Philosophy 405: Knowledge, Truth and Mathematics
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M, W: 1-2:15pm

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Class 2 (1/23)

I. Sign up for seminar papers.

II. Apriorty, analyticity, necessity: some characterizations

Consider again Brown's mathematical image:

1. Mathematical results are certain.
2. Mathematics is objective.
3. Proofs are essential.
4. Diagrams are psychologically useful, but prove nothing.
5. Diagrams can even be misleading.
6. Mathematics is wedded to classical logic.
7. Mathematics is independent of sense experience.
8. The history of mathematics is cumulative.
9. Computer proofs are merely long and complicated regular proofs.
10. Some mathematical problems are unsolvable in principle.

One element of Brown's characterization of mathematics is that mathematics is supposed to be independent of sense experience, or a priori.
Shapiro quotes Blackburn's *Dictionary of Philosophy* on the characterization of the a priori:

> A proposition is known a priori if the knowledge is not based on any 'experience of the specific course of events of the actual world' (22).

Brown presents the discovery of the square root of two as an illustration of an a priori process.

Proof: The square root of two is irrational.

Lemma 1: Every rational number can be reduced, uniquely, to lowest terms.
Lemma 2: The square of even numbers are even; the squares of odd numbers are odd.

Suppose $\sqrt{2}$ is rational.
Express it in lowest terms: p/q

$$2 = p^2/q^2$$
$$2q^2 = p^2$$

So, p^2 must be even.
And p is even, by Lemma 2.
Which just means that $p=2r$, for some $r<p$.

$$2q^2 = (2r)^2$$
$$2q^2 = 4r^2$$
$$q^2 = 2r^2$$

So, q^2 must be even, and q is even, again by Lemma 2.

But, then p/q is reducible to lower terms, violating our assumption and Lemma 1.
So, $\sqrt{2}$ is not rational.
So, it is irrational.
QED
(Note: this proof contains what may be the first recorded use of a reductio argument.)

I mentioned on Monday that this proof establishes a result a priori, since we could never discover that the square root of two is irrational by measuring.
The density of the rationals means that we can always find one which will fulfill our measurement needs.

So, we have a definition of apriority, from Blackburn via Shapiro, and an example.
Any belief not held a priori will be a posteriori, or empirical.
And, an example would be a geometric length based on measurement.
Given a circle of radius 2, a priori, it's circumference would be 4π.
Empirically, the circumference would be about 12.5.
This characterization is contentious, but should get us started.

We do use our senses to perceive proofs, of course.
But we should not confuse names for objects with the objects themselves.
See Frege on Mill.

There has been a long confusion of apriority with necessity, which is coming to an end.
Though, Shapiro still calls them "twin notions" (23).
One problem with the traditional notion of apriority, on which anything believed a priori must be true, can be seen by considering Kant's claim that Euclidean space is the result of the a priori application of our concepts on the noumenal world.
Since space turns out to be non-Euclidean, according to special relativity, what seemed a priori turned out to be false.
If we take apriority to entail necessity, that is if we assume that any proposition which was believed on a priori principles must be true, then if the statement turns out false, it must never have been held a priori.
Kant's entire metaphysical system depended on the application of a priori concepts to the noumenal world.
When space turned out to be non-Euclidean, Kant's system fell apart.

If we adopt a fallibilistic a priori, on which we can be wrong about a proposition, even if we hold it a priori, then a discovery that a proposition is false need not impugn the methods we used to acquire that belief.
That is, we can believe a proposition independently of experience, and still be wrong about that belief.
A wonderful example of this is found in Cantor and Frege, who held contradictory axioms of comprehension in their set theories.
Had Kant held a fallibilistic a priori, he might have been able to salvage some of his approach, though not the necessity of mathematics.
The fallibilist can, alternatively, hold that statements believed on the basis of a priori reasoning are necessarily true, *if true*.
(And if they are false, they are necessarily false.)
We will return to these topics.

The twentieth century dominance by philosophy of language has confused the matter worse by admixing analyticity, as an explanation of apriority.
Analyticity is a semantic notion, about meanings of terms.
For example, 'Bachelors are unmarried' and 'We walk with those with whom we stroll' are analytic.
Apriority is an epistemic notion, about belief and knowledge.
Necessity is a metaphysical notion, about the nature of the universe, broadly conceived.

Certainty is an epistemic notion, masquerading as a metaphysical notion.
I can be certain about something non-necessary, like that I am here now (skepticism aside).
I can be uncertain about something necessary, like whether Goldbach's conjecture is true.
I think Brown is liable at #1 for confusing certainty and necessity.

III. Metaphysics of mathematics

Realism: numbers exist, objectively
Idealism: numbers are mental constructs
Nominalism: numbers do not exist
The nominalist denies that there are any types corresponding to number tokens, inscriptions.

One achievement of the Greeks in mathematics, as opposed to earlier or contemporaneous civilizations, like the Babylonians or the Egyptians, was to recognize that realism about mathematics entails believing in an unseen world.
The Pythagoreans seem to be among the first to recognize that the mathematical realm goes beyond the senses.
Furthermore, they demanded proofs of mathematical theorems, rather than mere practical utility.

IV. Chaos, order, and mathematics

"The civilizations that preceded the Greek or were contemporaneous with it regarded nature as chaotic, mysterious, capricious, and terrifying" (Kline, 146).
Consider Koyaanisqatsi.

The world, as we perceive it, is orderly and predictable.
The odd surprise (a tsunami, a terrorist attack, Clinton winning New Hampshire) is always explicable, in hindsight.
We complain about our weather forecasters and political scientists.
But the alien-robot-from-the-future weather person/sociologist would have known.
In contrast, if Kline is right, this order may be unnatural.
Kline's claim is that the Greeks, through a variety of methods, tamed the chaos.
Further, the Pythagoreans used mathematics to bring order to chaos.

What are we to make of Kline's claim?
Is it like the teacher calling the pupils to line up after recess?
What would it mean to regard the world as chaotic (mysterious, capricious, and terrifying)?
Is it like Koyaanisqatsi? Or Guernica?
It can't be quite like that, because some one needs to make the artifacts in those representations.

V. The child development analogy

One way to make sense of Kline's claim is to draw an analogy between the development of civilization and the development of the individual.
We can extend the evolutionary 'ontogeny recapitulates phylogeny' claim.
The individual begins life by perceiving a chaotic world, and learns to order and organize that world around him/her.
This was Kant's contention, as well: the noumenal world, if we can say anything about it, is completely unordered.
We impose the order from our concepts.

The basis for the analogy seems acceptable.
We don't really know what the world is like for the baby, in immediate apprehension.
Perhaps, the baby is given a chaotic world, which would explain its screaming and crying.
So, child, as it grows, progresses through Piagetian stages, and learns to make order.

The Four Piagetian Stages

1. Sensorimotor stage (birth - age 2): The child builds concepts about the external world and how it works, correlating sense experiences with external objects. The child lacks, and learns, object permanence.
2. Pre-operational stage (ages 2 - 7): The child is not able to think abstractly. The child lacks and learns conservation of quantity.
3. Concrete operations (ages 7 - 11): The child starts to reason logically about concrete events. Some limited abstract problem-solving is possible, but only applied to concrete phenomena.
4. Formal operations (ages 11 - 15): The child's develops abstract reasoning.

The argument on the table is a metaphor.
Early civilizations are like babies, seeing the world as chaotic.
The development of civilization, like the development of a child, progresses through stages.
The Greek advance, then, is analogous to humanity entering the formal operations stage.

Can we really expect that the civilizations prior to the Greeks were all like babies?
Maybe, this is plausible in earlier stages of evolutionary development.
But, once we have something like mythology, we would need to think that the people had enough leisure, and freedom from fear and chaos, to develop the myths.
The child development analogy is uncharitable, as well as unhelpful.

It may not be helpful at all to see the pre-Greek cultures as childlike.
Kline argues that the difference between the Greeks and earlier cultures is that the Greeks provided a rational view of nature.
He points out that the mythology of earlier cultures makes life and death the whims of the gods.
But, the gods are presumably rational, too.
Their reasons are just potentially hidden from our view.
The Greeks provided scientific reasons, as opposed to mythological reasons.

VI. Explanations

Examining the differences between science and mythology helps explain the achievement of the Greeks.
We can understand Kline's claim as that the Greeks allowed us to see reasons in nature.
That is, what the Greeks were doing was not giving order to chaos, but providing natural explanations where no explanations were available.

Explanations are useful, but they do not solve the underlying mystery of the universe.
If we want to know why A fell off of a cliff, it is useful to know that B pushed her.
It begs the question of why B pushed her.
We might find out that C pushed B.
And that D pushed C.
Et cetera.
The ultimate causes get pushed back, but are not disappeared.
We do get an order to some portion of the universe, which may be Kline's point.

In any case, we do have some explanations of causes.
And, the explanations replace a visible world with a less-visible world.
Thales gave order and organization to the chaos by positing a single element, water.
Empedocles, and other Greeks following him, posited four elements: earth, air, fire, and water.
The Pythagoreans posited numbers as the fundamental constituent of all that we experience.
Numbers, as Kline says, were to the Pythagoreans as atoms are to us.
That is, the Pythagoreans did not distinguish between mathematics and science.

VII. Pythagoreanism

The Pythagoreans thought that the world was made of numbers.
Prima facie, the Pythagorean claim is absurd.
The Pythagoreans were a secret cult.
They had some odd beliefs, independently of their metaphysics.

One of their beliefs was that every geometric measure would be commensurable (i.e. rational).
The Pythagoreans were convinced that whole numbers and their ratios were the essences of everything.
The discovery of the irrationality of two, say from an isosceles triangle with unit legs, was devastating.
There is a story, perhaps apocryphal, that the person who discovered the proof that the square root of two is irrational was on a ship at the time, and he was thrown overboard.
One problem with examining the Pythagoreans is that there is very little extant writing.
We know about them only on the basis of what others, like Aristotle, wrote.
Many of their beliefs were likely not the beliefs of Pythagoras, anyway, but of his later followers.

Still, is there anything to be made of Pythagoreanism?
Is there any sense that we can make of the claim that the world is mathematical.

VIII. Galileo's Pythagoreanism

"Philosophy is written in this grand book of the universe, which stands continually open to our gaze. But the book cannot be understood unless one first learns to comprehend the language and to read the alphabet in which it is composed. It is written in the language of mathematics, and its characters are triangles, circles and other geometric figures, without which it is humanly impossible to understand a single word of it; without these, one wanders in a dark labyrinth." *The Assayer*, 1623

Consider, for example, Coulomb's law: $F = k\ |q_1 q_2| / r^2$.
It refers to a real number, k, the Coulomb's Law constant.
It includes two functions, absolute value and squaring, as well as multiplication and division.
Furthermore, the function maps real numbers, measurements of charge and distance, to other real numbers, which measure force.
We presume that the particles are not mathematical.
But, their representations surely are.

IX. Quine's Pythagoreanism

We start thinking of bodies as physical objects, but these have vague boundaries, and puzzling identity conditions over time.

We avoid some of the problems by taking bodies to be composed of smaller particles.
We can think of the world as composed of four-dimensional aggregates of these atomic elements.

But, atomism has its problems, too.
Electrons, for example, do not seem to have great identity conditions.
It is arbitrary, at times, to say whether two point events are moments in the career of one electron, or two different ones.

Another option is a field theory, of distributions of states over space-time.
So we are back to space-time, and its states.
The objects are the space-time regions themselves, and their properties.
But, we can identify space-time regions with Cartesian coordinates, making an arbitrary choice of coordinate axes.

"Predicates that formerly attributed states to points or regions will now apply rather to quadruples of numbers, or to sets of quadruples... I seem to have ended up with this as my ontology: pure sets" ("Whither Physical Objects", 501-2).