Hamilton College
Russell Marcus
rmarcus1@hamilton.edu

Class 27: Fictionalism and Plenitudinous Platonism

I. Some last words about structuralism

I don't want to revisit the ante rem/in re or places-as-offices/places-as-officeholders problems.
Here is what I take to be the claim of the structuralist that is relevant to our central questions of knowledge of mathematics and its main challenge.
The Benacerrafian problem from "Mathematical Truth" was that we seem to lack access to the objects of mathematics.
For Benacerraf the problem was causal access.
For Field, and we will pursue this line again today, the problem is an explanation of the reliability of claims that if mathematicians believe that p then p.
That is the point of the Nepalese village argument.

Can the structuralist solve the access problem?
Resnik and Shapiro seem to think that if mathematical claims are not about objects, then we might have an easier time explaining our knowledge of mathematics.
"My view is that, extensionally speaking, there is no difference, or at any rate no philosophically illuminating difference [between mathematical structures and other kinds of structures]" (Shapiro, "Mathematics and Reality," *Philosophy of Science*, December 1983, p 542).
Resnik says that our ancestors made a leap from sense experience of concrete patterns to the existence of abstract patterns.
It seems to me that these explanations of our access to mathematical objects repeat the same unhelpful story we have seen before, with Locke's doctrine of abstraction, for example.
We experience concrete objects, and somehow have knowledge of abstract ones.
The question is how we make that leap.
Whether we leap from concrete objects to abstract objects or from concrete systems to abstract structures, there is still a leap.
A system is an object, a structure is an object.
It seems to me that the structuralist provides nothing to help with the access problem.

II. Field's fictionalism

Field's claim: sentences like 5+7=12, taken at face value, claim that there are mathematical objects, and so are false, since there are none.

Field's argument:

F1. We should take mathematical sentences at face value.
F2. If we take (some of them) to be non-vacuously true, then we have to explain our access to them.
F3. The only good account of access is the indispensability argument.
F4. But, the indispensability argument fails.
FC: So, we should take the non-vacuous ones to be false.

Support for F1:
See Q4, below, and the Santa Claus analogy.

Support for F2:
Distinguish between vacuous and non-vacuous truth.
'If the square root of 2 is irrational, then the square root of 5 is irrational' is vacuously true.
The conditional is true, but only because the antecedent is false.
'The square root of 2 is irrational' is non-vacuously true (or false).
The platonist says that it is true because there is a 2, and it is irrational.
The fictionalist says that it is false because there is no 2.
(Jonathan's comment about Field's connection to Russell's definite descriptions is appropriate here.
I wasn't so quick on the uptake during class, I think.
I take Jonathan's point to be that 2 looks like a name, and there looks to be a problem of failure of presupposition.
That is, the fictionalist will want to say that 'the square root of 2 is rational' is also false.
If we re-cast the sentence, a la Russell, so that we have a definite description in the place of the name, we can call both of them false because of the non-existence of 2.
We are still taking the sentence at face value, even if we re-cast the semantics.)
Balaguer responds to Field's argument here, and at P3.

Support for F3:
Field thinks that Gödel-style platonism is desperate, we saw.

Support for F4:
Field shows how to nominalize NGT, sort of.
He emphasizes the conservativeness of analysis (presuming it is consistent) over his nominalist version of physics, as we discussed in our previous class.
M is conservative over (nominalist) P iff P + M yields nothing nominalist that P can't yield by itself.
Balaguer has some steps to nominalizing quantum mechanics.
I haven't seen any one working on general relativity, with its curved space-time, which seems to be a problem:

> "But the most obvious obstacle to developing an elegant, synthetic, pure, natural-looking, invariant, straightforward version of such a theory at present is the circumstance that so far no one has developed even an inelegant, analytic, coordinate, artificial-looking arbitrary-choice-dependent, devious version of such a theory." (Burgess and Rosen (1997) p 118)

Here's the quote from Burgess and Rosen I mentioned in class:

> "As a consequence of nominalism's being mainly a philosopher's concern, this open research problem is...one that has so far been investigated only by amateurs - philosophers and logicians - not professionals - geometers and physicists; and the failure of amateurs to surmount the obstacles is no strong grounds for pessimism about what could be achieved by professionals." (Burgess and Rosen (1997) p 118)

Field's Conclusion: fictionalism
See Q1, below.
Reconstruals or mathematical number terms, as referring to, say, possible mental objects, make one a

fictionalist about numbers and a realist about mental objects and modality.
Speaking of modality, Field thinks, as Balaguer does, that we need an object-level modal operator.
In order to establish conservativeness, we need consistency.
To get consistency, you normally use metalogic, which is done using set theory, and mathematical induction and the like.
We use set theory to show that a group of sentences, say, is consistent.
So, Field suggests that we take our knowledge of consistency not as metalogical, but as plain logical, as a primitive term, like conjunction.
Balaguer is nice about this, p 72.

So, Field's claim is that '5+7=12' should be understood as a falsehood, like the claim that Sherlock Holmes lived at 221B Baker Street, London.

III. Comments on the Field Questions

1. What is mathematical realism? What is fictionalism? What is the role of face-value readings of mathematical sentences, for each?

Comments:

Benacerraf argued for a face-value construal on the basis of a desire for uniform semantics. Field does not go that route.

The fictionalist can allow for non-face-value construals of mathematical terms. So, '2' could refer to some possible physical objects, or to mental constructs. Still, if one is reconstruing these terms, then one is denying that the mathematical objects they name exist, so one is still a fictionalist.

2. According to Field, what makes mathematics a good story? How does this account undermine the Oliver Twist analogy?

Comments:

The goodness of mathematics, for Field, consists in its applicability to empirical science. *Oliver Twist* does not apply to the real world, or help us solve problems in the real world.

3. "[Fictionalism] commits one to abjuring all appeal to mathematical entities in explanations when the chips are down: it must be possible...to develop theoretical physics without any appeal to mathematical entities" (6). Explain.

Comments:

Field denies the indispensability argument, by re-writing gravitational theory. But, general relativity, with its curved space-time, and quantum mechanics, with its probability spaces, resist the same kind of treatment. Still, "As a consequence of nominalism's being mainly a philosopher's concern, this open research problem is...one that has so far been investigated only by amateurs - philosophers and logicians - not professionals - geometers and physicists; and the failure of amateurs to surmount the obstacles is no strong grounds for pessimism about what could be achieved by professionals." (Burgess and Rosen (1997) p 118)

4. How does Field defend a face-value reading of mathematics? How does this defense relate to the Santa Claus analogy, and the defense of mathematics as a body of conceptual truths?

Comments:

The Santa Claus analogy says that nothing would count as Santa Claus unless it had the well-known properties of Santa Claus. Similarly, nothing should count as 2 unless it has traditional (face-value) properties of 2. Still, the fact that nothing counts as 2 unless it has those properties can not yield the existence of the object, as Kant showed.

Return to Shapiro, 76, and Jonathan, p 2.

5. What does Field think mathematical proofs show? What do they not show?

Comments:

Proofs are derivations. They don't tell us the status of the axioms from which we derive a theorem.

6. Does the initial plausibility of mathematical claims make them as certain as ordinary empirical claims? Explain.

Comments:

Jonathan's q on p 4.
p 11.
Jonathan, pp 3-4

7. What is the principle of intrinsic explanation? How does it support fictionalism over indispensability platonism?

Comments:

I have a paper on this topic. See:
http://www.thatmarcusfamily.org/philosophy/Papers/intrinsic_explanation.pdf

IV. Comments on the Balaguer questions

1. What is FBP? How does FBP interpret mathematical knowledge as logical knowledge?

Comments:

FBP says that all logically possible mathematical objects exist. Another formulation: every consistent mathematical theory truly describes an independently existing realm of abstract objects. See Katherine, p 1.

Truth (simpliciter) is supposed to apply only to statements that are true in every model. Is choice true? Well, there is a universe in which it is and another universe in which it is not. So, it is neither true nor false.

Consider the parallel postulate, p 64. Balaguer is saying that we could develop intuitions about set theoretic universes to parallel (sic) our intuitions about geometric spaces.

There is also truth as the intended model. There are non-standard models. Mathematicians are mainly concerned with the standard, or intended model. They are not concerned with problems of modeling. Balaguer proposes to explain the interest in the standard model sociologically, pp 64-5.

Discuss KK4, and how Balaguer is trying to explain the interest in the standard model, not the justification for mathematical truth.

2. What is Field's Nepalese villages argument? How does Balaguer respond? Consider thin and thick beliefs and the Santa Claus analogy.

Comments:

Balaguer's central argument is on p 49.

One quick objection seems to be whether I can imagine any possible mathematical objects. Perhaps the argument is a non-starter. That is, I can imagine all possible Nepalese villages, but can I imagine any mathematical objects? Certainly not, in the strict sense of imagine.

Balaguer says that we need not have thick beliefs about mathematical objects, merely thin ones. That is, we can have thin beliefs about non-existent objects, like Santa Claus, so it would seem that having thin beliefs about mathematical objects is not such a challenge.

3. How does FBP respond to the Benacerraf challenge to platonism? How does it accept the central premise of that challenge? What does Balaguer think of the Gödel view?

Comments:

Balaguer agrees that we need an account of access, and that intuition won't cut it.

4. Are ZFC and ZF+~C contradictory theories? How does FBP embrace them both? How does FBP embrace different sizes of the continuum?

Comments:

They just describe different realms. Katherine is good on this topic, p 1.

5. In what sense is FBP equivalent to EWA? Are the hypotheses of the existence of an external world and the existence of any mathematical objects posited by a consistent theory equally plausible?

Comments:

pp 56, 69

6. How is FBP presumed by mathematical practice? How does FBP support mathematical freedom?

Comments:

Mathematicians pursue the claims that they see are interesting, but they seem mostly interested in any theories that are consistent. Some consistent theories are too dull to interest mathematicians, but as long as a theory is consistent, mathematicians may explore it.

7. What is the difference between internalist and externalist accounts of the reliability of one's beliefs? Do we need an internalist account of mathematical knowledge?

Comments:

See Nelson's paper, and compare to the KK thesis. The KK thesis says that to know something we must know that we know it. Internalist justifications require that we know that we know, whereas externalist justifications require just that we know.

Does FBP put platonism on par with the theory that claims that there is an external world?

I don't really get the objection: 'If FBP, then T truly describes some part of the mathematical realm' seems like a reasonable response to Benacerraf. Maybe Balaguer was just trying to set that up. (This part seems poorly written by Balaguer.)

8. What are standard models? How does FBP have a problem with standard models? How does Balaguer explain our interest in them?

Comments:

This is the central problem with FBP. There are too many consistent theories, and we need some explanation for our interest in certain ones. Balaguer can explain our interest in the standard model, for limited mathematical theories, but their applicability to empirical science. But, once mathematical theories become purely theoretical, or un-applied, this justification is no longer useful. Alternatively, Balaguer can try to explain our interest in the standard model on the basis of sociological considerations. Perhaps our interest is just the result of community standards. Still, one has to ask about the origins of those community standards.