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Class 22: Benacerraf and Quine

I. Can $2\pi r_1 = 2\pi r_2$ when $r_1 > r_2$

We talked a bit about the so-called Aristotle paradox.
Walter sent these three links, one for the path traced by a point in the interior of the wheel (the curtate cycloid); one for the path traced by a point on the circumference of the wheel (the cycloid); and one for the path traced by a point whose distance from the center of the wheel is greater than the radius of the wheel (the prolate cycloid).
http://mathworld.wolfram.com/CurtateCycloid.html
http://mathworld.wolfram.com/Cycloid.html
http://mathworld.wolfram.com/ProlateCycloid.html

Thinking that this is a paradox may just be an indication that philosophers should not try to do mathematics!

II. Paper comments

Most of you are trying to do too much, trying to develop new theses.
It is wonderful if you can do so.
But, the challenge is to read some material closely, and to develop a new thesis on the basis of scholarly consideration of given texts.
Slow down.
Carefully explain the arguments.

III. Benacerraf

We talked a lot about theories of knowledge and Tarski's theory of truth.
The problem is whether we can match our epistemic capabilities with standard semantics, in particular with the platonist mathematics that the standard semantics seems to generate, via Tarski's theory of truth and standard model theory.
So, consider the sentences 1, 2, and 3.
The truth of an existential sentence depends on whether there are objects in the domain of quantification that can substitute for the variables in the sentence so that the properties ascribed to those objects hold.
That is, as Quine says, "to be is to be the value of a variable."
We'll get back to Quine after we finish with the Benacerraf.
The claim which is relevant to Benacerraf's argument is that the standard semantic theory will require objects to satisfy the sentences.
Benacerraf 670.

The problem for standard semantics is that we lack causal connection to mathematical objects.
We might appeal to the provability of mathematical theorems, but we still don't know why the axioms are true, why truth is conducive to proof.
Benacerraf 673.

The other option is to give up standard semantics, as the combinatorial views do.
Who are the combinatorialists?
Hilbert
Brouwer
Wittgenstein and the conventionalists, though Brown 150.
Modalism: reformulate mathematical claims as claims about possible physical objects.
Devitt argues that we should never allow our semantics to lead our metaphysics.
So, perhaps we should pursue this route.

Field takes, as many do, the Benacerraf argument to be a challenge to the realist.
While there are problems with the CTK, Field reformulates the challenge, pp 25-6.

IV. Benacerraf questions and comments

1. What is a semantic theory? What is a homogeneous semantic theory? How would a homogeneous (or standard) semantic theory treat mathematical sentences?

Comments:

See last class for some comments about semantic theories. A standard semantic theory says that two sentences with the same grammatical structure are to be analyzed in the same, formal way. Their content might differ, but the structure should not depend on the content.

Benacerraf considers two sentences of the form: ‘There are at least three FGs that bear R to a’.

Large cities older than NYC: $(\exists x)(\exists y)(\exists z)(Lx \bullet Ly \bullet Lz \bullet Oxn \bullet Oyn \bullet Ozn)$

Perfect numbers greater than 17: $(\exists x)(\exists y)(\exists z)(Px \bullet Py \bullet Pz \bullet Gxs \bullet Gys \bullet Gzs)$

These logical regimentations are liable to further analysis, since the be a large city is not just to be large, and to be a city; similarly for perfect numbers. Benacerraf’s point is that however we resolve that question, the resolution will be parallel for the two cases. Note the similar logical structure. Benacerraf’s idea is that they should have parallel truth conditions. So, the first sentence is true only if there are three cities, and the second sentence is true only if there are perfect numbers.

2. What does Benacerraf call the standard view of mathematics? How does it admit a standard semantics?

Comments:

The standard view is platonist, since sentences like ‘here is a number between 4 and 6’ have the same grammar as sentences like ‘there is a chair between the desk and the door’. Both of those sentences say that there is an object, and the best way to understand the mathematical objects is platonistically.

3. What is a combinatorial view of mathematics? Does it admit a standard semantics?

Comments:

Benacerraf uses ‘combinatorial’ to refer to Hilbert’s programme, formalism, intuitionism, and conventionalism. All of these philosophies of mathematics are united by their demand to start accounts of mathematics with the manipulation of some kinds of objects accessible to humans, whether inscriptions, or symbols, or mental constructs.

4. Explain Benacerraf's first condition for accounts of mathematics. How does it relate to Tarski's theory of truth? Why should we want a standard semantics?

Comments:

Benacerraf's first condition is to have a standard semantics. See p 666 and p 670.

5. How do combinatorial accounts of mathematics violate Benacerraf's first condition? Consider both Hilbert and the conventionalist. What other views violate Benacerraf's first condition?

Comments:

Hilbert takes mathematical terms to refer to inscriptions. So, we can not take '5' as referring to a five, for example. The conventionalist makes numerical terms refer to, what? Here, Benacerraf is really aiming at Carnap, rather than Wittgenstein.

Note that Brown reads Wittgenstein (150) as a platonist, though not a realist. So, Brown's Wittgenstein admits of the standard semantics. Maybe he solves Benacerraf's conundrum?

Consider Quine's arguments about convention, that for logic to be conventional, we would have to adopt a framework including it. But, the adoption of a framework is itself guided by logical laws. So, the logic has to be presupposed. Similar claims might be made for mathematics.

Benacerraf's point, on 676-7, extending Quine's point, is that a theory of truth demands a theory of reference. We have not only to partition the set of statements of a theory into two classes, we have to know what those two classes are, and why we put some terms in one class and some in the other. So, as long as we are using standard semantics, we need a standard theory of reference for mathematical terms.

Consider for one more case, the modalist who thinks that 2+2=4 is a statement about possible physical objects. Here again, we can see the violation of standard semantics.

6. Explain Benacerraf's second condition. How does the standard view violate the second condition?

Comments:

The second condition is that we should have epistemic access to the objects to which our theory is committed. We have to have some account of how we can know about the things we think exist, which seems to be absent in the case of platonistic entities. Field will cast this problem more generally, in terms of the reliability of mathematical beliefs. (See below.)

7. What is the causal theory of knowledge (CTK)? How is Benacerraf's defense of CTK doubly causal? How is CTK in tension with standard semantics for mathematics? Consider Gödel's realism.

Comments:

Semantic theories include theories of truth, meaning, and reference. CTK says that for me to know a claim, there must be a causal connection between me and the things to which the claim refers. Further, Benacerraf holds that the theory of reference is also causal, that my connection to any object to which a term I know refers must be based in some causal link between me and the object.

Standard semantics posits objects outside the causal realm, p 673.

For Gödel, see p 674.

8. How does Benacerraf argue for CTK? (See 672.)

Benacerraf argues that we need some sort of account of our connection to the objects of our theories. CTK explains knowledge of laws.

V. Field questions and comments

1. How does Field criticize Benacerraf's argument? What argument does he provide against CTK?

Comments:

Field just says that no one believes CTK anymore. I mentioned the problems for CTK arising from our visit to fake barn country in our last class.

2. Describe Field's reconstruction of Benacerraf's argument in terms of reliability of mathematical beliefs.

Comments:

Field points out that whatever epistemic theory we have, whether it is CTK or any other, we have to explain why our beliefs are reliable. That is, we have to explain why it is the case that if mathematicians believe p, then p. See pp 25-6.

3. Why can't the platonist take the reliability of mathematical beliefs as brute? Explain the Nepal problem.

Comments:

If we take the reliability of mathematical beliefs as brute, we might as well take the reliability of mystical insight as brute, or the reliability of our beliefs about what is going on in Nepal right now (if we have no causal or otherwise connection to Nepal) as brute.

4. How does Field think that one could argue that Benacerraf's challenge is insurmountable? How do idealists accept that the Benacerraf challenge is insurmountable?

Comments:

Field is saying that the Benacerraf argument relies on an inference to the best explanation; it is not a knock-down argument against platonism. A knock-down argument would provide some explanation of why it is impossible to explain the reliability of beliefs about platonistic entities. And, non-existence proofs are difficult. The idealist gives up on platonistic entities (in despair?), but posits mental entities, which Field thinks are just as obscure.

5. How is the Gödel platonist desperate?

Comments

Field, p 28, is worried about mathematical intuition being mystical; see Q 3, above.

6. How does the indispensabilist attempt to answer the Field/Benacerraf challenge?

Comments:

The indispensabilist argues that we have reliable beliefs about the physical world, and that reliability transfers to the mathematical objects.