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Class 24: Benacerraf, "What Numbers Could Not Be"

I. Quine's indispensability argument and nominalism

We looked in our previous class, at Quine's indispensability argument.

(QI) QI.1: We should believe the (single, holistic) theory which best accounts for our sense experience.
QI.2: If we believe a theory, we must believe in its ontic commitments.
QI.3: The ontic commitments of any theory are the objects over which that theory first-order quantifies.
QI.4: The theory which best accounts for our sense experience first-order quantifies over mathematical objects.
QI.C: We should believe that mathematical objects exist.

I wanted to look a bit more closely at QI.3, and Quine's claim that to be is to be the value of a variable.
Note the odd paragraph on p 99 of 'Existence and Quantification'.
Quine thinks that the nominalist has some recourse to using names for mathematical objects, as long as s/he avoids using them in quantified places.
We can always move between names and predicates specially designed to satisfy only the bearer of a particular name.
Given the problems of names we discussed last time, it seems odd to use names at all.
Quine defends the use of Russell's theory of definite descriptions to avoid the problems of names.

II. Names and predicates

Let's step back to talk a bit about being the value of a variable, again.
A paradox is any sentence of the form 'P • ~P'.
Consider, 'The King of America is bald'.
If we regiment 'the king of America' as a name, a constant, then we are led to the following paradox:

P: ~Bk • ~ ~Bk

We assert '~Bk' because the sentence 'the king of America is bald' seems false.
We assert '~ ~Bk' because '~Bk' seems to entail that the king of America has hair, and that claim must be false, too.
Russell argued that we should regiment the sentence as a definite description, so the paradox disappears.
'The king of America is bald' becomes: (∃x)[Kx • (y)(Ky ⊃ y=x) • Bx]
'The king of America is not bald' becomes: (∃x)[Kx • (y)(Ky ⊃ y=x) • ~Bx]
Conjoining their negations, as we did in P, leads to no paradox.
You can derive the non-existence of a unique king of America, though, which is a desired result.

In order to use Russell's technique on 'Pegasus', we have to turn it into a definite description.
"The singular noun in question can always be expanded into a singular description, trivially or otherwise,

and then analyzed out *á la* Russell" (OWTI, 8).
Quine mentions the equivalence of 'Pegasus' and 'the winged horse captured by Bellerophon', and he introduces the predicate 'pegasizes'.
We can regiment 'Pegasus does not exist' as '~(∃x)Px'.
Quine further thinks that we have solved a problem, that we no longer have any temptation to think that there is a Pegasus in order to claim '~(∃x)Px'.

We regiment our best theory.
It will include, or entail, a sentence like: ~(∃x)Px
That sentence is logically equivalent to: (x)~Px
If we want to know whether this sentence is true, we look inside the domain of quantification.
The domain of quantification is just a set of objects.

III. Substitutional quantification

Heather (p 3) points to Quine's comments about substitutional quantification.
Standard model theory takes objects from the domain of quantification as substitutes for the variables in first-order logic.
Substitutional quantification writes the interpretation of a theory in a meta-language.
The substitutes for variables are now names, instead of objects.
So, an existentially quantified sentence is true, on substitutional quantification, if there is a name, in the metalanguage, such that the substitution of that name for the quantifier variable yields a true (meta-linguistic) sentence.

There are two kinds of problems with names.
The first is the problem of fictional names.
We have now found a way to talk about Pegasus without McX's ideas or Wyman's subsistence.
If there is no object with the property of being Pegasus, we call this sentence true in the interpretation.
We construct our best theory so that everything in the world is in our domain of quantification, and nothing else is.
Using substitutional quantification, we refuse to admit 'Pegasus' as a name in our metalanguage.
(Though, we can have a name for the object-language term, 'Pegasus', so that we can talk about every term of the object language. This gets complicated.)

The second problem of names can be seen by considering the real numbers.
There are more reals than names.
We can, as the class urged, name any particular real number.
Though, I suppose it is tricky with irrationals, since we can never complete their names.
But, we know that any list of real numbers is necessarily incomplete.
So, there is a problem with substitutional quantification, unless our theory is restricted to a denumerable universe.

How does all of the above help with the paragraph on p 99?
Sarah's p 2, refers to Benacerraf's logical replacement for 'There are seventeen lions in the zoo'.
Does that help?

IV. Second-order logic

One more thing about Quine's criterion, re predicates.
Second order logic requires predicate variables.
First-order logic does not.
See the last paragraph in OWTI, on deviant logic.
Second-order logic, which quantifies over properties, looks like set theory.
(Quine calls it, "Set theory in sheep's clothing," in his *Philosophy of Logic*.)

In first-order logic, predicates are not names.
Heather, p 1, seems to conflate first- and second-order logics.
Jazmine has an example about turkeydom, on p 1.

Quine focuses on redness.
The profligate ontologist thinks there are abstract objects in addition to the concrete objects which have their properties.
There is redness in addition to fire engines and apples.
The issues concerning universals lead directly into Quine's discussion of three schools of philosophy of mathematics: logicism, intuitionism, and formalism, in OWTI.

V. Quiz on OWTI:

http://www.jcu.edu/philosophy/gensler/ap/quine-00.htm

VI. Benacerraf's "What Numbers Could Not Be"

1. Explain the difference between postulating Peano's axioms and the operation of addition, and deriving them. How does Ernie have an advantage over the rest of us in this regard?

Comments:
We got embroiled, in earlier classes with the question of the nature of our decisions about the axioms. Ernie doesn't have to ask those questions about arithmetic, though he will have to ask them about set theory.

2. What is the difference between transitive and intransitive counting? How does Benacerraf argue that one precedes the other? Why does he call this result, "mildly significant" (275)?

Comments:

Transitive counting uses numbers as adjectives. Intransitive counting uses them as objects. Benacerraf argues that intransitive counting precedes transitive counting; pp 274-5. This result, if accepted, refutes Mill, and makes numbers objects. We will start, on Monday, with that question.