Philosophy 405: Knowledge, Truth and Mathematics Spring 2008 M, W: 1-2:15pm Hamilton College Russell Marcus rmarcus1@hamilton.edu

Class 18-19: Wittgenstein

I. From Carnap to Wittgenstein

Last week, we talked about Carnap and positivism.

I expressed skepticism that Carnap's internal/external distinction helped solve the problem about access to abstract objects.

We may not be able merely to wish away the commitments to mathematics that come with the acceptance of the linguistic framework that includes mathematical objects.

Carnap's claim may come down to whether the empiricist criterion for meaning is acceptable.

In turn, we must consider whether metaphysical questions are really meaningless.

If purported external claims like 'There are numbers' are not meaningless, if they have content, then Carnap's proposal does not succeed.

Another way to see the problem: if 'There are numbers' is taken as an internal question, it is supposed to be analytic, and devoid of any experiential evidential basis.

But, if there is no analytic/synthetic distinction, then, again Carnap's distinction collapses, too.

I think that Quine is wrong about the analytic/synthetic distinction.

But, Carnap is also wrong that external questions are meaningless.

Carnap's meaninglessness stance derives, as we said, from Wittgenstein's concern to clear the world of metaphysics.

But, the doctrine of meaninglessness seems to be metaphysical, as well.

The terms which are supposed to be meaningless seem to have content.

They combine in nice ways; admit of compositionality in language.

Further, verifiability seems harder than Carnap and the positivists thought it was.

It does not seem that the project to derive all empirical claims from sense experience is defensible.

As Ayer notes, and Jazmine cites, the empiricist is in a bit of a bind, p 73.

The empiricist must either deny the necessity of mathematics, as Mill does, or explain how claims which are not factual, not derived from experience, have content.

We seem to be moving to rationalism, but we will return to the possibility of an empirical basis for mathematics with Quine.

Ayer's criticism of Mill points us back to Carnap, and to Wittgenstein.

Ayer considers potential refutations of mathematical claims.

We are back to the foxes and the chickens, or the salt and the water.

Clear up Jazmine's confusion.

We make decisions about what to hold onto, Ayer 77.

These decisions reflect Carnap's pragmatic attitude to external questions.

They more sharply reflect the later Wittgenstein's conventionalism.

Recall that we said that Carnap replaced the old apriority of mathematics with conventionality. Consider the claim that nothing can be both blue and green all over. We might be willing to see this claim as a matter of convention, since we could take borderline cases between blue and green to be both Knowledge, Truth, and Mathematics, Class Notes, April 2, Prof. Marcus, page 2

blue and green. But, we are much more reluctant to see 'nothing can be blue and red all over' as conventional; it looks like a necessary truth.

Wittgenstein goes wholly over to convention, by providing skepticism about all rules and rule-following. The necessity of 5+7=12 is just that we do not count anything as a counter-example. We could always count something as according to a rule, or not according to a rule, since all sorts of rules can be devised.

Consider a proof of Goldbach's conjecture. No one has seen it, yet. So, it is not a proof, yet. It will be up to us whether to count it as a proof.

II. Questions and Comments

 Do we know how to continue the series defined by "add two." Contrast with Brown 134.
 §I. 3
 §VI.24
 Contrast with §V.10: Does the picture begin to flitter?

Comments:

The central point for Wittgenstein is that any list of instances is compatible with a wide range of potential rules. So, we have to start to question the basis for extending current practice to novel cases. Whether we start with just a list, and want to go on, or whether we give a rule, which seems to determine how to go on, there will always be novel cases.

2. How is following a rule like a custom?
§VI.21
§VI.41
Relate to Carnap's linguistic frameworks.
Consider Matt's example of the sun rising twice a day.

Comments:

Carnap argued that the choice of a linguistic framework is guided by pragmatic considerations, by questions of utility and fruition. Similarly, Wittgenstein thinks that how to follow a rule is a matter of choice, a matter of decision. With Matt's example, we could choose to extend our notion of 'day' to what we ordinarily call two days. But, there seems to be something incoherent about keeping the notion of 'day' steady and alleging that the sun rises twice in it.

Wittgenstein believes that language is essentially a public, and conventional, object. In the *Philosophical Investigations*, he is taken to argue that there can be no private language. In our selection, VI.21, he makes the same claim. And since language, and rules, are essentially public, there is something essentially conventional about them.

3. What determines whether I am following a rule? Matt on apples and oranges §VI.38§VII.15 (on failing to notice a change in the rule)

Comments:

Here, Wittgenstein is arguing, again, that we might get confused about how to follow rules. We might not even recognize that the rule has shifted, or that we have changed the rule that we are using, or that the rule is indeterminate.

4. How do I determine now what I mean, by a rule, in the future? §VI.16 §VI.38 §VI.47

Comments:

If we are still thinking that there is such a rule as 'add two', then there must be some basis for settling on that rule. Wittgenstein derides hopes that we might ground those rules in our feelings and our intuitions. These supposed foundations are no kind of foundation at all, especially if language and rules are essentially public.

Consider the attempts to establish permanent markers of nuclear waste. (I have links on the website; see p 29 of Permanent markers; for the Sandia report, see p 150 et seq. and p 262 et seq.) For Wittgenstein, there could be no hope of communicating danger to a community completely disconnected from our own. The exercise is pointless.

On the other hand, we might think that the project is not crazy. If we think that there are ways to communicate such facts, then there might be some basis on which to ground our rules, some basis for objective truth in mathematics.

5. Does Wittgenstein want to replace mathematical truth with utility?
§I.4
§I.148-9, on logs
§VI.30, on bringing a stone
How is mathematics like alchemy (§V.16)?

Comments:

For Wittgenstein, there are conventions for how to measure, logs, say. There are conventions for how to command, and how to follow rules, and how to infer. All of these have similar groundings, in our public uses of language. We can choose to use different conventions, not individually, or wantonly, but if different conventions become useful. Knowledge, Truth, and Mathematics, Class Notes, April 2, Prof. Marcus, page 4

6. Is the fancy (§VII.15) that $a^2 + b^2 = (a + b)^2$ the same as the fancy about trisecting an angle? How do we account for the error? Do these errors show a problem with mathematical methods?

Comments:

Wittgenstein is extending his general skeptical conclusions about rule following to mathematical theorems. We could, if we found it useful, take $a^2 + b^2$ to be the same as $(a + b)^2$, as long as we adjusted other rules. We could accept contradictions, or abandon the law of the excluded middle. Or, we could give up more local principles.

7. Do proofs convince us? What do proofs do for us?
§III.25
§VII.74 - How is the proof in the background of a proposition?
Consider §III.39. Are there different conceptions of proof at play?

Comments:

Frege and Russell attempted to ground mathematics in formal proofs. Wittgenstein is arguing that such proofs obscure, rather than enlighten us. "[T]he intrusion of Russell's symbolism into the proofs has done a great deal of harm" (III.25). Consider the proof that 2+2=4. It is non-explanatory, despite its rigor.

8. How does mathematics compel us? (Or, what is the status of logical, or mathematical, necessity?)
§VI. 30
§VI.39
§VI.41
§VI. 46 (Compare to Carnap's replacement of apriority with convention)
§VI.48 (Compare to 'go away!')
§VI.49 (and conventions, though not opinions)
§VII.66-7
Relate 'What the Tortoise said to Achilles" to §I.33, §I.118 and the law analogy, and §VI.48.
Do we just give up inference and necessity?

Comments:

How can we show some one that they must follow a certain rule that we prescribe? We can always, like the Tortoise in Lewis Carroll's "What the Tortoise Said to Achilles", refuse to infer the way we are supposed to infer. Achilles says, "Logic would take you by the throat and *force* you to do it!" But Wittgenstein can demur, as the Tortoise does.

Whitman said, "Do I contradict myself? Very well, then I contradict myself."

"...and there was a touch of sadness in his tone."

The moral we can take from Lewis Carroll is that there is an important difference between logical truths and rules of inference. Ayer points out (p 81) that this distinction comes from Aristotle, and that the logicists abandon it. Wittgenstein's point is that the rules of inference are open to convention, and they can not compel us if we do not wish them to compel us.

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9. What is the problem of Russell's paradox? How is it a cancerous growth? Is the problem of contradiction just a problem within the calculus? Is Wittgenstein over-reacting to the paradox? §VII.11§VII.15

Comments:

Consider Wittgenstein's observation that Russell's paradox seems to come out of nowhere. We are unproblematically working with a formal system, when we notice a foundational problem. In a way, Wittgenstein wants to say that the paradox should not make us question the results we have already achieved, that the problem is not as foundational as it seems. But, given the structure of Russell's system, the paradox has to take the whole system down with it. So, Wittgenstein is forced to give up even more fundamental beliefs about the nature of proof and formal systems. If the paradox can destroy the system, maybe the system is not what it seemed.

10. Is Wittgenstein agreeing or disagreeing with Frege, on the role of psychology in mathematics? See §III.25

Comments:

Frege urged us to rid philosophy of mathematics, from any psychological component, such as those that Kant had introduced. Wittgenstein's questions about proof re-introduce a psychological component.

11. Look at places in which Wittgenstein asks us to imagine something. Can you really imagine it? (Is conceivability a guide to possibility?) See especially §III.85, §VII.11, §VII.15, §VII.43, §VII.61.

Comments:

I leave to you this concern of mine. The question of whether conceivability is a guide to possibility is a live one in the philosophy of mind. For example, I might be able to conceive of a creature physically identical to me, but without any conscious mental states, without the feelings that I have. Does this mean that my mental states are not reducible to physical states?