be used as a proper name of an animal species, but it may also be used merely as a general term true of each of various individuals and naming no one object at all; so the presence of the word is of itself no evidence that species are being assumed as objects. Again even "Pegasus," which is inflexibly a proper name grammatically speaking, can be used by persons who deny existence of its object. It is even used in denying that existence.

What would count then as evidence that an expression is

used in a theory as a name of an object? Let us represent the expression as "*a*." Now if the theory affirms the existentially quantified identity "$(\exists x)(x = a)$," certainly we have our answer: "*a*" is being used to name an object. In general we may say that an expression is used in a theory as naming if and only if the existentially quantified identity built on that expression is true according to the theory.

Of course we could also say, more simply, that "*a*" is used to name an object if and only if the statement "*a* exists" is true for the theory. This is less satisfactory only insofar as the meaning of "exists" may have seemed less settled than quantifiers and identity. We may indeed take "$(\exists x)(x = a)$" as explicating "*a* exists." John Bacon has noted a nice parallel here: [3] just as "*a* eats" is short for "*a* eats something," so "*a* is" is short for "*a* is something."

An expression "*a*" may occur in a theory, we saw, with or without purporting to name an object. What clinches matters is rather the quantification "$(\exists x)(x = a)$." It is the existential quantifier, not the "*a*" itself, that carries existential import. This is just what existential quantification is for, of course. It is a logically regimented rendering of the "there is" idiom. The bound variable "*x*" ranges over the universe, and the existential quantification says that at least one of the objects in the universe satisfies the appended condition—in this case the condition of being the object *a*. To show that some given object is required in a theory, what we have to show is no more nor less than that that object is required, for the truth of the theory, to be among the values over which the bound variables range.

Appreciation of this point affords us more than an explication of "*a* exists," since the existentially quantified identity "$(\exists x)(x = a)$" is one case of existential quantification among

[3] J. Bacon, *Being and Existence*, dissertation, Yale, 1966.

many. It is a case where the value of the variable that is said to exist is an object with a name; the name is "*a*." This is the way with singular existence sentences generally, sentences of the form "*a* exists" or "There is such a thing as *a*," but it is not the way with existence sentences generally. For instance, under classical set theory there are, given any interpreted notation, some real numbers that are not separately specifiable in that notation. The existence sentence "There are unspecifiable real numbers" is true, and expressible as an existential quantification; but the values of the variable that account for the truth of this quantification are emphatically not objects with names. Here then is another reason why quantified variables, not names, are what to look to for the existential force of a theory.

Another way of saying what objects a theory requires is to say that they are the objects that some of the predicates of the theory have to be true of, in order for the theory to be true. But this is the same as saying that they are the objects that have to be values of the variables in order for the theory to be true. It is the same, anyway, if the notation of the theory includes for each predicate a complementary predicate, its negation. For then, given any value of a variable, some predicate is true of it; viz., any predicate or its complement. And conversely, of course, whatever a predicate is true of is a value of variables. Predication and quantification, indeed, are intimately linked; for a predicate is simply any expression that yields a sentence, an open sentence, when adjoined to one or more quantifiable variables. When we schematize a sentence in the predicative way "*Fa*," or "*a* is an *F*," our recognition of an "*a*" part and an "*F*" part turns strictly on our use of variables of quantification; the "*a*" represents a part of the sentence that stands where a quantifiable variable could stand, and the "*F*" represents the rest.

Our question was: what objects does a theory require? Our answer is: those objects that have to be values of variables for the theory to be true. Of course a theory may, in this sense, require no objects in particular, and still not tolerate an empty universe of discourse either, for the theory might be fulfilled equally by either of two mutually exclusive universes. If for example the theory implies "$(\exists x)(x$ is a dog$)$," it will not tolerate an empty universe; still the theory might be fulfilled by a universe that contained collies to the exclusion of spaniels, and also vice versa. So there is more to be said of a theory, ontologically, than just saying what objects, if any, the theory requires; we can also ask what various universes would be severally sufficient. The specific objects required, if any, are the objects common to all those universes.

I think mainly of single-sorted quantification; i.e., a single style of variables. As remarked, the many-sorted is translatable into one-sorted. Generally such translation has the side effect of admitting as meaningful some erstwhile meaningless predications. E.g., if the predicate "divisible by 3" is henceforth to be trained on general variables instead of number variables, we must make sense of calling things other than numbers divisible by 3. But this is easy; we may count such attributions false instead of meaningless. In general, thus, the reduction of many-sorted quantification to one-sorted has the effect of merging some substitution classes; more words become meaningfully interchangeable.

Carnap's reservations over *Allwörter* now cease to apply, and so his special strictures against philosophical questions of existence lapse as well. To what extent have we meanwhile become clearer on such questions of existence? On the higher-order question, what things a theory assumes there to be, we have gained a pointer: look to the behavior of quantified vari-

ables and don't cavil about names. Regarding the meaning of existence itself our progress is less clear.

Existence is what existential quantification expresses. There are things of kind F if and only if $(\exists x)Fx$. This is as unhelpful as it is undebatable, since it is how one explains the symbolic notation of quantification to begin with. The fact is that it is unreasonable to ask for an explication of existence in simpler terms. We found an explication of singular existence, "*a* exists," as "$(\exists x)(x = a)$"; but explication in turn of the existential quantifier itself, "there is," "there are," explication of general existence, is a forlorn cause. Further understanding we may still seek even here, but not in the form of explication. We may still ask what counts as evidence for existential quantifications.

To this question there is no simple, general answer. If the open sentence under the quantifier is something like "x is a rabbit" or "x is a unicorn," then the evidence, if any, is largely the testimony of the senses. If the open sentence is "x is a prime number between 10 and 20," the evidence lies in computation. If the open sentence is merely "x is a number," or "x is a class," or the like, the evidence is much harder to pinpoint. But I think the positivists were mistaken when they despaired of evidence in such cases and accordingly tried to draw up boundaries that would exclude such sentences as meaningless. Existence statements in this philosophical vein do admit of evidence, in the sense that we can have reasons, and essentially scientific reasons, for including numbers or classes or the like in the range of values of our variables. And other existence statements in this metaphysical vein can be subject to counter-evidence; we can have essentially scientific reasons for excluding propositions, perhaps, or attributes, or unactualized bodies, from the range of our variables. Numbers and classes are fa-

voured by the power and facility which they contribute to theoretical physics and other systematic discourse about nature. Propositions and attributes are disfavored by some irregular behaviour in connection with identity and substitution. Considerations for and against existence are more broadly systematic, in these philosophical examples, than in the case of rabbits or unicorns or prime numbers between 10 and 20; but I am persuaded that the difference is a matter of degree. Our theory of nature grades off from the most concrete fact to speculations about the curvature of space-time, or the continuous creation of hydrogen atoms in an expanding universe; and our evidence grades off correspondingly, from specific observation to broadly systematic considerations. Existential quantifications of the philosophical sort belong to the same inclusive theory and are situated way out at the end, farthest from observable fact.

Thus far I have been playing down the difference between commonsense existence statements, as of rabbits and unicorns, and philosophical existence statements, as of numbers and attributes. But there is also a curious difference between commonsense existence statements and philosophical ones that needs to be played up, and it is one that can be appreciated already right in among the rabbits. For let us reflect that a theory might accommodate all rabbit data and yet admit as values of its variables no rabbits or other bodies but only qualities, times, and places. The adherents of that theory, or *immaterialists*, would have a sentence which, as a whole, had the same stimulus meaning as our sentence "There is a rabbit in the yard"; yet in the quantificational sense of the words they would have to deny that there is a rabbit in the yard or anywhere else. Here, then, prima facie, are two senses of existence of rabbits, a common sense and a philosophical sense.

A similar distinction can be drawn in the case of the prime

numbers between 10 and 20. Suppose someone has for reasons of nominalism renounced most of mathematics and settled for bodies as sole values of his variables. He can still do such part of arithmetic as requires no variables. In particular he can still subscribe to the nine-clause alternation "11 is prime or 12 is prime or 13 is prime or . . . or 19 is prime." In this sense he agrees with us that there are primes between 10 and 20, but in the quantificational sense he denies that there are primes or numbers at all.

Shall we say: so much the worse for a quantificational version of existence? Hardly; we already found this version trivial but undebatable. Are there then two senses of existence? Only in a derivative way. For us common men who believe in bodies and prime numbers, the statements "There is a rabbit in the yard" and "There are prime numbers between 10 and 20" are free from double-talk. Quantification does them justice. When we come to the immaterialist, and we tell him there is a rabbit in the yard, he will know better than to demur on account of his theory; he will acquiesce on account of a known holophrastic relation of stimulus synonymy between our sentence and some sentence geared to his different universe. In practice he will even stoop to our idiom himself, both to facilitate communication and because of speech habits lingering from his own benighted youth. This he will do when the theoretical question is not at issue, just as we speak of the sun as rising. Insofar we may say, I grant, that there are for him two senses of existence; but there is no confusion, and the theoretical use is rather to be respected as literal and basic than deplored as a philosophical disorder.

Similar remarks apply to our nominalist. He will agree that there are primes between 10 and 20, when we are talking arithmetic and not philosophy. When we turn to philosophy he will condone that usage as a mere manner of speaking, and

offer the paraphrase. Similar remarks apply to us; many of our casual remarks in the "there are" form would want dusting up when our thoughts turn seriously ontological. Each time, if a point is made of it, the burden is of course on us to paraphrase or retract.

It has been fairly common in philosophy early and late to distinguish between being, as the broadest concept, and existence, as narrower. This is no distinction of mine; I mean "exists" to cover all there is, and such of course is the force of the quantifier. For those who do make the distinction, the existent tends to be on the concrete or temporal side. Now there was perhaps a reminder of the distinction in the case of the rabbit and the immaterialist. At that point two senses of "there is," a common and a philosophical, threatened to diverge. Perhaps the divergence which that sort of case suggests has been one factor in making philosophers receptive to a distinction between existence and being. Anyway, it ought not to. For the point there was that the rabbit was not a value of the immaterialist's variables; thus existence, if this were the analogy, would not be a species of being. Moreover, we saw that the sensible materiality of the rabbit was inessential to the example, since the prime numbers between 10 and 20 sustained much the same point.

Along with the annoying practice of restricting the term "existence" to a mere species of what there is, there is Meinong's bizarre deviation of an opposite kind. *Gegenstände* or objects, for him, comprised more even than what there was; an object might or might not be. His notion of object was, as Chisholm puts it, *jenseits von Sein und Nichtsein.*[4] Oddly enough I find

[4] Roderick M. Chisholm, "Jenseits von Sein und Nichtsein," K. S. Guthke, ed., *Dichtung und Deutung* (Bern and Munich: Francke, 1961).

this idea a good one, provided that we bolster it with Bentham's theory of fictions. Contextual definition, or what Bentham called paraphrasis, can enable us to talk very considerably and conveniently about putative objects without footing an ontological bill. It is a strictly legitimate way of making theories in which there is less than meets the eye.

Bentham's idea of paraphrasis flowered late, in Russell's theory of descriptions. Russell's theory affords a rigorous and important example of how expressions can be made to parade as names and then be explained away as a mere manner of speaking, by explicit paraphrase of the context into an innocent notation. However, Russell's theory of descriptions was less a way of simulating objects than of contextually defining terms to designate real objects. When the description fails to specify anything, Russell accommodates it grudgingly: he makes its immediate sentential contexts uniformly false.

Where we find Russell exploiting paraphrasis for simulation of objects is not in his theory of descriptions but rather in his contextual theory of classes. There are really no such things as classes, according to him, but he simulates discourse about classes by contextual definition, and not grudgingly; not just by making all immediate contexts false.

There is a well-known catch to Russell's theory of classes. The theory depends on an unheralded but irreducible assumption of attributes as values of bound variables. Russell only reduces classes to attributes, and this can scarcely be viewed as a reduction in the right direction unless for wrong reasons.

But it is possible by paraphrasis to introduce a certain amount of class talk, less than Russell's, without really assuming attributes or any other objects beyond the ones wanted as members of the simulated classes. I developed this line somewhat under the head of virtual classes, long ago, and Richard

Martin was at it independently at that time.[5] Lately I made much use of it in *Set Theory and Its Logic*. What it yields is substantial enough to implant new hopes, in many breasts, of making do with a nominalist ontology. Unfortunately these would have to be breasts unmindful of the needs of mathematics. For of itself the virtual theory of classes affords no adequate foundation for the classical mathematics even of the positive integers. However, it is handy still as a supplementary technique after we have bowed to the need of assuming real classes too; for it enables us to simulate further classes beyond those assumed. For that reason, and also because I think it good strategy in all subjects to postpone assumptions until needed, I am in favor of exploiting the virtual theory for all it is worth.

Virtual classes do not figure as values of bound variables. They owe their utility partly to a conventional use of schematic letters, which, though not quantifiable, behave like free variables. The simulated names of the virtual classes are substitutable for such letters. We could even call these letters free variables, if we resist the temptation to bind them. Virtual classes can then be seen as simulated values of these simulated variables. Hintikka has presented a logic, not specifically of classes but of entities and non-entities generally, in which the non-entities figure thus as values only of free variables.[6] Or, to speak less figuratively, the singular terms which fail to designate can be substituted only for free variables, whereas singu-

[5] R. M. Martin, "A homogeneous system of formal logic," *Journal of Symbolic Logic* 8 (1943), 1–23.

[6] Jaakko Hintikka, "Existential presuppositions and existential commitments," *Journal of Philosophy* 56 (1959), 125–137. For a bigger venture in this direction see Henry S. Leonard, "Essences, attributes, and predicates," *Proceedings and Addresses of the American Philosophical Association* 37 (1964), 25–51.

lar terms which do designate can be used also in instantiating quantification.

So much for simulated objects. I want now to go back and pick up a loose end where we were considering the immaterialist. I said he would fall in with our statement "There is a rabbit in the yard" just to convey agreement on the stimulus content, or even out of habit carried over from youth. But what about the alternative situation where the immaterialist is not a deviant Western intellectual, but a speaker of an unknown language which we are bent on construing? Suddenly the conditions themselves become problematical. In principle there is no difficulty in equating a sentence of his holophrastically, by stimulus meaning, with our sentence "There is a rabbit in the yard." But how could it ever be determined, even in probabilistic terms, that his ontology includes qualities, times, and places, and excludes bodies? I argued in *Word and Object* that such ontological questions regarding a radically alien language make no objective sense. In principle we could devise any of various sets of analytical hypotheses for translating the language into ours; many such sets can conform fully to all evidence and even be behaviorally equivalent to one another, and yet disagree with one another as to the native's equivalents of our predicates and quantifiers. For practical translation we fix on one of the adequate sets of analytical hypotheses, and in the light of it we report even on the native's ontology; but what to report is uniquely determined neither by evidence nor by fact. There is no fact of the matter.

Consider, in contrast, the truth functions. We can state substantial behavioral conditions for interpreting a native sentence connective as, say, alternation. The requirement is that the natives be disposed to dissent from any compound statement, formed by the connective in question, when and only when

disposed to dissent from each of the component statements, and that they be disposed to assent to the compound whenever disposed to assent to a component. These conditions remain indeed less than definitive on one point: on the question of a native's assenting to the compound but to neither component. For instance we may affirm of two horses that one or the other will win, and still not be prepared to affirm of either one that he will win.[7] Still, the two conditions do much toward identifying alternation; more than any behavioral conditions can do for quantification. And it is easy to do as well for the other truth functions as for alternation.

There is indeed a variant of quantification, favored by Leśniewski and by Ruth Marcus,[8] which does admit behavioural criteria of translation as substantial as those for the truth functions. I shall call it *substitutional* quantification. An existential substitutional quantification is counted as true if and only if there is an expression which, when substituted for the variable, makes the open sentence after the quantifier come out true. A universal quantification is counted as true if no substitution makes the open sentence come out false. Behavioral conditions for interpreting a native construction as existential substitutional quantification, then, are readily formulated. We fix on parts of the construction as candidates for the roles of quantifier and variable; then a condition of their fitness is that the natives be disposed to dissent from a whole quantified sentence when and only when disposed to dissent from each of the sentences obtainable by dropping the quantifier and substituting for the variable. A second condition is that the natives

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[7] In *Word and Object*, p. 58, I gave only the condition on dissent and so overlooked this limitation on the assent side. Conjunction suffered in equal and opposite fashion.

[8] See above, p. 63n.

be disposed to assent to the whole whenever disposed to assent to one of the sentences obtainable by dropping the quantifier and substituting for the variable. As in the case of alternation, the behavioural conditions do not wholly settle assent; but they go far. Analogous criteria for universal substitutional quantification are equally evident.

Naturally we never expect mathematical certainty as to whether such a behavioral criterion is fulfilled by a given construction in the native language. For any one choice of native locutions as candidates for the role of quantifier and variable, an infinite lot of quantified sentences and substitution instances would have to be tested. The behavioral criteria for the truth functions are similar in this respect. Empirical induction is all we have to go on, and all we would ask.

Substitutional quantification and the truth functions are, in brief, far and away more recognizable behaviorally than classical quantification, or what we may call *objectual* quantification. We can locate objectual quantification in our own language because we grow up using those very words: if not the actual quantifiers, then words like "exists" and "there is" by which they come to be explained to us. We can locate it in other languages only relative to chosen or inherited codes of translation which are in a sense arbitrary. They are arbitrary in the sense that they could be materially different and still conform to all the same behavior apart from the behavior of translation itself. Objectual quantification is in this sense more parochial than substitutional quantification and the truth functions.

In his substitutional quantification Leśniewski used different styles of variables for different substitution classes. Substitutional quantification in the substitution class of singular terms, or names, is the sort that comes closest to objectual quantifica-

tion. But it is clearly not equivalent to it—not unless each of our objects is specifiable by some singular term or other in our language, and no term of that substitution class fails to specify an object. For this reason substitutional quantification gives no acceptable version of existence properly so-called, not if objectual quantification does. Moreover, substitutional quantification makes good sense, explicable in terms of truth and substitution, no matter what substitution class we take—even that whose sole member is the left-hand parenthesis.[9] To conclude that entities are being assumed that trivially, and that far out, is simply to drop ontological questions. Nor can we introduce any control by saying that only substitutional quantification in the substitution class of singular terms is to count as a version of existence. We just now saw one reason for this, and there is another: the very notion of singular terms appeals implicitly to classical or objectual quantification. This is the point that I made earlier about analyzing sentences according to the scheme "*Fa.*" Leśniewski did not himself relate his kind of quantification to ontological commitments.

This does not mean that theories using substitutional quantification and no objectual quantification can get on *without* objects. I hold rather that the question of the ontological commitment of a theory does not properly arise except as that theory is expressed in classical quantificational form, or insofar as one has in mind how to translate it into that form. I hold this for the simple reason that the existential quantifier, in the objectual sense, is given precisely the existential interpretation and no other: there are things which are thus and so.

It is easy to see how substitutional quantification might be translated into a theory of standard form. Consider a substitutional quantification whose quantifier is existential and contains the variable v and governs the open sentence S. We can

9 Leśniewski's example, from a conversation of 1933 in Warsaw.

paraphrase it in syntactical and semantical terms, with objectual quantification, thus: there is an expression which, put for v in S, yields a truth. Universal quantification can be handled similarly. For this method the theory into which we translate is one that talks about expressions of the original theory, and assumes them among its objects—as values of its variables of objectual quantification. By arithmetized syntax, natural numbers would do as well. Thus we may look upon substitutional quantification not as avoiding all ontological commitment, but as getting by with, in effect, a universe of natural numbers.

Substitutional quantification has its points. If I could see my way to getting by with an all-purpose universe whose objects were denumerable and indeed enumerated, I would name each object numerically and settle for substitutional quantification. I would consider this an advance epistemologically, since substitutional quantification is behaviorally better determined than objectual quantification. Here then is a new reason, if one were needed, for aspiring to a denumerable universe.

In switching at that point to substitutional quantification we would not, as already stressed, reduce our denumerable universe to a null universe. We would, however, turn our backs on ontological questions. Where substitutional quantification serves, ontology lacks point. The ontology of such a theory is worth trying to elicit only when we are making translations or other comparisons between that theory and a theory which, because of an indenumerable or indefinite universe, is irreducibly committed to something like objectual quantification. Indenumerable and indefinite universes are what, in the end, give point to objectual quantification and ontology.[10]

10 The foregoing reflections on substitutional quantification were elicited largely by discussions with Burton Dreben. On the pointlessness of ontology at the denumerable level see also my *Ways of Paradox*, p. 203.

I urged that objectual quantification, more than substitutional quantification, is in a sense parochial. Then so is the idea of being; for objectual existential quantification was devised outright for "there is." But still one may ask, and Hảo Wang has asked, whether we do not represent being in an unduly parochial way when we equate it strictly with our own particular quantification theory to the exclusion of somewhat deviant quantification theories. Substitutional quantification indeed would not serve as an account of being, for reasons already noted; but what of intuitionistic quantification theory, or other deviations? [11] Now one answer is that it would indeed be a reasonable use of words to say that the intuitionist has a different doctrine of being from mine, as he has a different quantification theory; and that I am simply at odds with the intuitionist on the one as on the other. When I try to determine the universe of someone else's theory, I use "being" my way. In particular thus I might come out with a different inventory of an intuitionist's universe than the intuitionist, with his deviant sense of being, would come out with. Or I might simply see no satisfactory translation of his notation into mine, and so conclude that the question of his ontology cannot be raised in terms acceptable to me.

But this answer misses an important element in Wang's question. Namely, how much better than arbitrary is our particular quantification theory, seen as one in some possible spectrum of quantification theories? Misgivings in this direction can be fostered by noting the following form of sentence, due essentially to Henkin: [12]

[11] One such, propounded by Leonard, "Essences, attributes, and predicates," p. 39, combines substitutional and objectual quantification.
[12] Leon Henkin, "Some remarks on infinitely long formulas," *Infinitistic Methods* (proceedings of a Warsaw symposium) (New York: Pergamon, 1961), pp. 167–183, specifically p. 181.

(1) Each thing bears P to something y and each thing bears Q to something w such that Ryw.

The best we can do for this in ordinary quantificational terms is:

(2) $\qquad (x)(\exists y)(Pxy \cdot (z)(\exists w)(Qzw \cdot Ryw))$

or equally:

(3) $\qquad (z)(\exists w)(Qzw \cdot (x)(\exists y)(Pxy \cdot Ryw))$.

These are not equivalent. (2) represents the choice of y as independent of z; (3) does not. (3) represents the choice of w as independent of x; (2) does not. Moreover there are interpretations of P, Q, and R in (1) that make both dependences gratuitous; for instance, interpretation of P as "is part of," Q as "contains," and R as "is bigger than."

(4) Each thing is part of something y and each thing contains something w such that y is bigger than w.

One may suspect that the notation of quantification is at fault in forcing a choice between (2) and (3) in a case like this.

By admitting functions as values of our bound variables, Henkin observes, we can escape the limitations of (2) and (3) as follows:

(5) $\qquad (\exists f)(\exists g)(x)(z)(Pxf_x \cdot Qzg_z \cdot Rf_xg_z)$.

But this move assumes higher-order objects, which may seem out of keeping with the elementary character of (1). Henkin then points out a liberalization of the classical quantification notation which does the work of (5) without quantifying over functions. Just allow branching quantifiers, thus:

$$(x)(\exists y)$$
(6) $\qquad (Pxy \cdot Qzw \cdot Ryw)$.
$$(z)(\exists w)$$

One may feel, therefore, that an ontological standard geared to classical quantification theory is overcritical. It would interpret (4) as assuming functions, by interpreting it as (5),

whereas the deviant quantification theory with its branching quantifiers would interpret (4) more plausibly as not talking of any functions. And it would do so without slipping into the inappropriate bias of (2), or that of (3).

One is tempted further by the following considerations. The second-order formula (5) is of a kind that I shall call *functionally existential,* meaning that all its function quantifiers are out in front and existential. Now there is a well-known complete proof procedure of Skolem's for classical quantification theory, which consists in showing a formula inconsistent by taking what I call its functional normal form and deriving a truth-functional contradiction from it.[13] Anyone familiar with the procedure can quickly see that it works not only for all first-order formulas, that is, all formulas in the notation of classical quantification theory, but all these functionally existential formulas as well. Any inconsistent formula not only of classical quantification theory, but of this functionally existential annex, can be shown inconsistent by one and the same method of functional normal forms. This makes the annex seem pretty integral. One is tempted to seek further notational departures, in the first-orderish spirit of the branching quantifiers, which would suffice to accommodate all the functionally existential formulas the way (6) accommodates (5). Henkin has in fact devised a general notation of this kind.

By considerations of duality, moreover, these reflections upon functionally existential formulas can be paralleled with regard to functionally universal formulas—those whose function quantifiers are out in front and universal. Skolem's method of proving inconsistency has as its dual a method of proving validity, and it works not only for all first-order formulas but for all these functionally universal formulas as well. Thus this

13 See my *Selected Logic Papers,* pp. 196 ff.

still further annex would be every bit as integral as the functionally existential one. We seem to see our way, then, to so enlarging classical quantification theory as to gain all the extra power that would have been afforded by assuming functions, so long as the function quantifiers were out in front and all existential or all universal. It would mean a grateful slackening of our ontological accountability.

These reflections encourage the idea that our classical logic of quantification is arbitrarily restrictive. However, I shall now explain what I think to be a still weightier counter-consideration. The classical logic of quantification has a complete proof procedure for validity and a complete proof procedure for inconsistency; indeed each procedure serves both purposes, since a formula is valid if and only if its negation is inconsistent. The most we can say for the functionally existential annex, on the other hand, is that it has a complete proof procedure for inconsistency; and the most we can say for the functionally universal annex is that it has a complete proof procedure for validity. The trick of proving a formula valid by proving its negation inconsistent, or vice versa, is not applicable in the annexes, since in general the negation of a functionally existential formula is not equivalent to a functionally existential formula (but only to a functionally universal one), and conversely. In fact there is a theorem due to Craig[14] whch shows that the negation of a functionally existential formula is never equivalent to a functionally existential formula, unless the functions were superfluous and the formula was equivalent to a first-order formula; and correspondingly for functionally universal formulas. Thus classical, unsupplemented quantification theory is on this score maximal: it is as far out as you can go and still

14 William Craig, "Three uses of the Herbrand-Gentzen theorem," *Journal of Symbolic Logic* 22 (1957), 269–285, specifically 281.

have complete coverage of validity and inconsistency by the Skolem proof procedure.

Henkin even shows that the valid formulas which are quantified merely in the fourfold fashion shown in (5), or (6), are already more than can be covered by any proof procedure, at any rate when identity is included.[15]

Here then is a reason to draw boundaries in such a way as to regard (6) as talking covertly of functions after all, and as receiving a just analysis in (5). On this view (1) is not the proper business of pure quantification theory after all, but treats of functions. That is, if the form (1) is not to be read with the bias (2) or the bias (3), it is to be explained as (5).

We may be somewhat reconciled to this conclusion by an observation of Jean van Heijenoort, to the effect that (1) is not after all very ordinary language; its grammar is doubtful. Can the "such that" reach back across the "and" to cover the "y"? If assignment of meaning to extraordinary language is what we are about, we may indeed assign (5) and not wonder at its being irreducibly of second order.

Since introducing (1), I have proved nothing. I have explained two sorts of considerations, one to illustrate how we might be led to see the classical state of quantification theory as arbitrary, and the other to illustrate how it is better than arbitrary. Classical quantification theory enjoys an extraordi-

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[15] Henkin, "Some remarks on infinitely long formulas," p. 182 and footnote. Henkin derives this conclusion from a theorem of Mostowski by an argument which he credits to Ehrenfeucht.

I am indebted to Peter Geach for first bringing the question of (1) to my attention, in January 1960; and I am indebted to my colleagues Burton Dreben and Saul Kripke and my pupil Christopher Hill for steering me to pertinent papers. Dreben's advice has been helpful also elsewhere.

nary combination of depth and simplicity, beauty and utility. It is bright within and bold in its boundaries. Deviations from it are likely, in contrast, to look rather arbitrary. But insofar as they exist it seems clearest and simplest to say that deviant concepts of existence exist along with them.