

SELECTIONS from  
HUME on  
MATHEMATICS

RUSSELL MARCUS

Hume, David  
AN ENQUIRY CONCERNING HUMAN UNDERSTANDING  
Indianapolis: Hackett 1982

NOTICE  
This material may be  
protected by copyright  
law (Title 17 U.S. Code.)

from the ENQUIRY

Section IV: *Skeptical Doubts Concerning  
the Operations of the Understanding.*

Part I.

All the objects of human reason or inquiry may naturally be divided into two kinds, namely, *relations of ideas* and *matters of fact*. Of the first kind are the sciences of geometry, algebra, and arithmetic, and, in short, every affirmation which is either intuitively or demonstratively certain. *That the square of the hypotenuse is equal to the squares of the two sides* is a proposition which expresses a relation between these figures. *That three times five is equal to the half of thirty* expresses a relation between these numbers. Propositions of this kind are discoverable by the mere operation of thought, without dependence on what is anywhere existent in the universe. Though there never were a circle or triangle in nature, the truths demonstrated by Euclid would forever retain their certainty and evidence.

Matters of fact, which are the second objects of human reason, are not ascertained in the same manner; nor is our evidence of their truth, however great, of a like nature with the foregoing. The contrary of every matter of fact is still possible, because it can never imply a contradiction and is conceived by the mind with the same facility and distinctness, as if ever so conformable to reality. *That the sun will not rise tomorrow* is no less intelligible a proposition and implies no more contradiction than the affirmation that *it will rise*. We should in vain, therefore, attempt to demonstrate its falsehood. Were it demonstratively false, it would imply a contradiction and could never be distinctly conceived by the mind.

It may, therefore, be a subject worthy of curiosity to inquire what is the nature of that evidence which assures us of any real existence and matter of fact beyond the present testimony of our senses or the records of our memory. This part of philosophy, it is observable, has been little cultivated either by the ancients or moderns, and, therefore, our doubts and errors in the prosecution of so important an inquiry may be the more excusable, while we march through such difficult paths without any guide or direction. They may even prove useful by exciting curiosity and

destroying that implicit faith and security which is the bane of all reasoning and free inquiry. The discovery of defects in the common philosophy, if there are any, will not, I presume, be a discouragement, but rather an incitement, as is usual, to attempt something more full and satisfactory than has yet been proposed to the public.

All reasonings concerning matter of fact seem to be founded on the relation of *cause and effect*. By means of that relation alone we can go beyond the evidence of our memory and senses. If you were to ask a man why he believes any matter of fact which is absent—for instance, that his friend is in the country or in France—he would give you a reason, and this reason would be some other fact: as a letter received from him or the knowledge of his former resolutions and promises. A man finding a watch or any other machine on a desert island would conclude that there had once been men on that island. All our reasonings concerning fact are of the same nature. And here it is constantly supposed that there is a connection between the present fact and that which is inferred from it. Were there nothing to bind them together, the inference would be entirely precarious. The hearing of an articulate voice and rational discourse in the dark assures us of the presence of some person. Why? Because these are the effects of the human make and fabric, and closely connected with it. If we anatomize all the other reasonings of this nature, we shall find that they are founded on the relation of cause and effect and that this relation is either near or remote, direct or collateral. Heat and light are collateral effects of fire and the one effect may justly be inferred from the other.

If we would satisfy ourselves, therefore, concerning the nature of that evidence which assures us of matters of fact, we must inquire how we arrive at the knowledge of cause and effect.

I shall venture to affirm, as a general proposition which admits of no exception, that the knowledge of this relation is not, in any instance, attained by reasonings *a priori*, but arises entirely from experience when we find that any particular objects are constantly conjoined with each other. Let an object be presented to a man of ever so strong natural reason and abilities; if that object is entirely new to him, he will not be able, by the most accurate examination of its sensible qualities, to discover any of its causes or effects. Adam, though his rational faculties are supposed entirely perfect at the very first, could not have inferred from

the fluidity and transparency of water that it would suffocate him, or from the light and warmth of fire that it would consume him. No object ever discovers, by the qualities which appear to the senses, either the causes which produced it or the effects which will arise from it; nor can our reason, unassisted by experience, ever draw any inference concerning real existence and matter of fact.

This proposition, *that causes and effects are discoverable, not by reason but by experience*, will readily be admitted with regard to such objects as we remember to have once been altogether unknown to us, since we must be conscious of the utter inability which we then lay under of foretelling what would arise from them. Present two smooth pieces of marble to a man who has no tincture of natural philosophy; he will never discover that they will adhere together in such a manner as to require great force to separate them in a direct line, while they make so small a resistance to a lateral pressure. Such events as bear little analogy to the common course of nature are also readily confessed to be known only by experience, nor does any man imagine that the explosion of gunpowder or the attraction of a lodestone could ever be discovered by *a priori* arguments. In like manner, when an effect is supposed to depend upon an intricate machinery or secret structure of parts, we make no difficulty in attributing all our knowledge of it to experience. Who will assert that he can give the ultimate reason why milk or bread is proper nourishment for a man, not for a lion or a tiger?

But the same truth may not appear at first sight to have the same evidence with regard to events which have become familiar to us from our first appearance in the world, which bear a close analogy to the whole course of nature, and which are supposed to depend on the simple qualities of objects without any secret structure of parts. We are apt to imagine that we could discover these effects by the mere operation of our reason without experience. We fancy that were we brought, all of the sudden, into this world, we could at first have inferred that one billiard ball would communicate motion to another upon impulse and that we did not need to have waited for the event in order to pronounce with certainty concerning it. Such is the influence of custom that where it is strongest

it not only covers our natural ignorance, but even conceals itself and seems not to take place, merely because it is found in the highest degree.

But to convince us that all the laws of nature and all the operations of bodies without exception are known only by experience, the following reflections may perhaps suffice. Were any object presented to us and were we required to pronounce concerning the effect which will result from it without consulting past observation, after what manner, I beseech you, must the mind proceed in this operation? It must invent or imagine some event which it ascribes to the object as its effect and it is plain that this invention must be entirely arbitrary. The mind can never possibly find the effect in the supposed cause by the most accurate scrutiny and examination. For the effect is totally different from the cause and consequently can never be discovered in it. Motion in the second billiard ball is a quite distinct event from motion in the first, nor is there anything in the one to suggest the smallest hint of the other. A stone or piece of metal raised into the air and left without any support immediately falls. But to consider the matter *a priori*, is there anything we discover in this situation which can beget the idea of a downward rather than an upward or any other motion in the stone or metal?

And as the first imagination or invention of a particular effect in all natural operations is arbitrary where we do not consult experience, so must we also esteem the supposed tie or connection between the cause and effect which binds them together and renders it impossible that any other effect could result from the operation of that cause. When I see, for instance, a billiard ball moving in a straight line towards another, even suppose motion in the second ball should by accident be suggested to me as the result of their contact or impulse, may I not conceive that a hundred different events might as well follow from that cause? May not both these balls remain at absolute rest? May not the first ball return in a straight line or leap off from the second in any line or direction? All these suppositions are consistent and conceivable. Why then should we give the preference to one which is no more consistent or conceivable than the rest? All our reasonings *a priori* will never be able to show us any foundation for this preference.

ENQUIRY  
§ 12

In a word, then, every effect is a distinct event from its cause. It could not, therefore, be discovered in the cause and the first invention or conception of it, *a priori*, must be entirely arbitrary. And even after it is suggested, the conjunction of it with the cause must appear equally arbitrary, since there are always many other effects which, to reason, must seem fully as consistent and natural. In vain, therefore, should we pretend to determine any single event or infer any cause or effect without the assistance of observation and experience.

Hence we may discover the reason why no philosopher who is rational and modest has ever pretended to assign the ultimate cause of any natural operation or to show distinctly the action of that power which produces any single effect in the universe. It is confessed that the utmost effort of human reason is to reduce the principles productive of natural phenomena to a greater simplicity and to resolve the many particular effects into a few general causes by means of reasonings from analogy, experience, and observation. But as to the causes of these general causes, we should in vain attempt their discovery, nor shall we ever be able to satisfy ourselves by any particular explication of them. These ultimate springs and principles are totally shut up from human curiosity and inquiry. Elasticity, gravity, cohesion of parts, communication of motion by impulse—these are probably the ultimate causes and principles which we shall ever discover in nature; and we may esteem ourselves sufficiently happy if, by accurate inquiry and reasoning, we can trace up the particular phenomena to, or near to, these general principles. The most perfect philosophy of the natural kind only staves off our ignorance a little longer, as perhaps the most perfect philosophy of the moral or metaphysical kind serves only to discover larger portions of it. Thus the observation of human blindness and weakness is the result of all philosophy and meets us at every turn in spite of our endeavors to elude or avoid it.

Nor is geometry, when taken into the assistance of natural philosophy, ever able to remedy this defect or lead us into the knowledge of ultimate causes by all that accuracy of reasoning for which it is so justly celebrated. Every part of mixed mathematics proceeds upon the supposition that certain laws are established

by nature in her operations and abstract reasonings are employed either to assist experience in the discovery of these laws or to determine their influence in particular instances where it depends upon any precise degree of distance and quantity. Thus, it is a law of motion, discovered by experience, that the moment or force of any body in motion is in the compound ratio or proportion of its solid contents and its velocity, and consequently that a small force may remove the greatest obstacle or raise the greatest weight if, by any contrivance or machinery, we can increase the velocity of that force so as to make it an overmatch for its antagonist. Geometry assists us in the application of this law by giving us the just dimensions of all the parts and figures which can enter into any species of machine, but still the discovery of the law itself is owing merely to experience and all the abstract reasonings in the world could never lead us one step towards the knowledge of it. When we reason *a priori* and consider merely any object or cause as it appears to the mind, independent of all observation, it never could suggest to us the notion of any distinct object, such as its effect, much less show us the inseparable and inviolable connection between them. A man must be very sagacious who could discover by reasoning that crystal is the effect of heat, and ice of cold, without being previously acquainted with the operation of these qualities.

### Part III.

There is, indeed, a more *mitigated* skepticism or *academic* philosophy which may be both durable and useful and which may, in part, be the result of this Pyrrhonism or *excessive* skepticism when its undistinguished doubts are in some measure corrected by common sense and reflection. The greater part of mankind are naturally apt to be affirmative and dogmatic in their opinions, and while they see objects only on one side and have no idea of any counterpoising argument, they throw themselves precipitately into the principles to which they are inclined, nor have they any indulgence for those who entertain opposite sentiments. To hesitate or balance perplexes their understanding, checks their passion, and suspends their action. They are, therefore, impatient, until they escape from a state which to them is so uneasy and they think that they can never remove themselves far enough from it by the violence of their affirmations and obstinacy of their belief. But could such dogmatic reasoners become sensible of the strange infirmities of human understanding, even in its most perfect state and when most accurate and cautious in its determinations, such a reflection would naturally inspire them with more modesty and reserve, and diminish their fond opinion of themselves and their prejudice against antagonists. The illiterate may reflect on the disposition of the learned who, amid all the advantages of study and reflection, are commonly still diffident in their determinations. And if any of the learned are inclined, from their natural temper, to haughtiness and obstinacy, a small tincture of Pyrrhonism might abate their pride by showing

them that the few advantages which they may have attained over their fellows are but inconsiderable, if compared with the universal perplexity and confusion which is inherent in human nature. In general, there is a degree of doubt and caution and modesty which, in all kinds of scrutiny and decision, ought forever accompany a just reasoner.

Another species of *mitigated* skepticism which may be of advantage to mankind and which may be the natural result of the Pyrrhonian doubts and scruples is the limitation of our inquiries to such subjects as are best adapted to the narrow capacity of human understanding. The *imagination* of man is naturally sublime, delighted with whatever is remote and extraordinary, and running without control into the most distant parts of space and time in order to avoid the objects which custom has rendered too familiar to it. A correct *judgment* observes a contrary method and, avoiding all distant and high inquiries, confines itself to common life and to such subjects as fall under daily practice and experience, leaving the more sublime topics to the embellishment of poets and orators or to the arts of priests and politicians. To bring us to so salutary a determination, nothing can be more serviceable than to be once thoroughly convinced of the force of the Pyrrhonian doubt and of the impossibility that anything but the strong power of natural instinct could free us from it. Those who have a propensity to philosophy will still continue their researches, because they reflect that, besides the immediate pleasure attending such an occupation, philosophical decisions are nothing but the reflections of common life, methodized and corrected. But they will never be tempted to go beyond common life so long as they consider the imperfection of those faculties which they employ, their narrow reach, and their inaccurate operations. While we cannot give a satisfactory reason why we believe, after a thousand experiments, that a stone will fall or fire burn, can we ever satisfy ourselves concerning any determination which we may form with regard to the origin of worlds and the situation of nature from and to eternity?

This narrow limitation, indeed, of our inquiries is in every respect so reasonable that it suffices to make the slightest examination into the natural powers of the human mind and to compare them with their

objects, in order to recommend it to us. We shall then find what are the proper subjects of science and inquiry.

It seems to me that the only objects of the abstract sciences or of demonstration are quantity and number and that all attempts to extend this more perfect species of knowledge beyond these bounds are mere sophistry and illusion. As the component parts of quantity and number are entirely similar, their relations become intricate and involved and nothing can be more curious, as well as useful, than to trace, by a variety of mediums, their equality or inequality through their different appearances. But as all other ideas are clearly distinct and different from each other, we can never advance further, by our utmost scrutiny, than to observe this diversity and, by an obvious reflection, pronounce one thing not to be another. Or if there is any difficulty in these decisions, it proceeds entirely from the indeterminate meaning of words, which is corrected by more just definitions. That *the square of the hypotenuse is equal to the squares of the other two sides* cannot be known, let the terms be ever so exactly defined, without a train of reasoning and inquiry. But to convince us of this proposition, *that where there is no property there can be no injustice*, it is only necessary to define the terms and explain injustice to be a violation of property. This proposition is, indeed, nothing but a more imperfect definition. It is the same case with all those pretended syllogistic reasonings which may be found in every other branch of learning except the sciences of quantity and number; and these may safely, I think, be pronounced the only proper objects of knowledge and demonstration.

All other inquiries of men regard only matter of fact and existence and these are evidently incapable of demonstration. Whatever *is* may *not be*. No negation of a fact can involve a contradiction. The nonexistence of any being, without exception, is as clear and distinct an idea as its existence. The proposition which affirms it not to be, however false, is no less conceivable and intelligible than that which affirms it to be. The case is different with the sciences, properly so called. Every proposition which is not true is there confused and unintelligible. That the cube root of 64 is equal to the half of 10 is a false proposition and

can never be distinctly conceived. But that Caesar, or the angel Gabriel, or any being, never existed may be a false proposition, but still is perfectly conceivable, and implies no contradiction.

The existence, therefore, of any being can only be proved by arguments from its cause or its effect and these arguments are founded entirely on experience. If we reason *a priori*, anything may appear able to produce anything. The falling of a pebble may, for all we know, extinguish the sun, or the wish of a man control the planets in their orbits. It is only experience which teaches us the nature and bounds of cause and effect and enables us to infer the existence of one object from that of another.<sup>35</sup> Such is the foundation of moral reasoning, which forms the greater part of human knowledge and is the source of all human action and behavior.

Moral reasonings are either concerning particular or general facts. All deliberations in life regard the former, as also all disquisitions in history, chronology, geography, and astronomy.

The sciences which treat of general facts are politics, natural philosophy, physics, chemistry, etc., where the qualities, causes, and effects of a whole species of objects are inquired into.

Divinity or theology, as it proves the existence of a Deity and the immortality of souls, is composed partly of reasonings concerning particular, partly concerning general facts. It has a foundation in *reason* so far as it is supported by experience. But its best and most solid foundation is *faith* and divine revelation.

Morals and criticism are not so properly objects of the understanding as of taste and sentiment. Beauty, whether moral or natural, is felt more properly than perceived. Or if we reason concerning it and endeavor to fix its standard, we regard a new fact, namely, the general taste of mankind or some such fact which may be the object of reasoning and inquiry.

When we run over libraries, persuaded of these

35. That impious maxim of the ancient philosophy, *Ex nihilo, nihil fit* [from nothing, comes nothing], by which the creation of matter was excluded, ceases to be a maxim according to this philosophy. Not only the will of the supreme Being may create matter, but, for all we know *a priori*, the will of any other being might create it or any other cause that the most whimsical imagination can assign.

principles, what havoc must we make? If we take in our hand any volume—of divinity or school metaphysics, for instance—let us ask: *Does it contain any abstract reasoning concerning quantity or number?* No. *Does it contain any experimental reasoning concerning matter of fact and existence?* No. Commit it then to the flames, for it can contain nothing but sophistry and illusion.

from TREATISE, BOOK I, PART I

### Section 7: Of Abstract Ideas

A very material question has been started concerning *abstract or general ideas, whether they be general or particular in the mind's conception of them.* A great philosopher<sup>5</sup> has disputed the received opinion in this particular and has asserted that all general ideas are nothing but particular ones annexed to a certain term which gives them a more extensive signification and makes them recall upon occasion other individuals which are similar to them. As I look upon this to be one of the greatest and most valuable discoveries that has been made of late years in the republic of letters, I shall here endeavor to confirm it by some arguments which I hope will put it beyond all doubt and controversy.

It is evident that, in forming most of our general ideas, if not all of them, we abstract from every particular degree of quantity and quality, and that an object does not cease to be of any particular species on account of every small alteration in its extension, duration, and other properties. It may, therefore, be thought that here is a plain dilemma that decides concerning the nature of those abstract ideas which have afforded so much speculation to philosophers. The abstract idea of a man represents men of all sizes and all qualities, which, it is concluded it cannot do but either by representing at once all possible sizes and all possible qualities or by representing no particular one at all. Now, it having been esteemed absurd to defend the former proposition as implying an infinite capacity in the mind, it has been commonly inferred in favor of the latter, and our abstract ideas have been supposed to represent no particular degree either of quantity or quality. But that this inference is erroneous, I shall endeavor to make appear, *first,*

5. Dr. Berkeley.

by proving that it is utterly impossible to conceive any quantity or quality without forming a precise notion of its degrees; and, *secondly,* by showing that though the capacity of the mind be not infinite, yet we can at once form a notion of all possible degrees of quantity and quality in such a manner at least as, however imperfect, may serve all the purposes of reflection and conversation.

To begin with the first proposition *that the mind cannot form any notion of quantity or quality without forming a precise notion of degrees of each,* we may prove this by the three following arguments. First, we have observed that whatever objects are different are distinguishable and that whatever objects are distinguishable are separable by the thought and imagination. And we may here add that these propositions are equally true in the *inverse*, and that whatever objects are separable are also distinguishable, and that whatever objects are distinguishable are also different. For how is it possible we can separate what is not distinguishable or distinguish what is not different? In order, therefore, to know whether abstraction implies a separation, we need only consider it in this view and examine whether all the circumstances which we abstract from in our general ideas be such as are distinguishable and different from those which we retain as essential parts of them. But it is evident at first sight that the precise length of a line is not different nor distinguishable from the line itself, nor the precise degree of any quality from the quality. These ideas, therefore, admit no more of separation than they do of distinction and difference. They are consequently conjoined with each other in the conception, and the general idea of a line, notwithstanding all our abstractions and refinements, has, in its appearance in the mind, a precise degree of quantity and quality, however it may be made to represent others which have different degrees of both.

Secondly, it is confessed that no object can appear to the senses, or in other words that no impression can become present to the mind, without being determined in its degrees both of quantity and quality. The confusion, in which impressions are sometimes involved, proceeds only from their faintness and unsteadiness, not from any capacity in the mind

to receive any impression which in its real existence has no particular degree nor proportion. That is a contradiction in terms and even implies the flattest of all contradictions, namely, that it is possible for the same thing both to be and not to be.

Now, since all ideas are derived from impressions and are nothing but copies and representations of them, whatever is true of the one must be acknowledged concerning the other. Impressions and ideas differ only in their strength and vivacity. The foregoing conclusion is not founded on any particular degree of vivacity. It cannot, therefore, be affected by any variation in that particular. An idea is a weaker impression, and as a strong impression must necessarily have a determinate quantity and quality, the case must be the same with its copy or representative.

Thirdly, it is a principle generally received in philosophy that everything in nature is individual and that it is utterly absurd to suppose a triangle really existent which has no precise proportion of sides and angles. If this, therefore, be absurd *in fact and reality*, it must also be absurd *in idea*, since nothing of which we can form a clear and distinct idea is absurd and impossible. But to form the idea of an object and to form an idea simply is the same thing, the reference of the idea to an object being an extraneous denomination of which in itself it bears no mark or character. Now, as it is impossible to form an idea of an object that is possessed of quantity and quality and yet is possessed of no precise degree of either, it follows that there is an equal impossibility of forming an idea that is not limited and confined in both these particulars. Abstract ideas are, therefore, in themselves individual, however they may become general in their representation. The image in the mind is only that of a particular object, though the application of it in our reasoning be the same as if it were universal.

This application of ideas beyond their nature proceeds from our collecting all their possible degrees of quantity and quality in such an imperfect manner as may serve the purposes of life, which is the second proposition I proposed to explain. When we have found a resemblance among several objects that often occur to us, we apply the same name to all of them, whatever differences we may observe in the degrees of their quantity and quality and whatever other differences may appear among them. After we have ac-

SELECTIONS from HUME on MATHEMATICS P. 5



quired a custom of this kind, the hearing of that name revives the idea of one of these objects and makes the imagination conceive it with all its particular circumstances and proportions. But as the same word is supposed to have been frequently applied to other individuals that are different in many respects from that idea which is immediately present to the mind, the word not being able to revive the idea of all these individuals only touches the soul, if I may be allowed so to speak, and revives that custom which we have acquired by surveying them. They are not really and in fact present to the mind but only in power, nor do we draw them all out distinctly in the imagination, but keep ourselves in a readiness to survey any of them as we may be prompted by a present design or necessity. The word raises up an individual idea, along with a certain custom, and that custom produces any other individual one for which we may have occasion. But as the production of all the ideas to which the name may be applied is in most cases impossible, we abridge that work by a more partial consideration and find but few inconveniences to arise in our reasoning from that abridgment.

For this is one of the most extraordinary circumstances in the present affair, that after the mind has produced an individual idea upon which we reason, the attendant custom, revived by the general or abstract term, readily suggests any other individual, if by chance we form any reasoning that does not agree with it. Thus, should we mention the word triangle and form the idea of a particular equilateral one to correspond to it, and should we afterwards assert that *the three angles of a triangle are equal to each other*, the other individuals of a scalenum and isosceles, which we overlooked at first, immediately crowd in upon us and make us perceive the falsehood of this proposition, though it be true with relation to that idea which we had formed. If the mind does not suggest always these ideas upon occasion, it proceeds from some imperfection in its faculties, and such a one as is often the source of false reasoning and sophistry. But this is principally the case with those ideas which are abstruse and compounded. On other occasions the custom is more entire and it is seldom we run into such errors.

No, so entire is the custom that the very same idea

may be annexed to several different words and may be employed in different reasonings without any danger of mistake. Thus the idea of an equilateral triangle of an inch perpendicular may serve us in talking of a figure, of a rectilinear figure, of a regular figure, of a triangle, and of an equilateral triangle. All these terms, therefore, are in this case attended with the same idea, but as they are wont to be applied in a greater or lesser compass, they excite their particular habits and thereby keep the mind in a readiness to observe that no conclusion be formed contrary to any ideas which are usually comprised under them.

Before those habits have become entirely perfect, perhaps the mind may not be content with forming the idea of only one individual, but may run over several, in order to make itself comprehend its own meaning and the compass of that collection which it intends to express by the general term. That we may fix the meaning of the word *figure*, we may revolve in our mind the ideas of circles, squares, parallelograms, triangles of different sizes and proportions, and may not rest on one image or idea. However this may be, it is certain that we form the idea of individuals whenever we use any general term, that we seldom or never can exhaust these individuals, and that those which remain are only represented by means of that habit by which we recall them whenever any present occasion requires it. This, then, is the nature of our abstract ideas and general terms, and it is after this manner we account for the foregoing paradox that *some ideas are particular in their nature, but general in their representation*. A particular idea becomes general by being annexed to a general term, that is, to a term which, from a customary conjunction, has a relation to many other particular ideas and readily recalls them in the imagination.

The only difficulty that can remain on this subject must be with regard to that custom which so readily recalls every particular idea for which we may have occasion and is excited by any word or sound to which we commonly annex it. The most proper method, in my opinion, of giving a satisfactory explication of this act of the mind is by producing other instances which are analogous to it and other principles which facilitate its operation. To explain the ultimate causes of our mental actions is impossible. It is sufficient if

we can give any satisfactory account of them from experience and analogy.

First, then, I observe that when we mention any great number, such as a thousand, the mind has generally no adequate idea of it, but only a power of producing such an idea by its adequate idea of the decimals under which the number is comprehended. This imperfection, however, in our ideas is never felt in our reasonings, which seems to be an instance parallel to the present one of universal ideas.

Secondly, we have several instances of habits which may be revived by one single word, as when a person who has, by rote, any periods of a discourse or any number of verses will be put in remembrance of the whole which he is at a loss to recollect by that single word or expression with which they begin.

Thirdly, I believe everyone who examines the situation of his mind in reasoning will agree with me that we do not annex distinct and complete ideas to every term we make use of and that in talking of *government*, *church*, *negotiation*, *conquest*, we seldom spread out in our minds all the simple ideas of which these complex ones are composed. It is, however, observable that notwithstanding this imperfection, we may avoid talking nonsense on these subjects and may perceive any repugnance among the ideas as well as if we had a full comprehension of them. Thus, if instead of saying that *in war the weaker have always recourse to negotiation*, we should say that *they have always recourse to conquest*, the custom which we have acquired of attributing certain relations to ideas still follows the words and makes us immediately perceive the absurdity of that proposition, in the same manner as one particular idea may serve us in reasoning concerning other ideas, however different from it in several circumstances.

Fourthly, as the individuals are collected together and placed under a general term with a view to that resemblance which they bear to each other, this relation must facilitate their entrance in the imagination and make them be suggested more readily upon occasion. And, indeed, if we consider the common progress of the thought, either in reflection or conversation, we shall find great reason to be satisfied in this particular. Nothing is more admirable than the readiness with which the imagination suggests its ideas

and presents them at the very instant in which they become necessary or useful. The fancy runs from one end of the universe to the other in collecting those ideas which belong to any subject. One would think the whole intellectual world of ideas was at once subjected to our view and that we did nothing but pick out such as were most proper for our purpose. There may not, however, be any present besides those very ideas that are thus collected by a kind of magical faculty in the soul which, though it be always most perfect in the greatest geniuses and is properly what we call a genius, is, however, inexplicable by the utmost efforts of human understanding.

Perhaps these four reflections may help to remove all difficulties to the hypothesis I have proposed concerning abstract ideas so contrary to that which has prevailed in philosophy up to now. But to tell the truth, I place my chief confidence in what I have already proved concerning the impossibility of general ideas according to the common method of explaining them. We must certainly seek some new system on this head and there plainly is none besides what I have proposed. If ideas be particular in their nature and at the same time finite in their number, it is only by custom they can become general in their representation and contain an infinite number of other ideas under them.

Before I leave this subject, I shall employ the same principles to explain that *distinction of reason* which is so much talked of and is so little understood in the schools. Of this kind is the distinction between figure and the body figured, motion and the body moved. The difficulty of explaining this distinction arises from the principle above explained that *all ideas which are different are separable*. For it follows from thence that if the figure be different from the body, their ideas must be separable as well as distinguishable; if they be not different, their ideas can neither be separable nor distinguishable. What then is meant by a distinction of reason, since it implies neither a difference nor separation?

To remove this difficulty, we must have recourse to the foregoing explication of abstract ideas. It is certain that the mind would never have dreamed of distinguishing a figure from the body figured, as being in reality neither distinguishable, nor different, nor

separable, did it not observe that even in this simplicity there might be contained many different resemblances and relations. Thus, when a globe of white marble is presented, we receive only the impression of a white color disposed in a certain form, nor are we able to separate and distinguish the color from the form. But observing afterwards a globe of black marble and a cube of white and comparing them with our former object, we find two separate resemblances in what formerly seemed and really is perfectly inseparable. After a little more practice of this kind, we begin to distinguish the figure from the color by a *distinction of reason*, that is, we consider the figure and color together, since they are, in effect, the same and undistinguishable, but still view them in different aspects, according to the resemblances of which they are susceptible. When we would consider only the figure of the globe of white marble, we form in reality an idea both of the figure and color, but tacitly carry our eye to its resemblance with the globe of black marble and in the same manner, when we would consider its color only, we turn our view to its resemblance with the cube of white marble. By this means we accompany our ideas with a kind of reflection of which custom renders us, in a great measure, insensible. A person who desires us to consider the figure of a globe of white marble without thinking on its color, desires an impossibility, but his meaning is that we should consider the color and figure together, but still keep in our eye the resemblance to the globe of black marble or that to any other globe of whatever color or substance.

## TREATISE, Book I

### Part III. Of Knowledge and Probability

#### Section 1: Of Knowledge

There are seven different kinds of philosophical relation,<sup>7</sup> namely, *resemblance, identity, relations of time and place, proportion in quantity or number, degrees in any quality, contrariety, and causation*. These relations may be divided into two classes: into such as depend entirely on the ideas which we compare together and such as may be changed without any

change in the ideas. It is from the idea of a triangle that we discover the relation of equality which its three angles bear to two right ones, and this relation is invariable as long as our idea remains the same. On the contrary, the relations of *contiguity* and *distance* between two objects may be changed merely by an alteration of their place, without any change on the objects themselves or on their ideas, and the place depends on a hundred different accidents which cannot be foreseen by the mind. It is the same case with *identity* and *causation*. Two objects, though perfectly resembling each other and even appearing in the same place at different times, may be numerically different and, as the power by which one object produces another is never discoverable merely from their idea, it is evident that *cause* and *effect* are relations of which we receive information from experience and not from any abstract reasoning or reflection. There is no single phenomenon, even the most simple, which can be accounted for from the qualities of the objects as they appear to us or which we could foresee without the help of our memory and experience.

It appears therefore that of these seven philosophical relations, there remain only four which, depending solely upon ideas, can be the objects of knowledge and certainty. These four are *resemblance, contrariety, degrees in quality, and proportions in quantity or number*. Three of these relations are discoverable at first sight and fall more properly under the province of intuition than demonstration. When any objects resemble each other, the resemblance will at first strike the eye, or rather the mind, and seldom requires a second examination. The case is the same with *contrariety* and with the *degrees* of any *quality*. No one can once doubt but existence and non-existence destroy each other and are perfectly incompatible and contrary. And though it be impossible to judge exactly of the degrees of any quality, such as color, taste, heat, cold, when the difference between them is very small, yet it is easy to decide that any of them is superior or inferior to another when their difference is considerable. And this decision we always pronounce at first sight, without any inquiry or reasoning.

We might proceed after the same manner in fixing the *proportions* of *quantity* or *number* and might at one view observe a superiority or inferiority between

7. Part IV, sec. 2.

8. Part I, sec. 5.

any numbers or figures, especially where the difference is very great and remarkable. As to equality or any exact proportion, we can only guess at it from a single consideration, except in very short numbers or very limited portions of extension which are comprehended in an instant and where we perceive an impossibility of falling into any considerable error. In all other cases we must settle the proportions with some liberty or proceed in a more *artificial* manner.

I have already observed that geometry, or the *art* by which we fix the proportions of figures, though it much excels the loose judgments of the senses and imagination both in universality and exactness, yet never attains a perfect precision and exactness. Its first principles are still drawn from the general appearance of the objects, and that appearance can never afford us any security when we examine the prodigious minuteness of which nature is susceptible. Our ideas seem to give a perfect assurance that no two right lines can have a common segment, but if we consider these ideas, we shall find that they always suppose a sensible inclination of the two lines and that where the angle they form is extremely small, we have no standard of a right line so precise as to assure us of the truth of this proposition. It is the same case with most of the primary decisions of the mathematics.

There remain, therefore, algebra and arithmetic as the only sciences in which we can carry on a chain of reasoning to any degree of intricacy and yet preserve a perfect exactness and certainty. We are possessed of a precise standard by which we can judge of the equality and proportion of numbers and, according as they correspond or not to that standard, we determine their relations without any possibility of error. When two numbers are so combined as that the one has always a unit answering to every unit of the other, we pronounce them equal, and it is for want of such a standard of equality in extension that geometry can scarcely be esteemed a perfect and infallible science.

But here it may not be amiss to obviate a difficulty which may arise from my asserting that though geometry falls short of that perfect precision and certainty which are peculiar to arithmetic and algebra, yet it excels the imperfect judgments of our senses and imagination. The reason why I impute any defect

to geometry is because its original and fundamental principles are derived merely from appearances, and it may perhaps be imagined that this defect must always attend it and keep it from ever reaching a greater exactness in the comparison of objects or ideas than what our eye or imagination alone is able to attain. I admit that this defect attends it so far as to keep it from ever aspiring to a full certainty, but since these fundamental principles depend on the easiest and least deceitful appearances, they bestow on their consequences a degree of exactness of which these consequences are singly incapable. It is impossible for the eye to determine the angles of a chiliagon to be equal to 1,996 right angles or make any conjecture that approaches this proportion, but when it determines that right lines cannot concur, that we cannot draw more than one right line between two given points, its mistakes can never be of any consequence. And this is the nature and use of geometry, to run us up to such appearances as by reason of their simplicity cannot lead us into any considerable error.

I shall here take occasion to propose a second observation concerning our demonstrative reasonings which is suggested by the same object of the mathematics. It is usual with mathematicians to pretend that those ideas which are their objects are of so refined and spiritual a nature that they do not fall under the conception of the fancy, but must be comprehended by a pure and intellectual view of which the superior faculties of the soul are alone capable. The same notion runs through most parts of philosophy and is principally made use of to explain our abstract ideas and to show how we can form an idea of a triangle, for instance, which shall neither be an isosceles nor scalenum, nor be confined to any particular length and proportion of sides. It is easy to see why philosophers are so fond of this notion of some spiritual and refined perceptions, since by that means they cover many of their absurdities and may refuse to submit to the decisions of clear ideas by appealing to such as are obscure and uncertain. But to destroy this artifice, we need but reflect on that principle so oft insisted on *that all our ideas are copied from our impressions*. For from thence we may immediately conclude that since all impressions are clear and precise, the ideas which are copied from them

must be of the same nature and can never, but from our fault, contain anything so dark and intricate. An idea is by its very nature weaker and fainter than an impression, but, being in every other respect the same, cannot imply any very great mystery. If its weakness render it obscure, it is our business to remedy that defect as much as possible by keeping the idea steady and precise, and until we have done so, it is in vain to pretend to reasoning and philosophy.