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Class 10: Mill

I. We have various debates on the table

1. Do the objects of mathematics exist? If so, how do they exist?
2. Are mathematical truths necessary?
3. Are there innate ideas?
4. Are mathematical claims truths of experience, or of reason?
5. Are mathematical statements synthetic, or analytic?

The first two questions are metaphysical.
The third and fourth are epistemological.
The fifth is semantic.

Answering the semantic question might help us answer the epistemological question.
For, if truths of mathematics are synthetic, we might be able to account for all mathematical knowledge on the basis of sense experience, and avoid appeals to innate ideas or pure reasoning.
If the truths of mathematics are analytic, then we might need to appeal to reason, in addition to sense experience, to explain our analytic knowledge.

There are other relations among the questions.
For example, if the answer to Q2 is affirmative, then Q4 seems answered, too.
A negative answer to Q2 might not lead us to an answer to Q4.

II. Avoiding mathematical objects

Plato and Descartes believe that mathematical objects exist.
Plato says that the soul is acquainted with them.
Descartes allows for innate ideas of mathematics.
But, mathematical objects are not accessible to our senses.
We have seen several attempts to avoid believing in them.
Mill, like Berkeley, denies traditional mathematical objects.
Also note that Mill's reasons are a lot like Berkeley's reasons, relying on the picture theory, p 169.

At the beginning of our selection, Mill surveys two positions which avoid mathematical objects: modalism and conceptualism.
He first dismisses modalism, which claims that mathematics is the study of possible objects.
We have not yet studied a modalist, but we will, later.
Against modalism, Mill argues that mathematical objects can not even possibly exist, for the same reasons that they do not exist.
It is "inconsistent with the physical constitution of our planet... if not of the universe" (168).

Mill next dismisses conceptualism, which claims that mathematics is the study of our ideas.
Locke thinks we have abstract ideas of mathematical objects.
Thus, mathematical objects are ideas in our minds.

Locke is thus a conceptualist.
Against conceptualism, Mill offers a Berkeleyan argument.
We can not form abstract ideas.
Thus far, Mill looks exactly like Berkeley.
Indeed, his position is quite close to Berkeley's view.
But, Mill would not have associated with Berkeley's idealism.
He is much closer to Leibniz and Hume.

Leibniz thinks that mathematical truths are logical truths.
So, we have innate ideas, but the objects are not special objects.
We start with immediately accessible truths, logical identities.
Then, we derive, through uses of symbols, further mathematical claims.

Mill agrees with Leibniz that definition is important to the development of mathematics, especially in deriving more complex theorems on the basis of simpler statements, deriving theorems from axioms.
At the end of §1, Mill professes a deductivist view, that mathematical certainty comes from the deduction of theorems from hypotheses.
The conclusions follow from the definitions.
Mill is ascribing necessity to the deductions, as logical truths.
Here, Mill and Hume agree, in the spirit of Walter's suggestion, from a few classes ago, that mathematics is all deductive.

Still, we have a question about the status of the axioms.
Hume and Leibniz are very close, here, despite their differences about innate ideas.
They both thought that all of mathematics derived from definitions, and so avoided the question about the axioms.
Leibniz, like Hume, took the foundational theorems, to be logical truths.
In the end, neither Hume nor Leibniz thinks that there are special mathematical objects.
So, I think they present an incomplete theory.
We might say that an existential axiom, like the empty set axiom ($\exists x \forall y\ y \notin x$) is a logical truth.
But, it asserts the existence of a set.
To deny that there are mathematical objects means that we have to find a way to understand that claim without taking it at face value.

There are three possible positions regarding the axioms.
First, they could be arbitrary definitions.
That is, we could just accept them without justification.
This position becomes known as formalism.
Perhaps, we could interpret Hume as a proto-formalist.

Second, they could be justified by intuition.
This is Kant's position, though of course Kant does not agree about the inferences being purely deductive.
Mill has already dismissed this possibility in dismissing conceptualism.

Third, they could be justified by their reference to real objects.
Notice that Descartes and Plato could agree to this part of the claim.
The question becomes, what are the real objects to which the axioms refer?

We have already seen Mill's claim that they can not be ideal objects, or platonic objects.
So, what is left?

Mill takes fundamental mathematical claims to be empirical generalizations.
They are inductions from sense experience, §4, p 172.
Mill thus improves on the Hume-Leibniz view, by giving referents to mathematical objects.

III. Enumerative Induction

The claim that mathematical theorems (including the axioms) are the results of enumerative induction seems most plausible for geometry.
For Mill, lines are just roads, or sticks, or streaks of clouds.
Circles are frisbees and pizzas.
Theorems of geometry, which seem to refer to ideal objects, actually refer to physical objects, given a theory of approximation (fn on p 174).
The claims of mathematics, like the existence of an irrational number, or the area of a plane figure, would be, strictly speaking, false.
He calls them "feigned" (169).
He allows a bit of rounding-out: "our liberty extends" (171).

Mill accounts for the difficulty extending the theory to arithmetic, p 188.
Numbers are numbers of something, 189.
We learn them from experiences with collections.

Mill argues that we can account for mathematics on the basis of sense experience.
He argues that the burden is on the traditionalists to support a priori knowledge of necessary truths, p 173.
The argument here relies on Ockham's razor: do not multiply entities unnecessarily.
If we can account for mathematical truth without positing platonic objects, or innate ideas, we should do so.

IV. Against apriority and necessity

Mill argues against a priori knowledge of necessary truths in §5-§6.
§5 concerns whether mere thought leads us to mathematics.
Mill assumes that mathematics is inductive.
Thus, we can not get it from pure reasoning.
The argument in §5 thus seems circular.

Mill's arguments against necessity are stronger.
He agrees that experience can not yield necessity.
Thus, either mathematical truths are not necessary, or they are not derived from experience.
Given his claims that mathematical truths are inductions from sense experience, they must not be necessary.
But, they seem necessary.
That, of course, was the foundation of Kant's whole metaphysical project.

So, Mill needs an argument against necessity.

Mill's argument against necessity mainly focuses on destroying Whewell's defense.
He ascribes to Whewell the belief that a statement is necessarily true if its negation is necessarily false.
That was, of course, the ground of Hume's relations of ideas.
But, how do we know if a statement is necessarily false?
Recall the question about conceivability and necessity we asked with Hume.
Hume says that we can conceive that the sun will not rise tomorrow, but we can not conceive of $2+2\neq4$.
So, it is not just Whewell who connects conceivability and necessity.
Mill denies that conceivability is a source of knowledge of necessity, p 178.
He says that conceivability is an accident of our limited minds.

Mill provides a wide range of examples where something people thought was inconceivable turned out not only to be conceivable, but true.
Also, he argues that what people thought is true may turn out to be false, and later perceived as inconceivable.
Newton's first law (181)
Newtonian gravitation (178)
the DeMorgan quote (178 fn)
Indestructibility of matter (184)

Still, note that all of these examples regard physical laws.
Does the attack translate to mathematics?
Mill does make something of a claim about logic, on 184.
Mill's defense of the failure of conceivability as a guide to necessity in mathematics is that mathematical statements are so firmly entrenched in our minds, from early on, that it is harder to see that they are just empirical generalizations, pp 181-2.

(Jonathan points out that Shapiro ascribes to Mill the view that mathematical statements are necessary.
Mill understands 'x is necessary' to mean 'we can not conceive of the falsity of x'.
Thus, Mill's necessity is not what we ordinarily mean by 'necessity'.)

V. Confirmation and disconfirmation

Let us turn to Mill's positive account.
If mathematical theorems are inductive generalizations, then they are confirmed from experience.
And, mathematical falsehoods are disconfirmed by experience.

Are the hypotheses of mathematics really empirically confirmed?
Mill, p 173, describes how our experience confirms the geometric theorem that lines that cross diverge as we travel away from their point of intersection.
But, what about elliptical geometry?
Consider the two lines diverging, p 175.
Consider traveling on the surface of a sphere (or a soft ball!)

Can we disconfirm mathematical statements empirically?
Is this what happened with Euclidean geometry?

Do we think of Euclidean geometry as a false theory, in the way that we think of Newtonian mechanics?
(According to Newtonian mechanics, there is no fastest speed.
According to relativity, there is.)

VI. Problems

Frege discusses several problems with Mill's account.
His attacks on Mill are famously amusing, especially his comment that it is a good thing that not everything is nailed down, since then 2+1 could not equal 3.
Shapiro rightly notes that Frege's criticisms are more thin mockery than substantial argument.

But, how can Mill explain the uses of every and all on 189?
Can inductions from sense experience be the right account?
Remember, we could never get to pi from sense experience.
We started the course with this, Pythagorean discovery.