Philosophy 405: Knowledge, Truth and Mathematics
Spring 2008
M, W: 1-2:15pm
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Classes 8-9: Kant

I. A hint for reading Kant

Kant's terminology is terribly difficult to master.
Just let it all wash over you: try to read it straight through, and go back for the details.
Most philosophy is worth reading three times, anyway.

II. Kant's metaphysics, epistemology, and semantics

Kant begins his *Critique*, indeed his whole metaphysical philosophy, with the claim that mathematical propositions (or judgments) are synthetic a priori.
Further, Kant's notion of apriority includes necessity.
So, his claim is both metaphysical and epistemological.
He is claiming both that mathematical propositions are necessary (metaphysics) and known a priori (epistemology).
In both of these claims, he is roughly in agreement with Hume, with a different terminology.
Hume does not call mathematical propositions necessary, but he does claim that they are relations of ideas, following from the principle of contradiction.
Nelson makes this point, on p 1.

Kant also claims a semantic thesis, that mathematical propositions are synthetic.
A proposition (in its most rough parsing) affirms a predicate of a subject.
All propositions are either synthetic or analytic.
A proposition is synthetic if the predicate adds something to the concept of the subject, if it augments or enlarges the subject.
A proposition is analytic if the predicate is contained within the subject.
For examples, consider Nelson's 'every mother has a child'; or 'p • q entails p'.
Hume's relations of ideas are all analytic.
See Nelson's chart.

So, the big claim is that 7+5=12 is not analytic, that the concepts of 7+5 are not found in the concept of 12.
Similarly, the concept of a circle does not in itself contain the concept of the tangent meeting the radius at right angles.

III. Conceptual containment

Kant's analytic/synthetic distinction depends on the notion of conceptual containment.
In 'p • q entails p', we can actually see the 'p' in the 'p • q'.
But, in 'every mother has a child' we can not see the having of a child in the term 'mother'.
Still, Kant is just claiming that we can unpack the concept of a mother and find the having of a child in there.

For Hume, the truths of mathematics were relations of ideas because they followed from definitions.
Kant disagrees that mathematical statements are true by definition.
See *Prolegomena* 268-9.
They have to be present in my thought.
Consider the construction of concepts in thought, A716-718 (pp 14-5)
See A164/B205 (p 23)
Still, we do not know what it means for a concept to contain another.
Does the concept of a cat contain the concept of being a mammal?
Kant believes that the concept of a triangle contains the concept of having three angles.
But, does the concept of a triangle contain the concept of having three sides?

More generally, is Kant right about the syntheticity of mathematics?

IV. Crazy talk

Further, is Kant's analysis of all four types of proposition correct?
Consider, as Nelson points out, that Kant claims that there are no analytic a posteriori claims.
That's right, for Kant.
See 'Gold is a yellow metal' in Prolegomena 267.
But consider:
'All tokens of C# have volume (loudness).'
Or, 'All material objects have weight.'

V. Why it is important to Kant that mathematical judgments be synthetic a priori

If mathematical judgments were all analytic, then they would all follow from the principle of contradiction.
But, Kant alleges, no contradiction will follow from the denial of some mathematical propositions.
'$7+5\neq12$' does seem like a contradiction, though.
This seems to be a real problem.
If the claim is synthetic, then it has to be logically possible for $7+5\neq12$.
It seems that Kant has two notions of necessity.
Mathematical truths are not necessary in the sense that they follow from the principle of contradiction.
But, they are necessary in the sense that they are a priori.
The denial of the parallel postulate does not seem problematic in the way that statements of arithmetic are.
See A718-9 (p 15)

If mathematical propositions were analytic, the scope of reason would narrow, and the possibility of metaphysics would diminish.
The synthetic a priori is the domain of philosophy, as Kant uses the term, or metaphysics, more specifically.
More importantly, Kant credits Hume with awakening him from his dogmatic slumbers.
Without the synthetic a priori, we end up as skeptics.
If all our empirical beliefs are matters of fact, and trace back to initial impressions, then we get the problem of induction.

We can never know scientific laws.
And, as Hume's position entails, we can never know to go out through the door rather than the window.
We have no reason to believe that the future will be like the past, that the laws of nature are uniform.
For, as Hume showed, it is not analytic in any of the concepts we trace back to impressions.
We can not find the effect in the cause.
And, it is not learned by instances.
So, it is not learned at all.

Hume showed that presuppositions about language and its relation to the world in Locke and Berkeley led to a deep skepticism.
But, Hume left mathematics alone.

Kant's solution to the skeptical problem is embodied in his transcendental method.
We start with Nelson's chart.
Then, we determine what the nature of our minds must be in order to create that chart.
What are the necessary conditions of the structure of our minds such that we get the necessary truths of mathematics, and laws of nature?
We impose a conceptual structure, built into the very fabric of our minds, which determines, a priori, the framework.
We are given a noumenal world, and we structure this world by imposing our concepts upon it.
In order to solve the skeptical problem, Kant opens up the synthetic a priori category, and mathematics falls in.

VI. Intuition

The heart of Kant's philosophy of mathematics involves the construction of objects of mathematics in intuition.
Nelson worries about the notion of intuition, p 3.
Kant is using that term as a technical term.
We are clearly given something, the passing show.
These are intuitions, in Kant's terminology.
Some of our intuitions are empirical.
Some of our intuitions are non-empirical.
Read A19-21.
They are structured; so the structure must come from somewhere.
If the structure came from outside of us, then we would forever be skeptics, with Hume.
But, if the structure is imposed by us, then we can know about it, by mere reflection.
Intuitions, in short, are whatever is left over after analytic knowledge is account for, after we've eliminated all entailments, including conceptual containments.

In particular, our intuitions of space and time are built into the conceptual structures we impose on the world, A713-4, p 14.
Here, Kant sounds a bit like Hume/Berkeley: we have empirical intuitions, but these stand for universals.
But note the difference.
We are also constructing, in pure intuition, the corresponding figure.
We draw the pure figure in our thoughts.
Or, for arithmetic, we construct stroke-symbols, and add them up.

Then, we have rules for adding symbols which depend on those symbols.
Using, for example, our fingers, we can construct empirical intuitions.
But, these only correspond to the pure intuitions, and do not constitute them.
There is no indication that the pure intuition has to be like a picture.
Consider A720 (p 16).
Note that the pure intuitions are formal, rather than material.
Compare with A723 (p 17).
Also see A140/B180 (p 11)
What is important about the pure intuition is the rule it dictates.

There is another part of Kant's argument for the syntheticity of mathematics lurking in the appeals to intuition.
Consider how we do mathematics.
We appeal to intuition, whether signs or symbols or diagrams.
If we were just analyzing concepts, definitions, we would have no need for such intuitions.

Walter had brought up the distinction between axioms and theorems.
We might focus our interest on the status of the axioms, and just allow that the theorems follow by rules of logic.
I mentioned in response to Walter that Kant's contention is that the axioms are synthetic a priori, so all the theorems are, too.
But, Kant also claims that the chain of reasoning which leads from axioms to theorems is guided by intuition, A717/B745.

Descartes had separated thought from sensation.
Locke, (and Berkeley and Hume) in his attempt to avoid innate ideas, had put sensation back into thought in the form of the representational theory of mind.
Ideas had to be like pictures, derived from some sensations.
But, Kant is again cleaving thought and sensation.
We have pure intuitions, which may be not be pictures.

VII. The Berkeley problem, revisited

Just to say that our intuitions are not pictures may not solve the Berkeley problem.
Couldn't Berkeley just ask the same question of Kant: Is the intuition of a triangle which you have constructed in your mind the intuition of a scalene, isosceles, or equilateral triangle?
Kant's response is to say that we can only use the parts of the triangle that we construct for the proof.
We rely on the rule which guides the construction in intuition.
If we are constructing a triangle, we have to construct three angles.
We need not construct three angles all of which are less than 90 degrees.
So, we must distinguish among the properties which belong to all triangles and those which belong to only some.

Kitcher distinguishes three types of properties:
R-properties, which all triangles have, analytically, e.g. having three angles.
S-properties, which are synthetic, arising from the construction in intuition for any triangle, e.g. the sum of two sides is greater than the third.

A-properties, like being scalene, which need not apply to every triangle.

A similar analysis can apply to arithmetic examples.
R-properties, like 12=12, or commutativity.
S-properties, like 5+7=12.
A-properties, like being prime.

We are free to construct the A-properties any way we like.
They do not follow from the nature of the triangle.
Their denials are logically possible.
We can construct a scalene triangle, or an equilateral triangle, as we wish.

It seems as if we might be free to construct the S-properties any way we like.
They do not follow from the nature of the triangle, and their denials are logically possible, too.
Recall, that it seems as if Kant is using two notions of necessity.
Still, there is a problem here, of distinguishing the S-properties from the A-properties.
Kitcher argues that Kant has no way to do that without already knowing the properties of space, which underlie my intuitions.

Kant approaches this problem by assuring us that all space is necessarily Euclidean.
He calls this the transcendental ideality of space (A19-30, but esp. A28)
We construct, necessarily, our intuitions in Euclidean space.
Mathematics is just the study of the pure forms of intuition.
Our knowledge of geometry is a priori knowledge of the necessary structure of space.
Our knowledge of arithmetic is a priori knowledge of the necessary structure of "combinatorial" aspects of space and time.
A724 (p 17).
(Also see Kitcher p 34)

Kitcher says, p 46, that to distinguish S-properties from A-properties is just to recognize the structure of space.
I'm not sure I understand Kitcher's point.
But there is a stronger problem here, to which Heather alludes.

VIII. Non-Euclidean geometry

Read Heather, p 2-3
Consider an interstellar triangle.
The sum of its angles will not be 180°, due to the curvatures of space-time corresponding to the gravitational pull of the stars, and other large objects.
Space-time is not Euclidean, but hyperbolic.

The difference between hyperbolic space-time and Euclidean space-time involves the parallel postulate.
Euclid's parallel postulate states that if a straight line falling on two straight lines makes the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles.
Playfair produced an equivalent, and nowadays more common, formulation: Given a line and a point

outside that line, there is exactly one line going through the point that is parallel to the given line.
In hyperbolic geometry, instead of there being one line that we can draw parallel to the given line, there are an infinite number of lines (not just the two to which Heather alludes).

Kant's claims that Euclidean geometry is known a priori and that a priori knowledge is infallible thus can not both hold.

IX. Looking forward

Looking at the historical sources we have explored, there is an interplay between metaphysics and epistemology.
Those philosophers who provided a substantial mathematical ontology (e.g. Plato, Descartes) seemed committed to an unacceptable epistemology.
Those philosophers who started with commonsense epistemologies (e.g. Aristotle, Locke) seemed committed to an unacceptably anemic mathematical ontology.
Where Kant denied that the empirical intuition constituted or sufficed for mathematical knowledge, Mill will deny that we can get anything more than that.
Mill severely restricts mathematical necessity and apriority.
But, he provides a much less controversial epistemology.

A different response to Kant, that of Frege, argues that Kant is wrong about mathematics being synthetic.
Frege will argue for a different notion of analyticity.