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Class 6: Modern Rationalism II, Locke and Leibniz

I. Descartes's foundationalism

Sarah asks a good question about the link between Descartes's own existence and mathematical truth.
The story is a long one, about the links between foundational truths and derivative ones.
We are often tempted to think that sensory knowledge is primary, and that mathematical knowledge should somehow derive from the senses.
Descartes inverts that order.
At the beginning of the *Meditations*, he discusses worries about some false beliefs.
He wonders how we know whether we are dreaming, or whether or not a powerful deceiver is putting false thoughts in our minds.
His real worry is that while some of our beliefs seem impervious to doubt, beliefs which are based on our senses seem less secure.
As Plato worried about the veracity of beliefs in the sensible world, Descartes worries about the sense properties of physical objects, what Locke calls the secondary properties.
Locke came after Descartes, but the primary/secondary distinction is present in much earlier scientists, including Galileo (1564-1642), who wrote:

> ...that external bodies, to excite in us these tastes, these odours, and these sounds, demand other than size, figure, number, and slow or rapid motion, I do not believe, and I judge that, if the ears, the tongue, and the nostrils were taken away, the figure, the numbers, and the motions would indeed remain, but not the odours, nor the tastes, nor the sounds, which, without the living animal, I do not believe are anything else than names.

Descartes is writing as a member of, and in response to, the scientific revolution.
The Copernican system was seen, especially by the Church, as antagonistic to religion.
Galileo's censorship by the Inquisition forced Descartes to suppress publication of *Le Monde*, his work on the philosophical implications of the new science.
Descartes's real goal is a system which will accommodate the new science of Galileo while not undermining the key elements of a religious world view.
That religious world view is essentially Aristotelian.
Aristotle had not placed the soul outside of the body, and thus had linked thought and sensation.
In particular Descartes presents the *Meditations* as a proof of the existence of God and the immortality of the soul.

Descartes presents a system of knowledge in the spirit of Euclid's *Elements*.
The geometric presentation, from the second set of objections and replies, makes his method clear.
We call Descartes a foundationalist because he wants to ground our knowledge on basic, indubitable truths.
In the *Meditations*, it looks like his foundational truth is the cogito.
In the geometric presentation, it looks like his foundation is the existence and goodness of God.

Leibniz, too, presents a foundational system.
For Leibniz, the foundations, or primary truths, are identities.
All other truths reduce to primary truths by definitions.
We will return to Leibniz.

II. Axiomatic theories

We looked just very briefly at a variety of axiomatic theories, just to get us in the mood.
We started with Hofstadter's MIU system

The MIU system

Any string of Ms Is and Us is a string of the MIU system.
MIU, UMI, and MMMUMUUUMUMMU are all strings.
Only some strings will be theorems.
The theorems will correspond to the true sentences of English.
Or, they could correspond to theorems of geometry.

Axioms and theorems

A theorem is any string which is either an axiom, or follows from the axioms by using some combination of the rules of inference.
The MIU system takes only one axiom: MI.
This means that MI is our foundational truth, as the cogito, or God, is the foundation for Descartes's epistemology.

Four rules of inference

R1. If a string ends in I you can add U.
R2. From Mx, you can infer Mxx.
That is, you can repeat whatever follows an M.
R3. If III appears in that order, then you can replace the three Is with a U
R4. UU can be dropped from any theorem.

We derived MIIIII.
I challenged you to derive MU.
For help, see Hofstadter's book, pp 259-261.
I hope that no one spent too much time on it!

We also looked at axiomatic versions of propositional logic, set theory, number theory (the Peano/Dedekind axioms), and geometry (Birkhoff's postulates).

As a response to the worries about doubts, Descartes, in the *Meditations*, presents the cogito as the foundation of our knowledge.
Descartes's method of clear and distinct ideas is like a rule of inference used in a formal system.
Note that Descartes's sixth and seventh postulates correspond to the method of clear and distinct ideas, on which Sarah focuses at the beginning of her paper.
Sarah asks on p3 whether the method suffices.
Leibniz agrees that there is a problem: "[O]ften what is obscure and confused seems clear and distinct to people careless in judgment" (*Meditations on Knowledge, Truth, and Ideas*, p 26).
Descartes says in the geometric presentation that we can see clearly that it does.
Does it?

III. God

In the geometric presentation, Descartes's first proposition is the existence of God.
The *Meditations* contains two arguments for the existence of God.
The first one is in the remainder of the third meditation, which I expunged.
The one we read, the ontological argument, is in the fifth meditation.
Both of these arguments appear up front in the geometric presentation.
While the arguments for the existence of God are not our concern, the underlying point of those arguments is essential to understand.
Plato denied the reality of the sensible world; it was a world of mere belief, not a world of truth.
Descartes only denies that our access to truth can come from the senses.
So, his arguments for the existence of God are a priori, from pure thought.
Descartes's central achievement was to distinguish thinking from sensation, to move away from the Aristotelian conception of thought as essentially sensible.

The Cartesian view, then, is that we can be most sure of the beliefs we have which are not based on confused sensory representations.
In response to worries about dreaming and demon deceivers, Descartes presents rational arguments for the existence of a good God, who protects us from error, as long as we use our reason in the proper way.
And the proper use of reason recognizes the frailties of the senses.

Sarah discusses the ontological argument pretty well.
Leibniz (p 25) has a nice objection: the argument must first show the concept to be possible.
Imagine the fastest motion.
(Let's start by ignoring relativity, to give the objection its due.)
It seems like 'the fastest motion' might be a consistent concept.
But, we can construct a faster motion than any real motion.
Consider a wheel spinning at the fastest motion.
Now, consider a point extended out beyond the rim of the wheel.
The extension will be moving at a faster speed than any point on the wheel.

Now, what does relativity do to the argument?
There is indeed a fastest motion, and Leibniz's thought experiment, according to special relativity, is impossible.
But note, the general point still stands.
It just turns out that the impossible notion is the impossibility of a fastest motion!

IV. Descartes and mathematics

We started with the question about the link between knowledge of our own existence and knowledge of mathematics, and the observation that Descartes has inverted the commonsense order of our knowledge.
For Descartes, knowledge of mathematics is more certain, more fundamental, than knowledge based on the senses.
Beliefs which depend on the senses are tenuous.
Mathematical beliefs, unsullied by sensation, are elevated to pure truths.
We can know them certainly, because they are the result of pure reason.

Descartes is making two related claims.
1. Descartes's metaphysical claim: Mathematical objects have a determinate, objective nature, independent of us. Mathematical truths are necessary truths.
His argument for the first part of the metaphysical claim is:
1. A thing's nature depends on me if I can make it any way I like.
2. A thing's nature is objective if I can not make it any way I like.
3. I can not make mathematical objects any way I would like.
So, mathematical objects are objective.

To show that mathematical truths are necessary truths, Descartes relies on the larger argument that error arises from the senses.
In fact, Descartes does not think that any truths are necessary in the sense that they are independent of God's will.

2. Descartes's epistemological claim: Our knowledge of the truths of mathematics can not come from the senses. So, it must be innate.
His argument for the epistemological claim is:
1. All ideas must be invented, acquired, or innate.
2. Mathematical truths can not be invented, by the metaphysical claim.
3. Mathematical truths can not be acquired, by the chiliagon claim.
So, they must be innate.

We certainly do not learn mathematics first.
Descartes is distinguishing between the order of knowledge as it comes to us and the order as it is justified.
Leibniz makes this point explicitly, in the *New Essays*.
"Although the senses are necessary... (p 49)
This may be the most important argument that we can take from Descartes and Leibniz.
There is a genetic fallacy in assuming that because evidence from the senses temporally precedes evidence for mathematics, such knowledge is more secure.

Kara points out that Locke is unhappy with this approach, p 2.
We will return to the point in discussing Locke, below.

V. Leibniz

Leibniz thought that we could trace all complex ideas back to foundational ones, too.
Leibniz criticizes Descartes's appeal to clear and distinct ideas, for not specifying what it means to be clear and distinct.
Remember, Descartes had reduced all knowledge to fundamental principles.
Leibniz also reduces knowledge to simple principles.
In addition, Leibniz thought that there were foundational objects.
That is, he provides twin reductions: an epistemological reduction and its sister metaphysical reduction.

Here we find the main disagreement between Leibniz and Descartes.
Descartes thought that matter was just geometry, made concrete.
The essential property of matter is its extension.

Thus, according to Descartes, matter should be infinitely divisible.
But, Leibniz realized that infinite divisibility won't allow him his metaphysical reductionism.
That is, we would not be able to get to foundational objects, as well as foundational propositions.
For Leibniz, the foundational objects were, like Aristotelian substances, active.
Kara points out (p3) the similarities between Leibniz and Plato.
Leibniz is certainly closer to Plato, but he attempted to synthesize the two.

Leibniz called the foundational substances monads.
Monads are soul-like, and they reflect the entire state of the universe at each moment.
"Can you really believe that a drop of urine is an infinity of monads, and that each of these has ideas, however obscure, of the universe as a whole?" (Voltaire, *Oeuvres complètes*, Vol. 22, p. 434)
(Leibniz also allows for something like unconscious thought.
Since the mind is always active, there is activity even when we are not aware of it.)
We will not pursue Leibniz's metaphysical foundationalism, as it is ancillary to the mathematical question.

The distinctions among kinds of knowledge point to the epistemological reduction.
Obscure knowledge does not allow us even to identify a thing.
For example, I see that something is a leaf, but I don't know what kind of tree it came from.
Obscure knowledge should not even be called knowledge, in our sense of the term.
It is mere belief.

Clear knowledge gives us a "means for recognizing the thing represented" (24).
Clear knowledge may be divided into confused or distinct knowledge, which are distinguished by our ability to distinguish things from each other.

Confused knowledge is working knowledge, like that of color.
Leibniz says that we know how colors appear to us, but not so well how they work, how they are composed.
Perhaps we have a better understanding of the physical bases for colors.
But, consider chicken sexers.
(I failed to mention them in class; you can ask me about them if you don't know already.)
We can not communicate confused knowledge well.

Distinct knowledge is connected with marks to distinguish an object from others.
We can communicate it, and start to discuss its component parts.
Still, distinct knowledge may be adequate or not, depending on how many of the component parts we understand.

Inadequate knowledge is when we do not know, and can not communicate, all of the component notions of a thing.
The assayer may know how to distinguish gold from iron pyrite, and aluminum from molybdenum.
But, part of that distinction has to do with atomic weight.
And, the assayer may know how to test for atomic weight, but not know what it is.

If I have adequate knowledge of p, then I have adequate knowledge of all components of p, all components of components of p, etc.
This seems like a tall order, and Leibniz admits that we may not have any adequate knowledge.

Though, “Knowledge of numbers approaches it” (24).
Leibniz, like Descartes, is thinking of the mathematical method, the axiomatic method, as primary.
In mathematics, we can trace any claim, via its proof, back to the axioms!
But, even adequate knowledge is not the ultimate foundation.
The mathematician uses definitions to make his work perspicuous.
When we give a proof in, say, linear algebra, we do not present it in its set-theoretic form.
Our finite minds have limited abilities to comprehend all the steps in a long and complex proof, or proposition.

Symbolic knowledge, then, is adequate knowledge which appeals to signs (definitions) to represent our knowledge of components.
The use of definitions prevents our knowledge from being fully intuitive.

Intuitive knowledge is of distinct primitive notions.
An infinite mind would be able to have intuitive knowledge of all propositions.

For Leibniz, the foundational truths are identities, laws of logic.
These would be known intuitively, or directly.
As Sarah rightly points out, we can consider all the component notions of the most perfect knowledge at the same time.
Note that the most perfect knowledge, intuitive and adequate knowledge, would be a priori, traced back to the component parts of its real definition (not just its nominal one, p 26).
Sarah claims, p 5, that there is something unfortunate for the mathematician, in our general inability to trace all the component parts back.
What is unfortunate?

VI. Locke on innate ideas

In the early Leibniz piece, (“Knowledge, Truth, and Ideas”) we do not see the use of ‘innate’.
But, in the later piece, we see him defend Descartes’s innate ideas from Locke’s attack.
Let us, with tradition, call defenders of innate ideas ‘rationalists’ and opponents of innate ideas ‘empiricists’.
These terms will not apply to contemporary rationalists and empiricists, but they will work for Descartes, Locke and Leibniz, as well as Hume and Berkeley.
(Once Kant enters the picture, all bets are off; I mean, those terms won’t be useful.)

Locke attempts to show that two maxims, which are purportedly innate, are not innate.
He focuses on ‘What is, is’ and ‘It is impossible for the same thing to be and not to be’.
Kara (fn1) wonders about Locke’s use of these two maxims.
Are there candidates for innateness which one could find more plausible?
Note that Locke defends his use of those two maxims at the very end of the piece, §28.

Kara is also unhappy with Locke’s argument in §5, about the rationalist’s recourse to an innate capacity.
Locke argues that since we are not born apprehending any ideas, the rationalist’s claim leads to a contradiction: these ideas are known (since they are built-in) and unknown (since they are not apprehended) at the same time.
Locke considers a rationalist response, to distinguish between innate maxims and innate capacities.

If only the capacity is innate, then the rationalist might avoid the above contradiction.
Locke points out that Descartes and Leibniz are committed not merely to capacities, but to significant content built-into the soul.
If they were only committed to innate capacities, then every thing we know, including empirical propositions, would be innate.
(In fact, I think that Descartes is committed to something like the claim that everything we know is innate. He is more similar to Plato than is ordinarily recognized.)

Consider also the 'words are not so soon got' argument in Locke, §16, that Kara mentions on p2.
Locke argues that our knowledge of mathematical ideas, and logical maxims, is just like our knowledge of empirical claims.
We don't know that, say, a whale is a mammal (not Locke's example) until we have knowledge of what those terms mean.
Once we learn the meanings of the terms, then we can see that 'whales are mammals' is true.
Similarly, according to Locke, we learn that 3+4=7 when we learn the meanings of 3, 4, 7, +, and =.
Locke says that empiricism (non-innateness) accounts for the temporal difference in learning 3+4=7 and 18+19=37.

One response for the rationalist, the one on which Kara focuses, is to use the temporal difference to his advantage.
So, Leibniz argues that some ideas are learned too quickly to be acquired.
This argument evokes Chomsky's poverty-of-the-stimulus argument.
Chomsky argues that universal grammar (UG) is innate, since children learn too much grammar too quickly to be explained by exposure to (experience with) language.
(See the website for some good Chomsky links!)

Another, more forceful, approach is to argue that the temporal order is irrelevant.
There is an empirical element in the learning of terms, and of associating terms with ideas.
But, justification is independent of the temporal order.
(Leibniz puts the problem in terms of intellectual ideas, which would be the innate ones, p 81.)
Kara mentions, on p3, Leibniz's response to Locke, in *New Essays*, p 78.
Leibniz agrees that some of the innate ideas are more difficult to apprehend than others.
But, if the point about the genetic fallacy holds, then the temporal issue is just a meaningless distraction.
The temporal, or natural, order of our learning is independent of the order of justification.