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# Class 4 (1/30)

## I. Aristotle's criticisms of Plato, and the forms

We had spoken on Monday about Plato's work, but did not emphasize the role of forms as explanations of causes, as in the *Phaedo*.
Aristotle denies that the forms are useful explications of causes.
Plato's theory of forms uses the forms as explanations of universals, of commonalities.
See Aristotle 1079a1-4.
If two things are both tall, there is some property that they share, tallness.
If two things are beautiful, there is some property that they share.
Plato reifies these properties, takes them to be objects.

There is some confusion about whether all commonalities are explicable by forms.
On the broad view, the view just mentioned, all commomalities lead to forms.
For any many, there is a one.
There is also a narrow view, on which only some commonalities have corresponding forms.
The forms, being perfect and unchanging, would not seem to admit muddiness.
Aristotle indicates doubts about whether there are forms of negations, and of perishable things.
I take Aristotle's 'argument for the sciences' (1079a8-9) to indicate the narrow view.
I think it is typical to say that early Plato accepted the broad view, but later Plato moved to a narrower view.

Aristotle rejects both views about the forms; see the handout from Book 1, Chapter 9 of the *Metaphysics*.
"To say that they are patterns and the other things share them is to use empty words and poetical metaphors" (*Metaphysics*, Book 1, Chapter 9, 991a21).

Aristotle presents several arguments in the starred paragraph:
1. Empty words
2. Several patterns
3. Pattern and copy

The several patterns argument seems just a bit of puzzle for the platonist.
It is not clear why Aristotle thinks it is a problem for one object to participate in many forms.
We will not pursue this criticism.

The 'empty words' argument is a question about whether one needs to reify commonalities, about whether there needs to be a one over any many.
As an alternative, Aristotle divides the world into substances and their properties.
People, animals, trees, and rocks are all substances.
We may call them natural kinds.
Tallness and beauty are not substances, but what is said of substances.
The substance and its attributes must be located together.
In essence, Aristotle is giving us the subject-predicate distinction, which remains in our grammar.
Given the subject/predicate distinction, we can understand the double-starred paragraph: we can not separate the forms from the substances in which they inhere.

Aristotle is thus presenting an adjectival use of numbers.
Roundness (circularity) and twoness (from counting) are properties of primary substances, not substances themselves.
Lear points out that number does not seem to be a property of an object.
One deck is 52 cards is 13 ranks is 4 suits; see Lear, p 183.
Geometric properties fare less poorly.

The last argument in the single-starred selection refers to the third man.
Aristotle also refers to the third man argument in our reading (1079a11-14).
Since forms are invoked to account for similarities, we must have some explanation of why the blue sky is similar to the form of blueness.
There are two options.
We can say that the form of blueness participates in itself, in which case the same object is both pattern and copy, as in the starred selection.
Or, we can posit a higher-level form (a third man) to explain the commonality between the sky and the form of blueness.
There are turtles all the way down.
That is the point of Aristotle's comments, at 1078b33-1079a4, that in explaining the sensible world, platonists posit many more objects (forms) than the sensible world itself.
Even if the platonist accepts all of the higher- and higher-level forms, the infinite number of forms lack an explanation of their similarity.
So, the theory of forms fails to explain what it set out to explain.
That is, either horn of the dilemma reduces to absurdity.

## II. The argument from division

We spent a bit of time in class talking about the argument from division (1076a38-1076b12), which interested Nelson.
The argument seems to show that the mathematical forms do not exist in sensible things.
And that seems to show that they can not exist at all, since Aristotle denies that they exist apart.
I agree that Aristotle provides a compelling argument that we can not divide a point, line, or point.
Perhaps it is just a semantic quibble.
But, wouldn't the argument contradict what Aristotle says at the end of Chapter 1, that we are to discuss not whether they exist, but how they exist?

Jonathan made a useful suggestion, that the argument from division shows that the mathematical forms can not be identical with any extension in matter, that we have to find a more subtle way of describing how the forms exist in matter.

## III. Aristotle, mathematics, and magnitudes

Let us start with Aristotle's metaphysics, with what mathematical objects are, for Aristotle.
Among anti-platonists, there are revolutionaries, who think mathematical statements are false and mathematical objects do not exist, and there are reinterpreters, who think that mathematical statements are true when reconstrued, and while platonist mathematical objects don't exist, we can understand mathematical terms as shorthand for other kinds of objects.

(You might want to compare my uses of these terms with the uses that Burgess and Rosen make of 'revolutionary' and 'hermeneutic' nominalism in *Subject with no Object*.)
Aristotle is a reinterpreter, not a revolutionary.
This is what, I believe, Walter meant by, "Aristotle believes (or assumes to be true) mathematical objects exist" at the beginning of his paper.

Since most of Aristotle's positive account of mathematics comes in Chapter 3 of Book M, we should take a moment with the first sentence of that chapter.
Here is Julia Annas's translation

> Just as general propositions in mathematics are not about separate objects over and above magnitudes and numbers, but are about these, only not *as* having magnitude or being divisible, clearly it is also possible for there to be statements and proofs about perceptible magnitudes, but not *as* perceptible but as being of a certain kind (1077b18-22).

Two questions seem to jump out.
First, what is a general proposition in mathematics?
Second, what is a magnitude?
Lear puts the opening line thus:

> The generalized theory of proportion need not commit us to the existence of any special objects - magnitudes - over and above numbers and spatial magnitudes (167).

So, magnitudes are actually not geometric lengths, as it might seem.
(And, as it understandably seemed to Walter, in his second paragraph.)
They are more general, the subject of Eudoxus's theory of proportions:

$$(x)(y)(z)(w)\ \{(x : y :: z : w) \equiv (v)(u)[(vx > uy \supset vz > uw) \bullet (vx < uy \supset vz < uw) \bullet (vx = uy \supset vz = uw)]\}$$

Since :: is an equivalence relation, we often replace it with '='.
From which it follows (I believe) that:

$$a:b::c:d \equiv ad=bc$$

Euclid presents the theory of proportions in Book V of the *Elements*.
It is generally thought to be one of Euclid's central achievements.
But, even prior to Euclid, we have an axiomatic treatment in philosophy.
Eudoxus developed the theory of proportions to handle the incommensurables we discussed when we talked about Pythagoreans.
The ratio, for example, of the length of a side of an isosceles right triangle to its diagonal, is incommensurable.
The Greeks could not imagine that such a division creates a number.
But, given Eudoxus's work, they could work with incommensurable proportions without committing themselves to irrational numbers.
The quantifiers range over magnitudes, which could be lengths, weights, volumes, areas, and times.
They do not range over numbers, for the Greeks, though we can see that they do.

Aristotle claims, in the first sentence of Chapter 3, that we do not think there are magnitudes in addition to lengths, weights, times, etc.
We should not reify magnitudes.
Aristotle's discussion of health (1077b34-1078a2) has the same point.
We do not reify health, in addition to the healthy or unhealthy person.
Again, we are looking at an adjectival view of properties, rather than a substantival view.

Similarly, consider a book.
We need not think that there is a shape of the book over and above the book itself.
There is just the book, considered more abstractly.

We know that Aristotle did not take mathematical objects to inhabit a separable realm, as Plato did.
We also know that Aristotle did not take the objects to be identical with substances, because of the argument from division.
To help us understand Aristotle's account of mathematics, it might be useful to consider his account of the soul.

## IV. Matter, form, and the soul

For Aristotle, every living thing, indeed every thing that we can name, has matter and form.
The matter is, roughly, the stuff out of which it is made.
The form is, roughly, the shape or function of the object.

Consider, the difference between a lump of clay, and a similar lump made into a statue.
The two lumps are made of the same kind of stuff, but have a different shape.
Plato would say that the lump that looks like a statue participates in the abstract form of the statue.
Aristotle calls the shape of the sensible statue itself its form.
Matter itself is mere potentiality; it is nothing in itself unless it has some form.
The form is what makes it what it is.
We call an object by a particular name according to its form.
Forms, for Aristotle, are thus just one aspect of a substance.
In the case of the statue, the form is related to its shape, though form is not merely shape.

Consider an eye.
It has matter, which it can share with a dead eye.
It has some properties in common with an eye of a statue, like its shape.
But, the real eye is able to see.
The function of seeing is what makes an eye a real eye.
So, the form of the eye is related to its function.
Similarly with my hand, which has particular functions.
All the parts of me: my heart, my lungs, my toes, have functions, and so both matter and form.

When we put all of these pieces together, we get a person.
We are all made out of the same kind of matter.
But, we have different properties.
The properties which make me what I am are my form.
Aristotle calls the form of a person his or her soul.

Since the form of something is what makes it what it is, the soul includes our biological aspects, like sensation and locomotion, as well as reason.
The soul is thus not separable from the body, though it is different from just the matter of the body.
Aristotle is thus a monist: there is only one realm.

Aristotle's account of the soul as the form of the human body makes the soul of a person seem a lot like the soul of an animal or plant.
For, plants and animals also have a matter and a form.
Each of these, thus, has a soul.
Plants have nutritive souls.
Animals also have sensitive souls.
While Plato identified several parts of the human soul, Aristotle mentions six faculties, though these are not to be taken as parts: nutrition and reproduction, sensation, desire (which cuts across all three parts of Plato's soul), locomotion, imagination (which we share with some animals), and reason.
Only humans have rational souls.
Thus, Aristotle defines human beings according the functions of their souls: rational animal.

## V. Mathematics, abstraction, and the qua operator

Aristotle is a monist regarding both the soul and mathematics.
In each case, he denies that there is a separate realm.
He is, essentially, a natural scientist about both questions.
The soul is an aspect of a person, apart from his or her matter, but tied to his or her functions.
Mathematical objects are just some aspects of physical objects.

Given Aristotle's no-separate-realm approach, Walter, in his third paragraph, wondered about the extraneous properties of objects
Aristotle deals with this worry by presenting an abstractionist philosophy of mathematics.
(Note that Nelson hinted at this abstraction repeatedly in his paper.)
The word 'abstract' may be used in at least two ways.
In one way, we refer to objects outside of space-time, or outside the sensible realm, as abstract objects.
That is a metaphysical interpretation of 'abstract'.
For Aristotle, abstraction is an epistemological notion.
Abstraction is a process we use on ordinary objects, to consider them as mathematical.
(Aristotle uses the term 'abstraction' in *Posterior Analytics* 74b1.)

To explain the process of abstraction, Lear introduces a qua operator.
My explanation of it differs from Lear's presentation; it is a bit simpler, avoiding metalogical notation.
(I may have oversimplified, but I do not see the problem; I suspect Lear made the qua operator too complicated.)

Aristotle's qua operator may thus be formalized in contemporary logic as:

$$G(b \text{ qua } F) \equiv F(b) \bullet (x)(Fx \supset Gx)$$

So,

Haman's hat has angles that sum to 180 degrees if and only if Haman's hat is a triangle and all triangles have angles which sum to 180 degrees.

The qua operator formalizes the process of abstraction that underlies Aristotle's positive account.
Lear says it acts as a filter to get rid of incidental, or accidental, properties, the properties about which Walter wondered.
Note that G(b qua hat) does not hold.
For, it is not the case that all hats have angles which sum to 180 degrees.

Aristotle's approach easily solves the problem of applicability.
Remember, Jonathan criticized Plato's account of mathematics because we did not know how objects in a separate realm could explain or affect anything in our realm.
For Aristotle, we can study a perceptible triangle qua triangle because it actually is a triangle.
It does not merely approach triangularity.

We are examining Aristotle's work because it sets a precedent for future empiricists in the philosophy of mathematics, especially Mill, in the 19th century, and Maddy, who was a contemporary Aristotelian.
(She has abandoned the Aristotelian project, but it is worth examining, anyway.)

Before we get to a couple of criticisms of Aristotle's particular approach, I want to mention one more topic we discussed in class.

## VI. Hypothetical objects

Walter called Aristotle's objects hypothetical, at the top of p 2.
This comment is related to Aristotle's claim at 1078a17 that platonists do not speak falsely.
Further, it is related to the questions which Walter asks at the end of p2: How can Aristotle say that we can claim without qualification that separable and non-separable things exist? And, why is it true without qualification?
See 1077b31.

Aristotle's point here, I think, is related to the earlier point about him not being a revolutionary.
Platonists do not get mathematics wrong.
The platonist's fiction is harmless.
It does not lead to errors in physics.
See Lear, p 175.
It is also compatible with what we know about beauty and goodness, to consider the last question in the paper.
That is, beauty and goodness are not separable from objects just as mathematics is not separable from objects.
Also note that Aristotle did not separate mathematics from optics or harmonics; see 1078a13-16.
The mathematical descriptions of substances are not separable from the aesthetic descriptions.
They all regard the same matter.

VII. Problems with Aristotle's positive account

1. Restriction

Even if we grant there are some physical circles, are there perfect examples of all the forms that the mathematician explores?
Moreover, there is only a finite world, and the mathematician studies infinitely many objects.
See Shapiro, p 66 fn6.
Aristotle limits the world to potential infinity
Part of his worry arises from paradoxes, specifically including Zeno's paradoxes.
Consider Achilles, having to complete an infinite series of motions before he catches the Tortoise.
See David Bostock's discussion, in "Aristotle, Zeno, and the Potential Infinite".

Also, consider Cantor's diagonal argument, which shows that there are far more numbers than physical objects.
Note that this means, for Plato, that there will be more sets of forms than forms themselves.

2. Separation/Transcendance
If matter were to disappear, there would be no more mathematics.
Mathematical statements seem to transcend the physical world.
The platonist account leads to difficulties about access.
But, it nicely deals with our intuitions about transcendence.

3. Precision
Physical objects do not actually have the mathematical properties they seem to approach.
Certainly, given modern-day science, we have a difficult time salvaging anything directly from Aristotle's approach.
But, Maddy tries to use it for sets, and we will look at her attempt later in the term.