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Class 25: Shapiro's Structuralism

(This set of notes just consists of comments in response to the questions I passed around.)

I. Benacerraf's "What Numbers Could Not Be"

(See Class 24 for some more on Benacerraf.)

2. What is the difference between transitive and intransitive counting? How does Benacerraf argue that one precedes the other? Why does he call this result, "mildly significant" (275)?

Comments:

Mill and other nominalists think that the transitive use is primary. That is, they take numbers as adjectives. Many of you have been uncomfortable taking numbers as objects, too.

The numbers as adjectives attitude is supported by logical representations of numbers, such as Benacerraf's "There are seventeen lions at the zoo," which Sarah cited in her paper. The logical reformulations of numbers makes them into a kind of quantifier. Quantifiers, like predicates, are modifiers.

Sarah had pointed out (p 2, p 3) that there is some relation between Benacerraf's discussion of Frege and his rejection of number words as predicates and his structuralism. (Frege: numbers are names of sets of sets, used as predicates.)

So, Benacerraf's argument that intransitive counting is prior puts numbers back into object places, rather than as modifiers of any sort.

3. Does 3 belong to 17? Explain the incompatible theorems on p 278.

Comments:

Sarah does a good job with this question, and the conflicting theorems, on p 1. Why would we think that numbers are sets in the first place? See Shapiro 78. We'll get back to Shapiro's criticism of Benacerraf in a moment.

4. Why is Option A (p 280) absurd? What are the three different versions of option B? Consider, "If the numbers constitute one particular set of sets, and not another, then there must be arguments to indicate which" (281).

Comments:

There are three options: 1. Ernie is right; 2. Johnny is right; 3. Neither Ernie nor Johnny is right. Benacerraf argues that since there are no arguments for 1 or 2, and since we can see the possibility of no such arguments, we must choose 3.

5. What is the relation between logicist reduction and Ernie's education? How does Benacerraf's denial that certain identity sentences are meaningful deny a central, logicist premise?

Comments:

Ernie is doing logicism backwards. The logicist reduced all of number theory to set theory. Ernie starts with set theory and develops number theory. So, the logicist presents a thesis that numbers are sets. And Ernie derives the numbers and their properties from his set theory.

Quine, among many others, insists that we can have no entity without identity; see Benacerraf 286. If we are wondering about whether to accept some object into our ontology, we have to be able to identify that object, to tell of any given thing whether it is that object.

Consider: Is this table the same thing as my thought that it is a beautiful day? Benacerraf says that we can not answer that question without presuming some sortal concept. Are they the same what? That is, we need a sort of object. In order to answer whether a number is the same as a certain set, we have to be able to say what category they belong to. Benacerraf's claim is that there is no category into which they fall.

Part of Benacerraf's claim arises from the obscurity of any possible answer. Here, Benacerraf sounds like a positivist, p 284-5. See also Shapiro 79. But, Benacerraf is also providing an explanation for the obscurity of the question.

6. How, according to Benacerraf, does the notion of an object vary from theory to theory? How does this variation allow identity to remain absolute?

Comments:

See pp 287-8. Relate this claim to Benacerraf's claim that there aren't any numbers, as objects. There are only structures, p 291. One problem: a structure is just another kind of object. Sarah thinks, p 4, that there may be a better way to choose 1 or 2, to deny that there are no arguments for 1 or 2.

Another option: maybe we can just take numbers as sui generis. The problem with a sui generis solution is that we don't have an explanation of the unity of mathematics. Reduction to sets is an explanation of the unity of mathematics, just as reduction to physics might explain the unity of science.

Further, there is a hope that structuralism might solve the problem from MT. We want an account of our access to mathematical objects. If mathematics is the science of structures, perhaps we can have access to the structures more easily than access to particular numbers. Shapiro denies that there are any important differences between abstract and concrete structures, for example. See James, p 2, on abstraction and direct description.

7.What is the difference between reduction and explication? How does it help Benacerraf?

Comments:

We can use the Zermelo or von Neumann identities to explicate the relations between numbers and sets without taking them as reductions. For explication, we would not identify numbers and sets, but we would show how they are related structures. For reduction, we are saying that the essence of the numbers is revealed by the sets.

8. How is arithmetic the science of structures? Explain the ruler analogy (p 292). Does Benacerraf deny that particular number terms lack meaning? Consider Benacerraf's final sentence.

Comments:

Benacerraf wants both to hold onto standard semantics, as he argues in MT, and deny that numbers are objects. But, standard semantics seems to indicate that numbers are some sorts of objects. So, he seems in some sort of bind. The solution to that tension is supposed to come from thinking of mathematics as the science of structure. So, perhaps turning to Shapiro would be helpful.

II. Shapiro's Structuralism

1. What is a system? What is a structure? What is the structuralist theory? What are numbers, on this theory?

Comments:

Shapiro is cashing out the Benacerraf project. pp 73-4. Or, James p 1. Mathematics is the science of structure. Numbers are structure-less position in the structure. See also James on misleading talk of numbers as objects.

2. How is Shapiro's structuralist a realist? How is he not a platonist? Consider the office/officeholder analogy on p 82.

Comments:

But, Shapiro does think that there are positions in structures. Those positions look like objects. Consider the baseball team. Baseball is the science, say, of the structure of a baseball team. But, any team has to have players. So, we need both offices, and officeholders.

3. Distinguish the places-are-offices perspective from the places-are-objects perspective. How does that distinction relate to the distinction between identity and predication? How does the places-as-offices perspective require a background ontology?

Comments:

If places are objects themselves, then the structuralist seems to fail to generate any solution to the ontological question, doubly. First of all, the structure itself is an object, and so we need an account of our knowledge of the object. Secondly, the positions in structures are objects, and so we need an account of which objects they are.

If places are offices, we need a background ontology, things to fill those offices. So, there have to be baseball players in order to field a baseball team, etc.

See James, p 4, and background ontology.

4. Distinguish in re realism from ante rem realism. Is Shapiro and in re realist or an ante rem realist? What are ontological eliminative structuralism and modal eliminative structuralism?

Comments:

See James, p 3 In re realism is Aristotle's position: universals depend on their particulars and do not exist without them. Ante rem realism is Plato: universals are independent of their particulars.

So, an ante rem structuralist says that there are no structures without the systems. An in re structuralist says that the structures are independent.

5. How does Shapiro argue that the natural number structure precedes the numbers themselves? Consider the question of the independence of numbers from one another.

Comments:

See James, p 2, on discussing the bishop independently of chess. Can I talk about 5 without talking about the rest of the number structure? Can I say, for example, that it is my son's favorite number. Or, that he likes it.

6. How does Benacerraf's problem in "What Numbers Could Not Be" relate to Frege's Caesar problem? How does the structuralist solve the problem?

Comments:

Frege's Caesar problem was that merely providing set-theoretic definitions of the numbers does not settle questions of the identity of the numbers. That is, we need firmer identity conditions. The structuralist is supposed to solve the problem by denying that there are numbers, by allowing that all models of the natural number structure are equivalent. Shapiro is denying that there is a Caesar problem.

7. Is the real 0 the same as the natural number 0? Is there nothing to be said for reduction? See p 81. Is the identity between the two 0s merely the result of stipulation?

Comments:

We did not discuss this interesting question. since the structures of the reals and that of the natural numbers diverge, it seems that the structuralist has to stipulate their identity. This problem for the structuralist is not in theory insuperable, but it is a problem.

8. How does Shapiro argue, with the mathematicians, that Field does not really eliminate mathematics (pp 75-7)? How does Shapiro see talk of numbers as a convenient shorthand (p 85)?

Comments:

Field argues that quantification over space-time points eliminates commitments to mathematical objects, even if the same theorems hold of space-time as hold of the real numbers. Shapiro thinks that if the same theorems hold, the same structures are described, and so no advance has been made.