

(for it is part of the mixture). Now even if it happens that the same thing is a principle both as matter and as mover, still *being* them is not the same. In which respect then is love a principle? It is paradoxical also that strife should be imperishable; strife is for him the nature of the bad.

Anaxagoras makes the good a motive principle; for thought moves things, but moves them for the sake of something, which must be something other than it, except according to *our* way of stating the case; for the medical art is in a sense health. It is paradoxical also not to suppose a contrary to the good, i.e. to thought. But all who speak of the contraries make no use of the contraries, unless we bring their views into shape. And why some things are perishable and others imperishable, no one tells us; for they make all existing things out of the same principles. Further, some make existing things out of the non-existent, and others to avoid the necessity of this make all things one.

Further, why should there always be becoming, and what is the cause of becoming?—this no one tells us. And those who suppose two principles must suppose another, a superior principle, and so must those who believe in the Forms; for why did things come to participate, or why do they participate, in the Forms? And all other thinkers are confronted by the necessary consequence that there is something contrary to Wisdom, i.e. to the highest knowledge; but *we* are not. For there is nothing contrary to that which is primary (for all contraries have matter and are potentially); and the ignorance which is contrary would lead us to a contrary object; but what is primary has no contrary.

Again, if besides sensible things no others exist, there will be no first principle, no order, no becoming, no heavenly bodies, but each principle will have a principle before it, as in the accounts of the mythologists and all the natural philosophers. But if the Forms or the numbers are to exist, they will be causes of nothing; or if not that, at least not of movement.

Further, how is extension, i.e. a *continuum*, to be produced out of unextended parts? For number will not, either as mover or as form, produce a *continuum*. But again there cannot be any contrary that is also a productive or moving principle; for it would be possible for it not to be. Or at least its action would be posterior to its capacity. The world then would not be eternal. But it is; one of these premises, then, must be denied. And we have said how this must be done. Further, in virtue of what the numbers, or the soul and the body, or in general the form and the thing, are one—of this no one tells us anything; nor can any one tell, unless he says, as we do, that the mover makes them one. And those who say mathematical number is first and go on to generate one kind of substance after another and give different principles for each, make the substance of the universe a series of episodes (for one substance has no influence on another by its existence or non-existence), and they give us many principles; but the world must not be governed badly.

'The rule of many is not good; let there be one ruler.'

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ARISTOTLE

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BOOK XIII (M)

1 . We have stated what is the substance of sensible things, dealing in the treatise on physics with matter, and later with the substance which has actual existence. Now since our inquiry is whether there is or is not besides the sensible substances any which is immovable and eternal, and, if there is, what it is, we must first consider what is said by others, so that, if there is anything which they say wrongly, we may not be liable to the same objections, while, if there is any opinion common to them and us, we shall not quarrel with ourselves on that account; for one must be content to state some points better than one's predecessors, and others no worse.

Two opinions are held on this subject; it is said that the objects of mathematics—i.e. numbers and lines and the like—are substances, and again that the Ideas are substances. And since some recognize these as two different classes—the Ideas and the mathematical numbers—and some recognize both as having one nature, while some others say that the mathematical substances are the only substances, we must consider first the objects of mathematics, not qualifying them by any other characteristic—not asking, for instance, whether they are Ideas or not, or whether they are the principles and substances of existing things or not, but only whether as the objects of mathematics they exist or not, and if they do, how they exist; then after this we must separately consider the Ideas themselves in a general way, and only as far as systematic treatment demands; for most of what we have to say has been repeatedly stated in popular works. And the greater part of our account must attack the inquiry already mentioned, viz. whether the substances and the principles of existing things are numbers and Ideas; for after the discussion of the Ideas this remains as a third inquiry.

If the objects of mathematics exist, then they must exist either in sensible objects, as some say, or separate from sensible objects (and this also is said by some), or if they exist in neither of these ways, either they do not exist, or they exist in some other way. So that the subject of our discussion will be not whether they exist but how they exist.

2 . That it is impossible for mathematical objects to exist *in* sensible things and at the same time that the doctrine in question is a fanciful one, has been said already in our discussion of difficulties,—the reasons being that it is impossible for two solids to be in the same place, and that according to the same argument all the other powers and characteristics also should exist in sensible things—none of them existing separately. This we have said already. But, further, it is obvious that on this theory it is impossible for any body whatever to be divided; for it would have to be divided at a plane, and the plane at a line, and the line at a point, so that if the point cannot be divided, neither can the line, and if the line cannot, neither can the plane nor the solid. What difference then does it make whether sensible things are of this kind, or, without being so themselves, have such things in them? The result will be

the same; if the sensible things are divided the others will be divided too, or else not even the sensible things can be divided.

But, again, it is not possible that such entities should exist *separately*. For if besides the sensible solids there are to be other solids which are separate from them and prior to the sensible solids, it is plain that besides the planes also there must be other and separate planes and points and lines; for consistency requires this. But if these exist, again besides the planes and lines and points of the mathematical solid there must be others which are separate. For the incomposite is prior to the compound; and if there are, prior to the sensible bodies, bodies which are not sensible, by the same argument the planes which exist by themselves must be prior to those which are in the motionless solids. Therefore these will be planes and lines other than those that exist along with the separate mathematical solids; for the latter exist along with the mathematical solids, while the others are prior to the mathematical solids. Again, there will be, belonging to these planes, lines, and prior to them there will have to be, by the same argument, other lines and points; and prior to these points in the prior lines there will have to be other points, though there will be no others prior to these. Now the accumulation becomes absurd; for we find ourselves with one set of solids apart from the sensible solids; three sets of planes apart from the sensible planes—those which exist apart from the sensible planes, and those in the mathematical solids, and those which exist apart from those in the mathematical solids; four sets of lines, and five sets of points. With which of these, then, will the mathematical sciences deal? Certainly not with the planes and lines and points in the motionless solid; for science always deals with what is prior. And the same account will apply also to numbers; for there will be another set of units apart from each set of points, and also apart from each set of realities, from the objects of sense and again from those of thought; so that there will be various classes of mathematical numbers.

Again, how is it possible to solve the questions which we enumerated in our discussion of difficulties? For besides the sensible things there will be, on similar principles, the things with which astronomy and those with which geometry deals; but how is it possible that a heaven and its parts—or indeed anything which has movement—should exist apart from the sensible heaven? Similarly also the objects of optics and harmonics will exist apart; for there will be voice and sight besides the sensible or individual voices and sights. Therefore it is plain that the other senses as well, and the other objects of sense, will exist apart; for why should one set of them do so and another not? And if this is so, animals also will exist apart, since the senses will.

Again, there are certain mathematical theorems of a universal character, extending beyond these substances. Here then we shall have another substance intermediate between, and separate from, the Ideas and the intermediates,—a substance which is neither number nor points nor spatial magnitude nor time. And if this is impossible, plainly it is also impossible that the *former* substances should exist separate from sensible things.

And, in general, conclusions contrary alike to the truth and to the usual views

follow, if one supposes the objects of mathematics to exist thus as separate entities. For if they exist thus they must be prior to sensible spatial magnitudes, but in truth they must be posterior; for the incomplete spatial magnitude is in the order of generation prior, but in the order of substance posterior, as the lifeless is to the living.

Again, what in the world¹ will make mathematical magnitudes one? For things in our perceptible world are one in virtue of soul, or of a part of soul, or of something else, reasonably enough; when these are not present, the thing is a plurality, and splits up into parts. But in the case of the objects of mathematics, which are divisible and are quantities, what is the cause of their being one and holding together?

Again, the modes of generation of the objects of mathematics show that we are right. For the dimension first generated is length, then comes breadth, lastly depth, and the process is complete. If, then, that which is posterior in the order of generation is prior in the order of substance, body will be prior to the plane and the line. And in *this* way also it is more complete and more whole, because it can become animate. How, on the other hand, could a line or a plane be animate? The supposition passes the power of our senses.

Again, body is a sort of substance; for it already has in a sense completeness. But how can lines be substances? Neither as a form or shape, as the soul perhaps is, nor as matter, like body; for we have no experience of anything that can be put together out of lines or planes or points, while if these had been a sort of material substance, we should have observed things which could be put together out of them.

Grant that they are prior in formula. Still not all things which are prior in formula are prior in substance. For those things are prior in substance which when separated from other things continue to exist, but those are prior in formula out of whose formulae the formulae of other things are compounded; and these two properties are not co-extensive. For if attributes, such as moving or white, do not exist apart from their substances, the white is prior to the white man in formula, but not in substance. For it cannot exist separately, but is always along with the compound thing; and by the compound thing I mean the white man. Therefore it is plain that neither is the result of abstraction prior nor that which is produced by adding posterior; for it is by adding to the white that we speak of the white man.

It has, then, been sufficiently pointed out that the objects of mathematics are not substances in a higher sense than bodies are, and that they are not prior to sensibles in being, but only in formula, and that they cannot in any way exist separately. But since they could not exist *in* sensibles either, it is plain that they either do not exist at all or exist in a special way and therefore do not exist without qualification. For 'exist' has many senses.

3 . Just as the universal part of mathematics deals not with objects which exist separately, apart from magnitudes and from numbers, but with magnitudes

¹Reading $\pi\sigma\tau$ for $\pi\acute{o}\tau$.

20 and numbers, not however *qua* such as to have magnitude or to be divisible, clearly
 it is possible that there should also be both formulae and demonstrations about
 sensible magnitudes, not however *qua* sensible but *qua* possessed of certain definite
 qualities. For as there are many formulae about things merely considered as in
 25 motion, apart from the essence of each such thing and from their accidents, and as it
 is not therefore necessary that there should be either something in motion separate
 from sensibles, or a separate substance in the sensibles, so too in the case of moving
 things there will be formulae and sciences which treat them not *qua* moving but
 only *qua* bodies, or again only *qua* planes, or only *qua* lines, or *qua* divisibles, or *qua*
 30 indivisibles having position, or only *qua* indivisibles.

Thus since it is true to say without qualification that not only things which are
 separable but also things which are inseparable exist—for instance, that moving
 things exist,—it is true also to say, without qualification, that the objects of
 mathematics exist, and with the character ascribed to them by mathematicians.
 And it is true to say of the other sciences too, without qualification, that they deal
 35 with such and such a subject—not with what is accidental to it (e.g. not with the
 white, if the white thing is healthy, and the science has the healthy as its subject),
 78*1 but with that which is the subject of each science—with the healthy if it treats
 things *qua* healthy, with man if *qua* man. So too is it with geometry; if its subjects
 happen to be sensible, though it does not treat them *qua* sensible, the mathematical
 sciences will not for that reason be sciences of sensibles—nor, on the other hand, of
 other things separate from sensibles.

5 Many properties attach to things in virtue of their own nature as possessed of
 some such property; e.g. there are attributes peculiar to the animal *qua* female or
qua male, yet there is no female nor male separate from animals. And so also there
 are attributes which belong to things merely as lengths or as planes. And in
 proportion as we are dealing with things which are prior in formula and simpler, our
 10 knowledge will have more accuracy, i.e. simplicity. Thus a science which abstracts
 from the magnitude of things is more precise than one which takes it into account;
 and a science is most precise if it abstracts from movement, but if it takes account of
 movement, it is most precise if it deals with the primary movement, for this is the
 simplest; and of this again uniform movement is the simplest form. The same
 account may be given of harmonics and optics; for neither considers its objects *qua*
 15 light-ray or *qua* voice, but *qua* lines and numbers; but the latter are attributes
 proper to the former. And mechanics too proceeds in the same way. Thus if we
 suppose things separated from their attributes and make any inquiry concerning
 them as such, we shall not for this reason be in error, any more than when one draws
 20 a line on the ground and calls it a foot long when it is not; for the error is not
 included in the propositions.

Each question will be best investigated in this way—by supposing separate
 what is not separate, as the arithmetician and the geometer do. For a man *qua* man
 is one indivisible thing; and the arithmetician supposes one indivisible thing, and
 25 then considers whether any attribute belongs to a man *qua* indivisible. But the
 geometer treats him neither *qua* man nor *qua* indivisible, but as a solid. For

evidently the attributes which would have belonged to him even if he had not been
 indivisible, can belong to him apart from these attributes. Thus, then, geometers
 speak correctly—they talk about existing things, and their subjects do exist; for
 being has two forms—it exists not only in fulfillment but also as matter. 30

Now since the good and the beautiful are different (for the former always
 implies conduct as its subject, while the beautiful is found also in motionless things),
 those who assert that the mathematical sciences say nothing of the beautiful or the
 good are in error. For these sciences say and prove a very great deal about them; for
 if they do not expressly mention them, but prove attributes which are their results or
 35 their formulae, it is not true to say that they tell us nothing about them. The chief
 forms of beauty are order and symmetry and definiteness, which the mathematical
 sciences demonstrate in a special degree. And since these (e.g. order and definite-
 1078*1 ness) are obviously causes of many things, evidently these sciences must treat this
 sort of cause also (i.e. the beautiful) as in some sense a cause. But we shall 5
 speak more plainly elsewhere about these matters.

4 . So much then for the objects of mathematics; we have said that they exist
 and in what sense they exist, and in what sense they are prior and in what sense not
 prior. Now, regarding the Ideas, we must first examine the ideal theory by itself, not
 connecting it in any way with the nature of numbers, but treating it in the form in
 10 which it was originally understood by those who first maintained the existence of
 Ideas. The supporters of the ideal theory were led to it because they were persuaded
 of the truth of the Heraclitean doctrine that all sensible things are ever passing
 away, so that if knowledge or thought is to have an object, there must be some other
 15 and permanent entities, apart from those which are sensible; for there can be no
 knowledge of things which are in a state of flux. Socrates occupied himself with the
 excellences of character, and in connection with them became the first to raise the
 problem of universal definitions—for of the natural scientists, only Democritus
 touched on the matter and defined, after a fashion, the hot and the cold; while the
 20 Pythagoreans had before this treated of a few things, whose formulae they
 connected with numbers—e.g. opportunity, justice, or marriage. But it was natural
 that Socrates should seek the essence. For he was seeking to deduce, and the essence
 is the starting-point of deductions. For there was as yet none of the dialectical power
 25 which enables people even without knowledge of the essence to speculate about
 contraries and inquire whether the same science deals with contraries. For two
 things may be fairly ascribed by Socrates—inductive arguments and universal
 definition, both of which are concerned with the starting-point of science. But
 Socrates did not make the universals or the definitions exist apart; his successors,
 30 however, gave them separate existence, and this was the kind of thing they called
 Ideas.

Therefore it followed for them, almost by the same argument, that there must
 be Ideas of all things that are spoken of universally, and it was almost as if a man
 wished to count certain things, and while they were few thought he would not be
 35 able to count them, but made them more and then counted them; for the Forms are

almost more numerous than the groups of sensible things, yet it was in seeking the causes of sensible things that they proceeded from these to the Forms. For to each set of substances there answers a Form which has the same name and exists apart from the substances, and so also in the other categories there is one character common to many individuals, whether these be sensible or eternal.

Again, of the ways in which it is proved that the Forms exist, none is convincing; for from some no inference necessarily follows, and from some it follows that there are Forms even of things of which they think there are no Forms.

For according to the arguments from the sciences there will be Forms of all things of which there are sciences, and according to the argument that there is one attribute common to many things there will be Forms even of negations, and according to the argument that thought has an object when the individual object has perished, there will be Forms of perishable things; for we can have an image of these. Again, of the most accurate arguments, some lead to Ideas of relations, of which they say there is no independent class, and others involve the difficulty of the third man. And in general the arguments for the Forms destroy that for whose existence the assertors of Forms are more anxious than for the existence of the Ideas; for it follows that not the dyad but number is first, and the relative is prior to that and prior to the self-dependent—and besides this there are all the other points on which certain people, by following out the opinions held about the Forms, have come into conflict with the principles of the theory.

Again, according to the assumption on which the belief in the Ideas rests, there will be Forms not only of substances but also of many other things; for the concept is single, not only in the case of substances, but also in that of non-substances, and there are sciences of other things than substance; and a thousand other such conclusions also follow. But according to the necessities of the case and the opinions about the Forms, if they can be shared in there must be Ideas of substances only. For they are not shared in incidentally, but each Form must be shared in as something not predicated of a subject. (E.g. if a thing shares in the double itself, it shares also in eternal, but incidentally; for the double happens to be eternal.) Therefore the Forms will be substance. And the same names indicate substance in this and in the ideal world (or what will be the meaning of saying that there is something apart from the particulars—the one over many?). And if the Ideas and the things that share in them have the same Form, there will be something common: for why should 2 be one and the same in all the perishable 2's, or in the 2's which are many but eternal, and not the same in the 2 itself as in the individual 2? But if they have not the same Form, they will have only the name in common, and it is as if one were to call both Callias and a piece of wood 'man', without observing any community between them.

But if we are to suppose that in other respects the common formulae apply to the Forms, e.g. that plane figure and the other parts of the formula apply to the circle itself, but that what it is must be added, we must inquire whether this is not absolutely empty. For to what will this be added? To 'centre' or to 'plane' or to all the parts of the formula? For all the elements in the substance are Ideas, e.g. animal

and two-footed. Further, the added notion must be an Idea, like plane, a definite entity which will be present as genus in all its species.

5 · Above all one might discuss the question what on earth the Forms contribute to sensible things, either to those that are eternal or to those which come into being and cease to be; for they cause neither movement nor any change in them. But again they help in no way towards the *knowledge* of other things (for they are not even the substance of these, else they would have been in them), nor towards their being, at least if they are not *in* the individuals which share in them—for in that case they might be thought perhaps to be causes, as white is for the white thing in which it is mixed. But this argument, which was used first by Anaxagoras, and later by Eudoxus in his discussion of difficulties and by certain others, is too easily upset; for it is easy to collect many insuperable objections to such a view.

But further all other things cannot come from the Forms in any of the ways that are usually suggested. And to say that they are patterns and the other things share in them is to use empty words and poetical metaphors. For what is it that works, looking to the Ideas? And any thing can both be and come into being without being copied from something else, so that, whether Socrates exists or not, a man like Socrates might come to be. And evidently this might be so even if Socrates were eternal. And there will be several patterns of the same thing, and therefore several Forms, e.g. animal and two-footed, and also man-himself, will be Forms of man. Again, the Forms are patterns not only of sensible things, but of things-themselves also, e.g. the genus is the pattern of the species of the genus; therefore the same thing will be pattern and copy.

Again, it might be thought impossible that substance and that whose substance it is should exist apart; how, therefore, could the Ideas, being substances of things, exist apart?

In the *Phaedo*² it is stated in this way—that the Forms are causes both of being and of becoming. Yet though the Forms exist, still things do not come into being, unless there is something to move them; and many other things come into being (e.g. a house or a ring), of which they say there are no Forms. Clearly therefore even the things of which they say there are Ideas can both be and come into being owing to such causes as produce the things just mentioned, and not owing to the Forms. But regarding the Ideas it is possible, both in this way and by more abstract and more accurate arguments, to collect many objections like those we have considered.

6 · Since we have discussed these points, it is well to consider again the results regarding numbers which confront those who say that numbers are separable substances and first causes of things. If number is a real thing and its substance is nothing other than just number, as some say, it follows that either there is a first in it and a second, each being different in kind, and³ this is true of the units

²*Phaedo* 100D.

³Omitting ἢ.

without qualification, and any unit is non-comparable with any unit, or they are all directly successive, and any of them is comparable with any, as they say is the case with mathematical number; for in mathematical number no unit is in any way different from another. Or some units must be comparable and some not, e.g. 2 is first after 1, and then comes 3 and then the other numbers, and the units in each number are comparable, e.g. those in the first 2 with one another, and those in the first 3 with one another, and so with the other numbers; but the units in the 2 itself are not comparable with those in the 3 itself; and similarly in the case of the other successive numbers. Therefore while mathematical number is counted thus—after 1, 2 (which consists of another 1 besides the former 1), and 3 (which consists of another 1 besides these two), and the other numbers similarly, ideal number is counted thus—after 1, a distinct 2 which does not include the first 1, and a 3 which does not include the 2, and the other numbers similarly. Or one kind of number is like the first that was named, one like that which the mathematicians speak of, and that which we have named last must be a third kind.

Again, these numbers must either be separable from things, or not separable but in sensible things (not however in the way which we first considered, but in the sense that sensible things consist of numbers which are present in them)—either some of them and not others, or all of them.

These are of necessity the only ways in which the numbers can exist. And of those who say that the 1 is the beginning and substance and element of all things, and that number is formed from the 1 and something else, almost every one has described number in one of these ways; only no one has said *all* the units are incomparable. And this has happened reasonably enough; for there can be no way besides those mentioned. Some say both kinds of number exist, that which has a before and after being identical with the Ideas, and mathematical number being different from the Ideas and from sensible things, and both being separable from sensible things; and others say mathematical number alone exists, as the first of realities, separate from sensible things.

Now the Pythagoreans, also, believe in one kind of number—the mathematical; only they say it is not separate but sensible substances are formed out of it. For they construct the whole universe out of numbers—only not numbers consisting of abstract units; they suppose the units to have spatial magnitude. But how the first 1 was constructed so as to have magnitude, they seem unable to say.

Another thinker says the first kind of number, that of the Forms, alone exists, and some say mathematical number is identical with this.

The case of lines, planes, and solids is similar. For some think that those which are the objects of mathematics are different from those which come after the Ideas; and of those who express themselves otherwise some speak of the objects of mathematics and in a mathematical way—viz. those who do not make the Ideas numbers nor say that Ideas exist; and others speak of the objects of mathematics, but not mathematically; for they say that neither is every spatial magnitude divisible into magnitudes, nor do any two units make 2. All who say the 1 is an element and principle of things suppose numbers to consist of abstract units, except

the Pythagoreans; but *they* suppose the numbers to have magnitude, as has been said before. It is clear from this statement, then, in how many ways numbers may be described, and that all the ways have been mentioned; and all are impossible, but some perhaps more than others.

7 . First let us inquire if the units are comparable or non-comparable, and if non-comparable, in which of the two ways we distinguished. For it is possible that any unit is non-comparable with any, and it is possible that those in the ideal 2 are non-comparable with those in the ideal 3, and, generally, that those in each primary number are non-comparable with one another. If all units are comparable and without difference, we get mathematical number and this alone, and the Ideas cannot be the numbers. For what sort of number will the ideal man or animal or any other Form be? There is one Idea of each thing, e.g. one of ideal man and another one of ideal animal; but the similar and undifferentiated numbers are infinitely many, so that *this* 3 is no more the ideal man than any other 3. But if the Ideas are not numbers, neither can they exist at all. For from what principles will the Ideas come? *Number* comes from the 1 and the indefinite dyad, and the principles and the elements are said to be principles and elements of number, and the Ideas cannot be ranked as either prior or posterior to the numbers.

But if the units are non-comparable, and non-comparable in the sense that none is comparable with any other, number of this sort cannot be mathematical number; for mathematical number consists of undifferentiated units, and the truths proved of it suit this character. Nor can it be ideal number. For 2 will not come first after 1 and the indefinite dyad, and be followed directly by the successive numbers, as we say '2, 3, 4' (for the units in the ideal 2 are generated at the same time, whether, as the first holder of the theory said, from unequals—coming into being when these were equalized—or in some other way).⁴ Besides, if one unit is to be prior to the other, it will be prior to the 2 composed of these; for when there is one thing prior and another posterior, the compound of these will be prior to one and posterior to the other.

Again, since the ideal 1 is first, and then there is a 1 which is first among the others and next after the ideal 1, and again a third which is next after the second and next but one after the first 1, the units must be prior to the numbers by which they are named in counting, e.g. there will be a third unit in 2 before 3 exists, and a fourth and a fifth in 3 before the numbers 4 and 5 exist.—None of these thinkers has said the units are non-comparable in this way, but according to their principles even this way is reasonable, though in truth it is impossible. For it is reasonable that the units should have priority and posteriority if there is a first unit and a first 1, and the 2's also if there is a first 2; for after the first it is reasonable and necessary that there should be a second, and if a second, a third, and so with the others successively. (And to say both at the same time, that a *unit* is first and another unit is second after the ideal 1, and that a 2 is first after it, is impossible.) But they make

⁴Retaining ἐκτα.

a first unit and 1, but not a second and a third, and a first 2, but not a second and a third.

10 Clearly, also, it is not possible, if all the units are non-comparable, that there should be an ideal 2 and 3; and similarly in the case of the other numbers. For whether the units are undifferentiated or each differs from each, number must be
15 counted by addition, e.g. 2 by adding another one to the one, 3 by adding another one to the two, and 4 similarly. This being so, numbers cannot be generated, as they generate them, from the dyad and the 1; for 2 becomes part of 3, and 3 of 4, and the
20 same happens in the case of the succeeding numbers, but for them 4 came from the first 2 and the indefinite 2,—which makes it two 2's *other* than the ideal 2; if not, the ideal 2 will be a part of 4 and one other 2 will be added. And similarly 2 will consist of the ideal 1 and another 1; but if this is so, the other element cannot be an
25 indefinite 2; for it generates a unit, but not a definite 2. Again, besides the ideal 3 and the ideal 2 how can there be other 3's and 2's? And how do they consist of prior and posterior units? All these doctrines are absurd and fiction, and there cannot be
30 a first 2 and then an ideal 3. Yet there must, if the 1 and the indefinite dyad are to be the elements. But if the results are impossible, it is also impossible that these are the principles.

If the units, then, are differentiated, each from each, these results and others
35 similar to these follow of necessity. But if those in different numbers are differentiated, but those in the same number are alone undifferentiated from one another, even so the difficulties that follow are no less. E.g. in the ideal 10 there are
1082¹ ten units, and the 10 is composed both of them and of two 5's. But since the ideal 10 is not any chance number nor composed of any chance 5's—or, for that matter,
5 units—the units in this 10 must differ. For if they do not differ, neither will the 5's of which the 10 consists differ; but since they differ, the units also will differ. But if they differ, will there be no other 5's in the 10 but only these two, or will there be
10 others? If there are not, this is paradoxical; and if there are, what sort of 10 will consist of them? For there is no other 10 in the 10 but itself. But it is also necessary that the 4 should not consist of any chance 2's; for the indefinite 2, as they say, took the definite 2 and made two 2's; for its nature was to double what it took.

15 Again, as to the 2 being a thing apart from the two units, and the 3 a thing apart from the three units, how is this possible? Either by one's sharing in the other, as white man is different from white and man (for it shares in these), or when one is a differentia of the other, as man is different from animal and two-footed. Again,
20 some things are one by contact, some by intermixture, some by position; none of which relations can belong to the units of which the 2 or the 3 consists; but as two men are not a unity apart from both, so must it be with the units. And their being
25 indivisible will make no difference to them; for points are indivisible, but yet a pair of them is nothing apart from the two.

30 But this consequence also we must not forget, that it follows that there are prior and posterior 2's, and similarly with the other numbers. For let the 2's in the 4 be simultaneous; yet these are prior to those in the 8, and as the 2 generated them, they generated the 4's in the ideal 8. Therefore if the first 2 is an Idea, these 2's also

will be Ideas. And the same account applies to the units; for the units in the first 2 generate the four in 4, so that all the units come to be Ideas and an Idea will be
35 composed of Ideas. Clearly therefore those things also, of which these are Ideas, will be composite, e.g. one might say that animals are composed of animals, if there are
1082¹ Ideas of them.

In general, to differentiate the units in any way is an absurd fiction; and by a fiction I mean that which is brought in forcibly to suit a hypothesis. For neither in quantity nor in quality do we see unit differing from unit, and number must be
5 either equal or unequal—all number but especially that which consists of abstract units—so that if one number is neither greater nor less than another, it is equal; but what is equal and in no wise differentiated we take to be the same when we are speaking of numbers. If not, even the 2's in the ideal 10 will be differentiated though
10 they are equal; for what reason will the man who says they are not differentiated be able to allege?

Again, if every unit plus another unit makes two, a unit from the ideal 2 and one from the ideal 3 will make a 2. Now this consists of differentiated units; and will
15 it be prior to the 3 or posterior? It rather seems that it must be prior; for one of the units is simultaneous with the 3, and the other is simultaneous with the 2. And we, for our part, suppose that in general 1 and 1, whether the things are equal or unequal, is 2, e.g. the good and the bad, or a man and a horse; but those who hold these views say that not even two *units* are 2.

If the number of the ideal 3 is not greater than that of the 2, this is surprising;
20 and if it is, clearly there is a number in it equal to the 2, so that this is not different from the ideal 2. But this is not possible, if there is a first and a second number. Nor will the Ideas be numbers. For in this particular point they are right who claim that the units must be different, if there are to be Ideas, as has been said before. For the
25 Form is unique; but if the units are not different, the 2's and the 3's also will not be different. Therefore they must say that when we count thus—'1, 2,' we do not add to the previous number; for if we do, neither will the numbers be generated from the
30 indefinite dyad, nor can a number be an Idea; for one Idea will be in another, and all the Forms will be parts of one Form. Therefore with a view to their hypothesis they are right, but absolutely they are wrong; for their view is very destructive, since they will admit that this question itself affords some difficulty—whether, when we count and say '1, 2, 3,' we count by addition or by partitions. But we do both; therefore it is
35 absurd to refer this to so great a difference of substance.

8 . First of all it is well to determine what is the differentia of a
1083¹ number—and of a unit, if it has a differentia. Units must differ either in quantity or in quality; and neither of these seems to be possible. But number *qua* number differs in quantity. And if the units also differed in quantity, number would differ from
5 number, though equal in number of units. Again, are the first units greater or smaller, and do the later ones increase or diminish? All these are irrational suppositions. But neither can they differ in quality. For no attribute can attach to them; for even to numbers quality is said to belong *after* quantity. Again, quality
10

could not come to them either from the 1 or from the dyad; for the former has no quality, and the latter gives *quantity*; for its nature is to cause things to be many. If the facts are really otherwise, they should above all state this at the beginning and determine if possible, regarding the differentia of the unit, why it must exist; otherwise, what do they mean by it?

Evidently then, if the Ideas are numbers, the units cannot all be comparable, nor can they be non-comparable in either of the two ways. But neither is the way in which some others speak about numbers correct. These are those who do not think there are Ideas, either without qualification or as identified with certain numbers, but think the objects of mathematics exist and the numbers are the first of real things, and the ideal 1 is the starting-point of them. It is paradoxical that there should be a 1 which is first of 1's, as they say, but not a 2 which is first of 2's, nor a 3 of 3's; for the same reasoning applies to all. If, then, the facts with regard to number are so, and one supposes mathematical number alone to exist, the 1 is not the starting point. For this sort of 1 must differ from the other units; and if this is so, there must also be a 2 which is first of 2's, and similarly with the other successive numbers. But if the 1 is the starting-point, the truth about the numbers must rather be what Plato used to say, and there must be a first 2 and 3, and the numbers must not be comparable with one another. But if on the other hand one supposes this, many impossible results, as we have said, follow. But either this or the other must be the case, so that if neither is, number cannot exist separately.

It is evident from this that the third view is the worst,—that ideal and mathematical number is the same. For two mistakes evidently meet in the one opinion. (1) Mathematical number cannot be of this sort, but the holder of this view has to spin it out by making suppositions peculiar to himself. And (2) he must also admit all the consequences that confront those who speak of numbers as Forms.

The doctrine of the Pythagoreans in one way affords fewer difficulties than those before named, but in another way has others peculiar to itself. For not thinking of number as capable of existing separately removes many of the impossible consequences; but that bodies should be composed of numbers, and that this should be mathematical number, is impossible. For it is not true to speak of indivisible magnitudes; and however much there might be magnitudes of this sort, units at least have no magnitude; and how can a magnitude be composed of indivisibles? But arithmetical number, at least, consists of abstract units, while these thinkers identify number with real things; at any rate they apply their propositions to bodies as if they consisted of those numbers.

If then it is necessary, if number is a self-subsistent real thing, that it should be conceived in one of these ways which have been mentioned, and if it cannot be conceived in any of these, evidently number has no such nature as those who make it separable construct for it.

Again, does each unit come from the great and the small, equalized, or one from the small, another from the great? If the latter, neither does each thing contain all the elements, nor are the units without difference; for in one there is the great and in another the small, which is contrary in its nature to the great. Again, how is it with the units in the ideal 3? There is one over. But perhaps it is for this

reason that they give the ideal 1 the middle place in odd numbers. But if each of the two units consists of both the great and the small, equalized, how will the 2, which is one thing, consist of the great and the small? Or how will it differ from the unit? Again, the unit is prior to the 2; for when it is destroyed the 2 is destroyed. It must, then, be the Idea of an Idea since it is prior to an Idea, and it must have come into being before it. From what, then? Not from the indefinite dyad, for its function was to double.

Again, number must be either infinite or finite; for these thinkers think of number as capable of existing separately, so that it is not possible that neither of those alternatives should be true. Clearly it cannot be *infinite*; for infinite number is neither odd nor even, but the generation of numbers is always the generation either of an odd or of an even number,—when 1 operates in one way on an even number, an odd number is produced, and when 2 (or an odd number) operates in the other way, the numbers got from 1 by doubling (or the other even numbers) are produced. Again, if every Idea is an Idea of something, and the numbers are Ideas, infinite number will be an Idea of something, either of some sensible thing or of something else. Yet this is not possible in view of their hypothesis any more than it is reasonable in itself, if they conceive of the Ideas as they do.

But if number is *finite*, how far does it go? With regard to this not only the fact but the reason should be stated. But if number goes only up to 10, as some say, firstly the Forms will soon run short; e.g. if 3 is man-in-himself, what number will be the horse-in-itself? The numbers which are Ideas of the several things go up to 10. It must, then, be one of the numbers within these limits; for it is these that are substances and Ideas. Yet they will run short; for the various kinds of animal will exceed them. At the same time it is clear that if in this way the 3 is the Idea of man, the other 3's are so also (for those in the same number are similar), so that there will be an infinite number of men, and if each 3 is an Idea, each of the men will be man-in-himself, and if not, they will at least be men. And if the smaller number is part of the greater (being number of such a sort that the units in the same number are comparable), then if the ideal 4 is an Idea of something, e.g. of horse or of white, man will be a part of horse, if man is 2. It is paradoxical also that there should be an Idea of 10, but not of 11, nor of the succeeding numbers. Again, there both are and come to be certain things of which there are no Forms; why, then, are there not Forms of them also? We infer that the Forms are not causes. Again, it is paradoxical if the number-series up to 10 is more of a real thing and a Form than 10 itself. There is no generation of the former as one thing, and there is of the latter. But they try to form a theory on the assumption that the series of numbers up to 10 is a complete series. At least they generate other things—the void, proportion, the odd, and the others of this kind—within the 10. For some things, e.g. movement, rest, good, bad, they assign to the principles, and the others to the numbers. This is why they identify 1 with the odd; for if the odd implied 3, how would 5 be odd? Again, magnitudes and all such things are explained without going beyond a definite number, e.g. the first indivisible line, then the 2, then the others up to 10.

Again, if number can exist separately, one might ask which is prior—1, or 2 or

3? Inasmuch as the number is composite, 1 is prior, but inasmuch as the universal
 5 and the form is prior, the number is prior; for each of the units is part of the number
 as its matter, and the number acts as form. And in a sense the right angle is prior to
 the acute, because it is definite and in virtue of its formula; but in a sense the acute
 is prior, because it is a part and the right angle is divided into acute angles. As
 10 matter, then, the acute angle and the element and the unit are prior, but as regards
 the form and the substance (in the sense of the formula), the right angle, and the
 whole consisting of the matter and the form, are prior; for the compound thing is
 nearer the form and the object of the formula, but in generation it is later. How then
 15 is 1 the starting-point? Because it is not divisible, they say. But both the universal,
 and the particular or the element, are indivisible; but in different ways, one in
 formula and the other in time. In which way then is 1 the starting-point? As has
 been said, the right angle is thought to be prior to the acute, and the acute to the
 right, and each is one. They make 1 the starting-point in both ways. But this is
 20 impossible. For one kind of starting-point is the form or substance, the other the
 part or matter. For each is in a way one—in truth, each unit exists potentially (at
 least if the number is a unity and not like a heap, i.e. if different numbers consist of
 different units, as they say), but not actually.

The cause of the mistake they fell into is that they conducted their inquiry at
 the same time from the standpoint of mathematics and from that of universal
 25 formulae, so that from the former standpoint they treated unity, their first
 principle, as a point; for the unit is a point without position. They put things
 together out of the smallest parts, as some others have done. Therefore the unit
 becomes the matter of numbers and at the same time prior to 2; and again posterior,
 30 2 being treated as a whole, a unity, and a form. But because their inquiry was
 universal they treated the unity which can be predicated of a number, as in this
 sense also a part of the number. But these characteristics cannot belong at the same
 time to the same thing.

If the ideal 1 must be merely without position⁵ (for it differs in nothing from
 other 1's except that it is the starting-point), and the 2 is divisible but the unit is not,
 35 the unit must be more like the ideal 1. But if so, *it* must be more like the unit than
 the 2; therefore each of the units must be prior to the 2. But they deny this; at least
 36¹ they generate the 2 first. Again, if the ideal 2 is a unity and the ideal 3 is one also,
 both form a 2. From what, then, is this 2 produced?

9 . Since there is not contact in numbers, but the units between which there
 5 is nothing, e.g. those in 2 or in 3, are successive, one might ask whether they succeed
 the ideal 1 or not, and whether, of the terms that succeed it, 2 or either of the units
 in 2 is prior.

Similar difficulties occur with regard to the classes of things posterior to
 number,—the line, the plane, and body. For some construct these out of the forms
 10 of great and small; e.g. lines from long and short, planes from broad and narrow;

⁵Ross marks this clause as corrupt.

masses from deep and shallow; which are forms of great and small. And the
 principle of these which answers to the 1 different men describe in different ways.
 And in these also the impossibilities, the fictions, and the contradictions of all
 15 probability are seen to be innumerable. For they are severed from one another,
 unless the principles of these imply one another in such a way that the broad and
 narrow is also long and short; but if this is so, the plane will be a line and the solid a
 20 plane. Again, how will angles and figures and such things be explained? And the
 same happens as in regard to number; for these things are attributes of magnitude,
 but magnitude does not *consist* of these, any more than the line consists of straight
 and curved, or solids of smooth and rough.

All these cases share a difficulty which occurs with regard to species of a
 genus, when one posits the universals, viz. whether it is the ideal animal or
 25 something other than the ideal animal that is in animals. True, if the universal is not
 separable, this will present no difficulty; but if the 1 and the numbers are separable,
 as those who express these views say, it is not easy to solve the difficulty, if one may
 call the impossible 'not easy'. For when we apprehend the unity in 2, or in general in
 30 a number, do we apprehend a thing-in-itself or something else?

Some, then, generate magnitudes from matter of this sort, others from the
 point—and the point is thought by them to be not 1 but something like 1—and from
 other matter like plurality, but not identical with it; about which principles none the
 35 less the same difficulties occur. For if the matter is one, line and plane and solid will
 be the same; for from the same elements will come one and the same thing. But if
 the matters are more than one, and there is one for the line and a second for the
 1085^b1 plane and another for the solid, they either imply one another or not, so that the
 same results will follow even so; for either the plane will not contain a line or it will
 be a line.

Again, how number can consist of the one and plurality, they make no attempt
 5 to explain; at least as they state the case, the same objections arise as confront those
 who construct number out of the one and the indefinite dyad. For the one view
 generates number from the universally predicated plurality, and not from a
 particular plurality; and the other generates it from a particular plurality, but the
 10 first; for 2 is said to be a first plurality. Therefore there is practically no difference,
 but the same difficulties will follow,—is it intermixture or position or fusion or
 generation? and so on. Above all one might press the question, if each unit is one,
 what does it come from? Certainly each is not the one-in-itself. It must, then, come
 15 from the one-in-itself and plurality, or a part of plurality. To say that the unit is a
 plurality is impossible, for it is indivisible; and to generate it from a part of plurality
 involves many other objections; for each of the parts must be indivisible (or it will be
 a plurality and the unit will be divisible) and the elements will not be the one and
 20 *plurality*; for the single units do not come from plurality and the one. Again, the
 holder of this view does nothing but produce another number; for his plurality of
 indivisibles is a number. Again, we must inquire, in view of this theory also, whether
 the number is infinite or finite. For there was at first, as it seems, a finite plurality,
 25 from which and from the one comes the finite number of units. And plurality in

itself is different from infinite plurality; what sort of plurality, then, is the element which co-operates with the one?

One might inquire similarly about the point, i.e. the element out of which they make magnitudes. For surely this is not the one and only point; at any rate, then, let them say out of what each of the other points is formed. Certainly not of some distance together with the point-in-itself. Nor again can parts of a distance be indivisible parts, as the parts of plurality out of which the units are said to be made are indivisible; for number consists of indivisibles, but magnitudes do not.

All these objections and others of the sort make it evident that number and magnitudes cannot exist apart from things. Again, the fact that the chief thinkers disagree about numbers is a sign that it is the incorrectness of the alleged facts themselves that brings confusion into the theories. For those who make the objects of mathematics alone exist apart from sensible things, seeing the difficulty about the Forms and their fictitiousness, abandoned ideal number and posited mathematical. But those who wished to make the Forms at the same time numbers, but did not see, if one assumed these principles, how mathematical number was to exist apart from ideal, made ideal and mathematical number the same—in name, since in fact mathematical number is destroyed; for they state hypotheses peculiar to themselves and not those of mathematics. But he who first supposed that the Forms exist and that the Forms are numbers and that the objects of mathematics exist, naturally separated the two. Therefore it turns out that all of them are right in some respect, but on the whole not right. And they themselves confirm this, for their statements conflict. The cause is that their hypotheses and their principles are false. And it is hard to make a good case out of bad materials, according to Epicharmus: 'as soon as 'tis said, 'tis seen to be wrong.' But regarding numbers the questions we have raised and the conclusions we have reached are sufficient; for he who is already convinced might be further convinced by a longer discussion, but one not yet convinced would not come any nearer to conviction.

But regarding the first principles and the primary causes and elements, the views expressed by those who discuss only sensible substance have been partly stated in the *Physics*, and partly do not belong to the present inquiry; but the views of those who say there are other substances besides the sensible must be discussed next after those we have been mentioning. Since, then, some say that the Ideas and the numbers are such substances, and that the elements of these are elements and principles of real things, we must inquire regarding these what they say and in what sense they say it.

Those who posit numbers only, and these mathematical, must be considered later; but as regards those who believe in the Ideas one might survey at the same time their way of thinking and the difficulties into which they fall. For they at the same time treat the Ideas as universal, and again as separable and individual. That this is not possible has been shown before. The reason why those who say substances are universal combined these two views in one, is that they did not make them identical with sensible things. They thought that the sensible particulars were in a state of flux and none of them remained, but that the universal was apart from these

and different. And Socrates gave the impulse to this theory, as we said before, by means of his definitions, but he did not separate them from the particulars; and in this he thought rightly, in not separating them. This is plain from the results; for without the universal it is not possible to get knowledge, but the separation is the cause of the objections that arise with regard to the Ideas. His successors, treating it as necessary, if there are to be substances besides the sensible and transient substances, that they must be separable, had no others, but gave separate existence to these universally predicated substances, so that it followed that universals and individuals were almost the same sort of thing. This in itself, then, would be one difficulty in the view we have mentioned.

10 . Let us now mention a point which presents a certain difficulty both to those who believe in the Ideas and to those who do not, and which was stated at the beginning among the problems. If we do not suppose substances to be separate, and in the way in which particular things are said to be separate, we shall destroy that sort of substance which we wish to maintain; but if we conceive substances to be separable, how are we to conceive their elements and their principles?

If they are individual and not universal, real things will be just of the same number as the elements, and the elements will not be knowable. For let the syllables in speech be substances, and their elements elements of substances; then there must be only one *ba* and one of each of the syllables, if they are not universal and the same in form but each is one in number and a 'this' and not homonymous (and again they suppose each thing-in-itself to be one). And if the syllables are unique, so are the parts of which they consist; there will not, then, be more *a*'s than one, nor more than one of any of the other elements, on the same principle on which none of the syllables can exist in the plural number. But if this is so, there will not be other things existing besides the elements, but only the elements. Again, the elements will not be even knowable; for they are not universal, and knowledge is of universals. This is clear both from demonstrations and from definitions; for we do not conclude that this triangle has its angles equal to two right angles, unless every triangle has its angles equal to two right angles, nor that this man is an animal, unless every man is an animal.

But if the principles are universal either the substances composed of them are universal too, or non-substance will be prior to substance; for the universal is not a substance, and the element or principle is universal, and the element or principle is prior to the things of which it is the principle or element.

All these difficulties follow naturally, when they make the Ideas out of elements and claim that there are separate unities apart from the substances which have the same form. But if, e.g., in the case of the elements of speech, the *a*'s and the *b*'s may quite well be many and there need be no ideal *a* and ideal *b* besides the many, there may be, as far as this goes, an infinite number of similar syllables. The statement that all knowledge is universal, so that the principles of things must also be universal and not separate substances, presents indeed, of all the points we have mentioned, the greatest difficulty, but yet the statement is in a sense true, although

15 in a sense it is not. For knowledge, like knowing, is spoken of in two ways—as
potential and as actual. The potentiality, being, as matter, universal and indefinite,
deals with the universal and indefinite; but the actuality, being definite, deals with a
definite object,—being a 'this', it deals with a 'this'. But *per accidens* sight sees
20 universal colour, because this individual colour which it sees is colour; and this
individual *a* which the grammarian investigates is an *a*. For if the principles must be
universal, what is derived from them must also be universal, as in demonstrations;
and if this is so, there will be nothing capable of separate existence—i.e. no
25 substance. But evidently in a sense knowledge is universal, and in a sense it is not.

BOOK XIV (N)

1 · Regarding this kind of substance, what we have said must be taken as
30 sufficient. All philosophers make the first principles contraries: as in natural things,
so also in the case of unchangeable substances. But since there cannot be anything
prior to the first principle of all things, the principle cannot be the principle as being
something else. To suggest this is like saying that the white is the first principle, not
qua anything else but *qua* white, but yet that it is predicable of a subject, and is
35 white as being something else; for then that subject will be prior. But all things are
generated from contraries as belonging to an underlying subject; a subject, then,
1087¹ must be present in the case of contraries, if anywhere. All contraries, then, are
always predicable of a subject, and none can exist apart. But appearances suggest
that there is nothing contrary to substance, and argument confirms this. No
contrary, then, is the first principle of all things in the full sense; the first principle is
something different.

5 But these thinkers make one of the contraries matter, some making the
unequal—which they take to be the essence of plurality—matter for the one, which
is the equal,¹ and others making plurality matter for the one. (The former generate
numbers out of the dyad of the unequal, i.e. of the great and small, and the other
thinker we have referred to generates them out of plurality, while according to both
10 it is generated by the substance of one.) For even the philosopher who says the
unequal and one are the elements, and the unequal is a dyad composed of the great
and small, treats the unequal, or the great and the small, as being one, and does not
draw the distinction that they are one in formula, but not in number. But they do
not describe rightly even the principles which they call elements, for some name the
15 great and the small with the one and treat these three as elements of numbers, two
being matter, one form; while others name the many and few, because the great and
the small are more appropriate in their nature to magnitude than to number; and
others name rather the universal character common to these—that which exceeds
and that which is exceeded. None of these varieties of opinion makes any difference

¹Retaining τῷ ἴσῳ.

to speak of, in view of some of the consequences; they affect only the abstract
objections, which these thinkers take care to avoid because their own demonstra-
20 tions are abstract,—with this exception, that if the exceeding and the exceeded are
the principles, and not the great and the small, consistency requires that number
should come from the elements before 2 does; for both are more universal than 2, as
the exceeding and exceeded are more universal. But as it is, they say one of these
25 things but do not say the other. Others oppose the different and the other to the one,
and others oppose plurality to the one. But if, as they claim, things consist of
contraries, and to the one either there is nothing contrary, or if there must be
something it is plurality, and the unequal is contrary to the equal and the different
to the same and the other to the thing itself, those who oppose the one to plurality
30 have most claim to plausibility, but even their view is inadequate, for the one would
on their view be a few; for plurality is opposed to fewness, and the many to the
few.

'One' evidently means a measure. And in every case it is some underlying thing
with a distinct nature of its own, e.g. in the scale a quarter-tone, in magnitude a
finger or a foot or something of the sort, in rhythms a beat or a syllable; and
35 similarly in weight it is a definite weight; and in the same way in all cases, in
qualities a quality, in quantities a quantity (and the measure is indivisible, in the
1088¹ former case in kind, and in the latter to the sense); which implies that the one is not,
in any instance, in itself a substance. And this is reasonable; for the one means the
measure of some plurality, and number means a measured plurality and a plurality
5 of measures. Thus it is natural that one is not a number; for the measure is not
measures, but both the measure and the one are starting-points. The measure must
always be something predicable of all alike, e.g. if the things are horses, the measure
is horse, and if they are men, man. If they are a man, a horse, and a god, the
10 measure is perhaps living beings, and the number of them will be a number of living
beings. If the things are man and white and walking, these will scarcely have a
number, because all belong to a subject which is one and the same in number, yet
the number of these will be a number of classes, or of some equivalent term.

Those who treat the unequal as one thing, and the dyad as an indefinite
compound of great and small, say what is very far from being probable or possible.
15 For these are modifications and accidents, rather than substrata, of numbers and
magnitudes—the many and few of number, and the great and small of magni-
tude—like even and odd, smooth and rough, straight and curved. Again, apart from
20 this mistake, the great and the small, and the like, must be relative to something;
but the relative is least of all things a real thing or substance, and is posterior to
quality and quantity; and the relatives are accidents of quantity, as was said, but not
25 its matter, since there is something else both for relative in general and for its parts
and kinds. For there is nothing either great or small, many or few, or, in general,
relative, which is many or few, great or small, or relative without being so as
something else. A sign that the relative is least of all a substance and a real thing is
30 the fact that it alone has no proper generation or destruction or movement, as in
quantity there is increase and diminution, in quality alteration, in place locomotion,

in substance simple generation and destruction. The relative has no proper change; for, without changing, a thing will be now greater and now less or equal, if that with which it is compared has changed in quantity. And the matter of each thing, and therefore of substance, must be that which is potentially of the nature in question; but the relative is neither potentially nor actually substance. It is strange, then, or rather impossible, to make non-substance an element in, and prior to, substance; for all the categories are posterior. Again, the elements are not predicated of the things of which they are elements, but many and few are predicated both apart and together of number, and long and short of the line, and both broad and narrow apply to the plane. If there is a plurality, then, of which the one term, viz. few, is always predicated, e.g. 2 (which cannot be many for if it were many, 1 would be few), there must be also one which is absolutely many, e.g. 10 is many (if there is no number which is greater than 10), or 10,000. How then, in view of this, can number consist of few and many? Either both ought to be predicated of it, or neither; but according to the present account only the one *or* the other is predicated.

2 · We must inquire generally, whether eternal things can consist of elements. If they do, they will have matter; for everything that consists of elements is composite. Since, then, a thing must have come into being out of that of which it consists (and if it is eternal, then if it *had* come into being it would have done so in that way), and since everything comes to be what it comes to be out of that which is it potentially (for it could not have come to be out of that which had not this capacity, nor could it consist of such elements), and since the potential can be either actual or not,—this being so, however everlasting number or anything else that has matter is, it must be capable of not existing, just like anything which is a single day or any number of years old; if this is capable of not existing, so is that which has lasted for a time so long that it has no limit. They cannot, then, be eternal, since that which is capable of not existing is not eternal, as we had occasion to show in another context. If that which we are now saying is true universally—that no substance is eternal unless it is actuality, and if the elements are matter that underlies substance, no eternal substance can have elements present in it, of which it consists.

There are some who describe the element which acts with the one as the indefinite dyad, and object to the unequal, reasonably enough, because of the ensuing difficulties; but they have got rid only of those objections which inevitably arise from the treatment of the unequal, i.e. the relative, as an element; those which arise apart from this opinion must confront even these thinkers, whether it is ideal number, or mathematical, that they construct out of those elements.

There are many causes which led them off into these explanations, and especially the fact that they framed the difficulty in an old-fashioned way. For they thought that all things that are would be one—viz. Being itself, if one did not join issue with and refute the saying of Parmenides:²

For never will this be proved, that things that are not are.

²Frag. 7 Diels-Kranz.

They thought it necessary to prove that that which is not is; for thus—of that which is and something else—could the things that are be composed, if they are many.

But firstly, if 'being' has many senses (for it means sometimes substance, sometimes quality, sometimes quantity, and at other times the other categories), what sort of one are all the things that are, if non-being is to be supposed not to be? Is it the substances that are one, or the affections and the other categories as well, or everything—so that the 'this' and the 'such' and the 'so much' and the other categories that indicate each some one thing will all be one? But it is strange, or rather impossible, that a single nature should bring it about that part of that which is is a 'this', part a 'such', part a 'so much', part somewhere.

Secondly, of what sort of non-being and being do the things that are consist? For 'non-being' also has many senses, since 'being' has; and not being a man means not being a certain 'this', not straight not being of a certain quality, not three cubits long not being of a certain quantity. From what sort of being and non-being, then, do the things that are come to be many? He means by the non-being, the union of which with being makes the things that are many, the false and the character of falsity. This is also why it was said that we must assume something that is false, as geometers assume the line which is not a foot long to be a foot long. But this cannot be so. For neither do geometers assume anything false (for the proposition in question is extraneous to the inference), nor are the things that are, generated from or resolved into non-being in this sense. But since non-being in the various cases has as many senses as there are categories, and besides this the false is said not to be and so is the potential, generation proceeds from the *latter*, man from that which is not man but potentially man, and white from that which is not white but potentially white, and this whether it is one thing that is generated or many.

The question evidently is, how being in the sense of *substances* is many; for the things that are generated are numbers and lines and bodies. It is strange to inquire how being in the sense of essence is many, and not how either qualities or quantities are many. For surely the indefinite dyad or the great and the small are not a cause of there being two kinds of white or many colours or flavors or shapes; for then these also would be numbers and units. But if they had attacked this point, they would have seen the cause of the plurality in substances also; for the cause is the same or analogous. This aberration is the reason also why in seeking the opposite of being and the one, from which and being and the one the things that are proceed, they posited the relative (i.e. the unequal), which is neither the contrary nor the contradictory of these, but is one kind of being as substance and quality are.

They should have inquired also how relatives are many and not one. But as it is, they inquire how there are many units besides the first 1, but do not go on to inquire how there are many unequals besides *the* unequal. Yet they use them and speak of great and small, many and few (from which proceed numbers), long and short (from which proceeds the line), broad and narrow (from which proceeds the plane), deep and shallow (from which proceed solids); and they speak of yet more kinds of relatives. What is the reason, then, why there is a plurality of these?

It is necessary, as we say, to presuppose for each thing that which is it

potentially; and the holder of these views further declared what that is which is potentially a 'this' and a substance but is not in itself being—viz. that it is the relative (as if he had said the qualitative), which is neither potentially the one or being, nor the contradictory of the one nor of being, but one among beings. And it was much more necessary, as we said, if he was inquiring how beings are many, not to inquire about those in the same category—how there are many substances or many qualities—but how beings as a whole are many; for some are substances, some modifications, some relations. In the categories other than substance there is another matter to give us pause, viz. how can there be many? For since they are not separable, qualities and quantities are many only because their substrate becomes and is many. Yet there *ought* to be a matter for each category; only it cannot be separable from substances. But in the case of a 'this', it is possible to explain how the 'this' is many things, unless a thing is to be treated as both a 'this' and a general character. The difficulty arising from these facts is rather this, how there are actually many substances and not one.

But further, if the 'this' and the quantitative are not the same, we are not told how and why the things that are many, but how quantities are many. For all number means a quantity, and so does the unit, unless it means merely a measure or the indivisible in quantity. If then the quantitative and essence are different, we are not told whence or how essence is many; but if any one says they are the same, he has to face many inconsistencies.

One might fix one's attention also on the question, regarding the numbers,—what justifies the belief that they exist. To the believer in the Ideas they provide a cause for existing things, since each number is an Idea, and the Idea is to other things somehow or other the cause of their being; for let this supposition be granted them. But as for him who does not hold this view because he sees the inherent objections to the Ideas (so that it is not for this reason that he posits numbers), but who posits *mathematical* number, why must we believe his statement that such number exists, and of what use is such number to other things? Neither does he who says it exists maintain that it is the cause of anything (he rather says it is a thing in itself), nor is it observed to be the cause of anything; for the theorems of arithmeticians will all be found true even of sensible things, as was said.

3 . Those who suppose the Ideas to exist and to be numbers, take each to be one thing by setting each out apart from the many—so that they try at least to explain somehow why numbers exist. Since their reasons, however, are neither conclusive nor in themselves possible, one must not, on this account at least, assert the existence of number. But the Pythagoreans, because they saw many attributes of numbers belonging to sensible bodies, supposed real things to be numbers—not separable numbers, however, but numbers of which real things consist. But why? Because the attributes of numbers are present in a musical scale and in the heavens and in many other things. But those who say that mathematical number alone exists cannot according to their hypotheses say anything of this sort; indeed, they used to say that those numbers could not be objects of the sciences. But we maintain that

they are, as we said before. And it is evident that the objects of mathematics do not exist apart; for if they existed apart their attributes would not have been present in bodies. The Pythagoreans in this point are open to no objection; but in that they construct natural bodies out of numbers, things that have lightness and weight out of things that have not weight or lightness, they seem to speak of another heaven and other bodies, not of the sensible. But those who make number separable assume that it exists and is separable because the axioms would not be true of sensible things, while the statements of mathematics *are* true and delight the soul; and similarly with the magnitudes of mathematics. It is evident, then, both that our contrary theory will say the contrary of this, and that the difficulty we raised just now, why if numbers are in no way present in sensible things their attributes are present in sensible things, is solved for those who hold our views.

There are some who, because the point is the limit and extreme of the line, the line of the plane, and the plane of the solid, think there must be real things of this sort. We must therefore examine this argument too, and see whether it is not remarkably weak. For extremes are not substances, but rather all these things are mere limits. For even walking, and movement in general, has a limit, so that on their theory this will be a 'this' and a substance. But that is absurd. Even if they are substances, they will all be the substances of particular sensible things; for it is to these that the argument applied. Why then should they be capable of existing apart?

Again, if we are not too easily satisfied, we may, regarding all number and the objects of mathematics, press this difficulty, that they contribute nothing to one another, the prior to the posterior; for if number did not exist, none the less magnitudes would exist for those who maintain the existence of the objects of mathematics only, and if magnitudes did not exist, soul and sensible bodies would exist. But the phenomena show that nature is not a series of episodes, like a bad tragedy. The believers in the Ideas escape this difficulty; for they construct magnitudes out of matter and number, lines out of 2, planes doubtless out of 3, solids out of 4, or they use other numbers, which makes no difference. But will these magnitudes be Ideas, or what is their manner of existence, and what do they contribute to things? These contribute nothing, as the objects of mathematics contribute nothing. But not even is any theorem true of them, unless we want to change mathematics and invent doctrines of our own. But it is not hard to assume any random hypotheses and spin out a long string of conclusions. These thinkers, then, are wrong in this way, in wanting to unite the objects of mathematics with the Ideas.

And those who first posited two kinds of number, that of the Forms and the other which is mathematical, neither have said nor can say in the least how mathematical number is to exist and of what it is to consist. For they place it between ideal and sensible number. If it consists of the great and small, it will be the same as the other—ideal number. (And from what other³ great and small can he

³Reading ἐξ ἄλλου δὲ τίνος.

1091¹ produce magnitudes?) And if he names some other element, he will be making his elements rather many. And if the principle of each of the two kinds of number is a 1, unity will be something common to these. And we must inquire how the one is these many things, while at the same time *number*, according to him, cannot be generated except from one and the indefinite dyad.

5 All this is absurd, and conflicts both with itself and with the probabilities, and we seem to see in it Simonides' 'long story'; for the long story comes into play, like those which slaves tell, when men have nothing sound to say. And the very elements—the great and the small—seem to cry out against the violence that is done to them; for they cannot in any way generate numbers other than those got from 1 by doubling.

10 It is strange also to attribute generation to eternal things, or rather this is one of the things that are impossible. There need be no doubt whether the Pythagoreans attribute generation to them or not; for they obviously say that when the one had been constructed, whether out of planes or of surface or of seed or of elements which they cannot express, immediately the nearest part of the unlimited began to be drawn in and limited by the limit. But since they are constructing a world and wish to speak the language of natural science, it is fair to make some explanation of their account of nature, but to let them off from the present inquiry; for we are investigating the principles at work in *unchangeable* things, so that it is numbers of *this* kind whose genesis we must study.

15 4 . These thinkers say there is no generation of the odd number, which evidently implies that there *is* generation of the even; and some say the even is produced first from unequals—the great and the small—when these are equalized. The inequality, then, must belong to them *before* they are equalized. If they had always been equalized, they would not have been unequal before; for there is nothing before that which is always. Therefore evidently they are not giving their account of the generation of numbers merely as a theoretical account.

20 A difficulty, and a reproach to any one who finds it *no* difficulty, are contained in the question how the elements and the principles are related to the good and the beautiful; the difficulty is this, whether any of the elements is such a thing as we mean by the good itself and the best, or this is not so, but these are later in origin. The mythologists seem to agree with some thinkers of the present day, who answer the question in the negative, and say that both the good and the beautiful appear only when nature has made some progress. This they do to avoid a real objection which confronts those who say, as some do, that the one is a first principle.

25 1091¹ (The objection arises not from their ascribing goodness to the first principle as an attribute, but from their making the one a principle—and a principle in the sense of an element—and generating number from the one.) And the old poets agree with this inasmuch as they say that not those who are first in time, e.g. Night and Heaven or Chaos or Ocean, reign and rule, but Zeus. These poets, however, speak thus only because they think of the rulers of the world as changing; for those of them who combine two characters in that they do not use mythical language throughout,

e.g. Pherecydes and some others, make the original generating agent the Best, and so do the Magi, and some of the later sages also, e.g. Empedocles and Anaxagoras, of whom one made friendship an element, and the other made thought a principle. Of those who maintain the existence of the unchangeable substances some say the one itself is the good itself; but they thought its substance lay mainly in its unity.

10 This, then, is the problem,—which of the two ways of speaking is right. It would be strange if to that which is primary and eternal and most self-sufficient this very quality—self-sufficiency and self-maintenance—belongs primarily in some other way than *as a good*. But indeed it can be for no other reason indestructible or self-sufficient than because its nature is good. Therefore to say that the first principle is good is probably correct; but that this principle should be the one or, if not that, an element, and an element of numbers, is impossible. Powerful objections arise, to avoid which some have given up the theory (*viz.* those who agree that the one is a first principle and element, but only of *mathematical* number). For all the units become identical with species of good, and there is a great profusion of goods. Again, if the Forms are numbers, all the Forms are identical with species of good. But let a man assume Ideas of anything he pleases. If these are Ideas only of goods, the Ideas will not be substances; but if the Ideas are also Ideas of substances, all animals and plants and all things that share in Ideas will be good.

15 These absurdities follow, and it also follows that the contrary element, whether it is plurality or the unequal, i.e. the great and small, is the bad-itself. (Hence one thinker avoided attaching the good to the one, because it would necessarily follow, since generation is from contraries, that badness is the fundamental nature of plurality; others say inequality is the nature of the bad.) It follows, then, that all things partake of the bad except one—the one itself, and that numbers partake of it in a more undiluted form than magnitudes, and that the bad is the space in which the good is realized, and that it partakes in and desires that which tends to destroy it; for contrary tends to destroy contrary. And if, as we said, the matter is that which is potentially each thing, e.g. that of actual fire is that which is potentially fire, the bad will be just the potentially good.

20 All these objections, then, follow, partly because they make every principle an element, partly because they make contraries principles, partly because they make the one a principle, partly because they treat the numbers as the first substances, and as capable of existing apart, and as Forms.

25 5 . If, then, it is equally impossible not to put the good among first principles and to put it among them in this way, evidently the principles are not being correctly described, nor are the first substances. Nor do we conceive the matter correctly if we compare the principles of the universe to that of animals and plants, on the ground that the more complete always comes from the indefinite and incomplete—which is what leads this thinker to say that this is also true of the first principles of reality, so that the one itself is not even an existing thing. For here too the principles from which these come are complete; for it is a man that produces a man, and the seed is not first.

It is strange, also, to generate place simultaneously with the mathematical solids (for place is peculiar to the individual things, and hence they are separable in place, but mathematical objects are nowhere), and to say that they must be somewhere, but not say what the place is.

Those who say that the things that are come from elements and that the first of things that are are the numbers, should have first distinguished the senses in which one thing comes from another, and then said in which sense number comes from its elements.

By intermixture? But not everything is capable of intermixture, and that which is produced by it is different, and on this view the one will not be separate or a distinct entity; but they want it to be so.

By juxtaposition, like a syllable? But then the elements must have position; and he who thinks of the one and plurality must think of them apart; number then will be this—a unit *and* plurality, or the one *and* the unequal.

Coming from certain things means in one sense that these are still to be found in the product and in another that they are not; in which sense does number come from these elements? Only things that are generated can come from elements which are present in them. Does number come from its elements as from seed? But nothing can come from that which is indivisible. Does it come from its contrary, its contrary not persisting? But all things that come in this way come also from something else which does persist. Since, then, one thinker places the 1 as contrary to plurality, and another places it as contrary to the unequal, treating the 1 as equal, number is treated as coming from contraries. There will then be something else that persists, from which and from one contrary the compound is or has come to be. Again, why in the world do the other things that come from contraries, or that have contraries, perish (even when all of the contrary is used to produce them), while number does not? Nothing is said about this. Yet whether present or not present in the compound the contrary destroys it, e.g. strife destroys the mixture (yet it should not; for it is not to that that it is contrary).

Once more, it has in no sense been determined in which way numbers are the causes of substances and of being—whether as limits (as points are of magnitudes). This is how Eurytus decided what was the number of what (e.g. of man, or of horse), viz. by imitating the figures of living things with pebbles, as some people bring numbers into the forms of triangle and square. Or is it because harmony is a ratio of numbers, and so is man and everything else? But how are the attributes—white and sweet and hot—numbers? Evidently the numbers are not the substance nor causes of the form; for the ratio is the substance, while the number is the matter. E.g. the substance of flesh or bone is number only in this way, 'three parts of fire and two of earth.' And a number, whatever it is, is always a number of certain things, either of fire or earth or of units; but the substance is that there is so much of one thing to so much of another in the mixture; and this is no longer a number but a ratio of mixture of numbers, whether these are corporeal or of any other kind.

Number, then, whether number in general or the number which consists of

abstract units, is neither the cause as agent, nor the matter, nor the formula and form of things. Nor, of course, is it that for the sake of which.

6 . One might also raise the question what the good is that things get from numbers because their composition is expressible by a number, either by one which is easily calculable or by an odd number. For in fact honey-water is no more wholesome if it is mixed in the proportion of three times three, but it would do more good if it were in no particular ratio but well diluted than if it were numerically expressible but strong. Again, the ratios of mixtures are expressed by the *adding* of numbers, not by mere numbers, e.g. it is three parts to two, not three times two. For the same genus must underlie things that are multiplied together; therefore the product $1 \times 2 \times 3$ must be measurable by 1, and $4 \times 5 \times 7$ by 4, and therefore all products into which the same factor enters must be measurable by that factor. The number of fire, then, cannot be $2 \times 5 \times 3 \times 7$, and at the same time that of water 2×3 .

If all things must share in number, it must follow that many things are the same, and the same number must belong to one thing and to another. Is number the cause, then, and does the thing exist because of its number, or is this not certain? E.g. the motions of the sun have a number, and again those of the moon, and so do the life and prime of each animal. Why, then, should not some of these numbers be squares, some cubes, and some equal, others double? There is no reason why they should not, and indeed they must be comprised within these descriptions, since all things were assumed to share in number and things that differed might fall under the same number. Therefore if the same number had belonged to certain things, these would have been the same as one another, since they would have had the same form of number; e.g. sun and moon would have been the same. But why are these numbers causes? There are seven vowels, the scale has seven strings, the Pleiades are seven, at seven animals lose their teeth (at least some, though some do not), and the champions who fought against Thebes were seven. Is it then because the number is what it is, that the champions were seven or the Pleias consists of seven stars? Surely the champions were seven because there were seven gates or for some other reason, and the Pleias we count as seven, as we count the Bear as twelve, while other peoples count more stars in both. They even say that Ξ , Ψ , and Z are concords, and because there are three concords, the double consonants also are three. They quite neglect the fact that there might be a thousand such letters; for one sign might be attached to ΓP . But if they say that each of these three is equal to two of the other letters, and no other is so, and if the cause is that there are three parts of the mouth and one letter is applied to Σ in each, it is for this reason that there are only three, not because the concords are three; since as a matter of fact the concords are more than three, but of double consonants there cannot be more. These people are like the old Homeric scholars, who see small resemblances but neglect great ones. Some say that there are many such cases, e.g. that the middle strings are represented by nine and eight, and that the epic verse has seventeen syllables, which is equal in number

to the two strings; and the scansion is, in the right half of the line nine syllables, and in the left eight. And they say that the distance in the letters from alpha to omega is equal to that from the lowest note of the flute to the highest, and that the number of this note is equal to that of the whole system of the heavens. We must observe that
5 no one could find difficulty either in stating such analogies or in finding them in eternal things, since they can be found even in perishable things.

But the celebrated characteristics of numbers and their contraries, and generally the mathematical relations, if we view them as some do, making them
10 causes of nature, seem to escape us; for none of them is a cause in any of the senses that have been distinguished in reference to the first principles. Yet if mathematical objects be conceived as these thinkers conceive them, evidently goodness is predicable of them, and the odd, the straight, the equal-by-equal, and the powers of certain numbers, are in the column of the beautiful. For the seasons and a particular
15 number go together; and the other agreements that they collect from the theorems of mathematics all have this meaning. Hence they are like coincidences. For they are accidents, but appropriate to one another, and one by analogy. For in each category of being an analogous term is found—as the straight line is in length, so is
20 the plane in surface, perhaps the odd in number, and the white in colour.

Again, it is not the *ideal* numbers that are the causes of musical phenomena and the like (for equal ideal numbers differ from one another in form; for even the units do); so that we need not assume Ideas for this reason at least.

These, then, are the results of the theory, and yet more might be brought
25 together. The fact that they have much trouble with the generation of ideal numbers and can in no way make a system of them, seems to indicate that the objects of mathematics are not separable from sensible things, as some say, and that they are not the first principles.