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Class 26: Mostly structuralism, though I wanted to start on fictionalism

This class was a bit confusing, on structuralism.

Benacerraf argues that mathematics is really concerned with structures, and not objects, p 291.
See also Shapiro, pp 73-4, or James, p 1.
Benacerraf is making an ontological claim.
The structuralist also may hope that structuralism can solve the epistemic problem from "Mathematical Truth."
We want an account of our access to mathematical objects.
Perhaps we can account for access to structures more easily than access to particular numbers.
Shapiro, for example, denies that there are any important differences between abstract and concrete structures.

There are really two steps to the structuralist's project.
First, s/he denies that there are mathematical objects, in the traditional sense.
But, structures are just another, more complex kind of object.
And, there are positions in structures, which look a lot like the traditional mathematical objects.
For example, we might call the third position in the sequence which satisfies the Peano Axioms 2.
Even if we think that 2 does not exist independently of the sequence, that sequence has a second position.
Consider the baseball team, and take baseball to be the science of the structure of a baseball team.
Any team has to have players.
So, we need both offices and officeholders.
If places are objects themselves, then the structuralist seems to fail to generate any solution to the ontological question, doubly.
The structure itself is an object, and so we need an account of our knowledge of the object.
And, the positions in structures are objects, and so we need an account of which objects they are.
If places are offices, we still need a background ontology, things to fill those offices.
So, there have to be baseball players in order to field a baseball team, etc.
See James, p 4, and background ontology.

Second, the structuralist argues that our experiences with systems somehow accounts for our access to structures.
See James, p 2, on abstraction and direct description.
But, this sounds just like the original doctrine of abstraction, from Locke for example.
Shapiro defends ante rem structuralism, but I'm not sure that it helps.
An ante rem structuralist says that there are no structures without the systems.

Lastly, Shapiro argues, with the mathematicians, that Field's fictionalism does not really eliminate commitments to mathematics, pp 76-7.
So, let us turn to Field's project.

Field defends the conservativeness of mathematics. Let A be any nominalistically statable assertion, N any body of such assertions, and S any mathematical theory. Take 'Mx' to mean that x is a mathematical object. Let A* and N* be restatements of A and N with a restriction of the quantifiers to non-mathematical objects.[1] This restriction yields an agnostic version of the nominalist theory; it does not rule out the existence of mathematical objects. S is conservative over N* if A* is not a consequence of N*+S+'$\exists x \sim Mx$' unless A is a consequence of N.[2]

The general idea of conservativeness is that mathematics is conservative over a purely nominalist physical theory if adding it to the nominalist theory allows us to derive no new nominalistically acceptable results. We don't get any more about the world with the math than we did without it.

If mathematics is conservative over nominalist versions of physics, and those nominalist versions of physics are adequate (i.e. yield the same results as the standard versions) then we can go ahead and use the standard, mathematized versions without a guilty conscience.

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[1] Restrict the quantifiers by inserting into every universal quantification of N, just after the quantifiers, '$\sim Mx_i \supset$', for all x_i in the formula; and into every existential quantification of N, just after the quantifiers, '$\sim Mx_i \bullet$'. This ensures that all statements will be neutral about the existence of mathematical objects.

[2] '$\exists x \sim Mx$', that there is at least one non-mathematical object, is a technical convenience. See Field (1980), pp 10-16.