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Class 17: Carnap

I. Historical background: a summary

The period from 1879-1931 saw intense interest and productivity in logic, mathematics, and philosophy of mathematics.
We have looked at the Big Three positions in the philosophy of mathematics: logicism, formalism, and intuitionism.
In fact, we really should talk about the Big Four, since Hilbert's programme is not entirely formalist, but finitist.

Logicism: mathematics is logic, analytic, we get all of classical mathematics.
Drawbacks: Russell's paradox, the (ugly) theory of types.
Plus, set theory really doesn't look like logic, purely.

Formalism: mathematics is an empty game, with no content.
Drawbacks: we don't get completeness or consistency.
Plus, the nominalism for ideal statements leaves open the question of how to explain the utility of mathematics in science.
Why would an empty game have mathematical relevance?
Hilbert doesn't really have this problem for real mathematical statements, but the questions about the formalist hold for his ideal statements.
Remember, even $(\forall x)(\forall y)(x+y=y+x)$ is an ideal statement.

Intuitionism: mathematics is synthetic, constructed.
Drawbacks: we lose many classical results.
Also, there seems to be a problem with intuitionist's logic.
For the intuitionist, '$A \supset B$' means that there is a constructive procedure leading from a proof of A to a proof of B.
In mathematics, we can, perhaps, adopt this logic.
In ordinary language, and in science, the '$\supset$' doesn't seem to mean that we have constructed proofs from one to the other.
Further, the intuitionist relies on the consistency of the mind, and its constructions, to secure the health of its mathematics.
But, couldn't the mind be inconsistent?
(That point comes from Putnam.)

Early Wittgenstein worked in the logicist tradition.
The *Tractatus*, is an attempt to work out a kind of logicist reduction of everything, not just mathematics, to statements of fact.
There are basic facts about the world, and these combine in ways guided by logical calculi.
After finishing philosophy by writing the Tractatus, Wittgenstein gave up and taught Elementary School.
In Vienna, though, people continued to work out his ideas.
These people became known as the logical positivists: Moritz Schlick, Rudolph Carnap, Otto Neurath, Hans Reichenbach; and visitors like A.J. Ayer, who brought positivism to the English-speaking world, and Quine, whose work was always a reaction to Carnap.

In part, positivism was an attempt to take seriously the mathematical and scientific advances, especially the theory of relativity and the upheaval of Newtonian physics to which it led.
The positivists were notoriously hostile to metaphysics, and traditional philosophical speculation.
They were descendants of Hume, especially methodologically; see the end of the *Enquiry*.
In order to eliminate metaphysics, the positivists took the meaning of a statement is its method of verification.
Since there were no way to verify claims about the existence of God, the took claims about god to be meaningless.

Now, what about mathematics?
In one way, there is no way to verify, empirically, the existence of numbers, or circles.
On the other hand, mathematical objects are used in science, which the positivists esteemed most highly.

Carnap's "Empiricism, Semantics, and Ontology" was published in 1950, after the end of the Vienna Circle.
He maintains some of the positivist's theses, though.

II. Questions and some comments

1. On p 205, Carnap discusses a person with an uneasy conscience. Why does that person have an uneasy conscience? What are the scientific contexts to which Carnap refers? Contrast this introductory passage with the end of the incomplete paragraph on p 215.

Comments:

There is a worry about double-talk. If we claim there are numbers (in science) at the same time as thinking that there are no numbers (in philosophy), we seem to be contradicting ourselves. In scientific contexts, we use real numbers as space-time coordinates, and in general for measurement purposes.

2. Where, besides mathematics, do we find commitments to abstract objects?

Comments:

In semantics, we find abstract objects as propositions. One might take properties to be abstract objects, as well.

3. How do we determine answers to internal questions? Consider how we answer questions about the existence of King Arthur and the Easter Bunny. Consider also the difference between logical and factual methods.

Comments:

Try to connect the Easter Bunny question with the problem of choosing a linguistic framework: we use pragmatic guidelines like simplicity and fruitfulness. But, can we choose to use an Easter Bunny language?

The answer to an internal question need not be empirical. In mathematics, the answers to internal questions are supposed to be analytic, using the logical method; Carnap is relying on Frege's account of mathematics.

4. What is a pseudo-question? How can we identify one?

Comments:

The external questions are supposed to be meaningless, since there is no method we can use to verify them. The question whether there are numbers, or whether there is a God, is supposed to be merely a question about the practical utility of adopting that language. If a statement has no method of verification, then it is meaningless, a pseudo-question.

5. How do we determine answers to external questions? (See pp 208, 214, and 218.)

Comments:

We have to distinguish between the question of whether a question is internal or external and the question of whether to adopt a framework. Heather is right that the question of whether to adopt a framework admits of degrees. The criteria for adopting a framework are pragmatic; a framework may be more or less useful depending on context. But, whether a question is internal or external does not admit of degrees. Either we have a method of verification or we do not. If there is a method to verify an answer, then the question has content, and can not be merely external.

It is important, for Quine's criticism of Carnap to succeed, that Carnap be able to draw a strict line between internal and external questions. For, Quine's most popular argument is that Carnap's internal/external distinction collapses with the analytic/synthetic distinction. (Actually, Quine's better argument is the double-talk claim, I believe.)

6. Especially, how are space-time coordinate systems a matter for external questions? Do we use real or rational numbers for coordinates, and why?

Comments:

We try to achieve overall simplicity of a theory. Real numbers can't come directly from measurement, but they can come from calculation. There was discussion, among the positivists, whether one could maintain Euclidean geometry in the light of special relativity. Poincare thought that the choice between Euclidean and hyperbolic geometries was merely conventional. My understanding is that general relativity prevents this, but I'm not sure.

7. What is the relationship between apriority and conventionality? Consider the problem that the theory of relativity raises for Kant's philosophy.

Comments:

This is a central point in understanding Carnap. He is trying to replace the old notion of apriority, for sureness, with a decision. He is replacing necessity with linguistic convention.

When Kant's claim that Euclidean geometry is a priori was seen to conflict with the curvature of space-time, the positivists could hold that Euclidean geometry is still conventional. By making geometry conventional, we lose content, but we do retain something like immunity from empirical revision. That is, whether we take geometry to be a priori true or conventional, it still remains distinct from empirical claims.

We have number terms, perhaps in our best science. We are faced with realists, who claim that there are numbers, propositions, and properties, like redness. And nominalists, who deny their reality. But, we are back to the man with the lame conscience.

8. Is Ryle's 'Fido'-Fido criticism (216) effective? Consider redness and five.

Comments:

Carnap does not quite agree with it, since it follows that numbers exist. Ryle doesn't give us a way to deny that number terms refer to numbers, without the guilty conscience.

9. Distinguish Carnap's positivist solution from the nominalistic/empiricist one. (See p 219.) Why does Carnap oppose the nominalist?

Comments:

Carnap says that the question of whether there are numbers is a pseudo-question. Ryle, the nominalist, is engaged in metaphysics, just like the platonist.

10. Does Carnap avoid platonism?

Comments:

I don't see how Carnap provides a solution. It seems like he's just avoiding the question. When we get to Quine's double-talk criticism, we will see the point made clearly. But, Carnap understands the double-talk criticism and still holds his position. So...

11. According to Carnap, is mathematics analytic or synthetic? (I.e. is Carnap more closely allied with Frege or with Kant?)

Comments:

Analytic, Frege: see comments on Q3

12. Consider the phrases referring to entities listed on p 220 (a little bit above the middle of the page). How do we explain their use? Does Carnap's internal/external distinction help us explain their use?

Comments:

I leave these to you as exercises.