Hamilton College
Russell Marcus
rmarcus1@hamilton.edu

Day 1

I. Course Introduction

Hand out syllabus
Talk a bit about its length; if things get overwhelming, we should talk.
The syllabus is not set in stone, though it may look like it.

All of philosophy is philosophy of mathematics: but some people are scared of mathematics.
Read the "burn in hell" passage from Brown.
That's a bit too strong, but the idea is right.

Plato's students were implored to excel in mathematics; a sign over the door to his Academy said, "Let no one enter who is ignorant of geometry."
Descartes helped to found analytic geometry.
Leibniz developed the calculus.
Frege and Russell made advances in the foundations of mathematics proper.
Quine, Kripke, Field and many others contribute to set theory and the foundations of mathematics.

Euclid's method has had a long and profound influence on the methods of philosophy.
Cantor's work on transfinite numbers transformed the philosopher's concept of infinity.
Hilbert, Gödel, von Neumann, and Tarski, are central philosophical figures.

Berkeley tried to debunk the calculus; check out *The Analyst* in the Ewald collection.
Kant's transcendental idealism begins with the question of what the structure of our reasoning must be in order to yield mathematical certainty.
Wittgenstein's *Remarks on the Foundations of Mathematics* contain core elements of his philosophical positions, specifically his skepticism about rule-following.

Wittgenstein: philosophy, "Leaves mathematics as it is, and no mathematical discovery can advance it." (*Philosophical Investigations*, §124)
Kripke: "There is no mathematical substitute for philosophy."

This course is going to be unique in its blend of history and contemporary approaches to the philosophy of mathematics.
Philosophy of mathematics is taught rarely.
When it is taught, it tends to focus either exclusively on twentieth century and contemporary readings or on a small period, like the 1920s when the debate between intuitionists and formalists was prominent.
We are going to do a survey of historical approaches to mathematics and the philosophy of mathematics.

There will be three parts to the course:

1. History to the late nineteenth century;
2. Frege to Gödel, 1879-1947, or so;
3. Contemporary approaches.

See the website syllabus for a schedule; a full schedule will be distributed next week.
Blackboard is only for grades, email, and ereserve.

Readings come in three levels:
- Primary - required; includes the seminar paper (or papers) for that day
  - Some, mostly historical sources, are selections.
  - I may provide some introductory passages.
- Secondary - suggested; almost required for seminar papers
  - The secondary readings are there to help you.
  - The two books I've ordered are mainly secondary sources, and good ones.
  - Brown is more fun.
  - Shapiro is more traditional.
  - Some of you are going to have to work a bit harder with the history.
  - I'm not presuming knowledge of the history of philosophy, but you will have an easier time if you already know some of the terminology, if you've already read Kant or Aristotle.
- Tertiary - see the course bibliography

Assignments and grading

Seminar papers are due, to the class, by noon Sunday/Tuesday
- See the Seminar Paper guideline sheet.
- You can use the blackboard email function.
- The seminar paper/papers will start the ball rolling in class.
- (Two on the same day might want to coordinate efforts.)
- I'm going to moderate discussion, ask questions.
- We will sign up for the first two seminar papers on Wednesday. (We'll wait on the third).
- You are likely to write your term paper on a topic you've begun as a seminar paper.

Term paper
- See the term paper assignment.
- Three stages:
  - Topic due on March 12
  - Draft due on April 14
  - Final version due at the end of the term
- See the Course Bibliography for some more references.

Final Exam/Course Prep
- The final will be comprehensive and difficult.
- It should be thought of as a punishment for blowing it during the term, not as an alternative to participation.
- I want you to participate, run this class.
- Come to class having done the primary readings, and read the seminar papers.
- You should have questions and comments on the seminar paper, as well as the readings.

Finish discussion of course website
I'll try to provide summaries of discussion, after class, on the website

II. What is a proof?

This example comes from the Kline reading for Wednesday, on Pythagoras.
In fact, there are four short, mostly easy readings for Wednesday.

Greek mathematics was essentially geometric.
Euclid's *Elements* contained number theory, but it was derived from geometric relations.
The Pythagoreans developed figurate numbers.
When we talk about square numbers, the Pythagoreans are presuming geometric squares.
The Pythagoreans considered other figurate numbers, e.g. pentagonal numbers and triangular numbers.

Consider the triangular numbers, which were especially interesting to the Pythagoreans: 1, 3, 6, 10, 15, 21, 28, 36...
We can see them arranged as triangles, using dots.
The formula for the sum is: (n/2)(n+1)

It turns out that the sum of two consecutive triangular numbers is a square number.
This is easily shown algebraically.

$(n/2)(n+1) + ((n+1)/2)((n+1)+1)$ resolves to $(n+1)^2$

Kline says: "That the Pythagoreans could prove this general conclusion, however, is doubtful" (30).
Is Kline's claim correct?

In Kline's defense, it is true that the Pythagoreans did not have algebra, which was not developed until the 9th century, by the Arabs.
Still, the Pythagoreans did have the picture in Kline, p 30, Figure 3.2.

What is a proof?
Why is the picture not a proof?
Consider Wittgenstein's proof of commutativity.
Some pictures are misleading, at best.
It is difficult to draw intuitively useful pictures of odd spaces, for example.

There was some discussion of the different demonstrations of the fact that the sum of two consecutive triangular numbers is a square number. Most of us were uncomfortable, in some way, with the picture proof. It seemed not, in itself, to generalize the result that the algebraic proof did generalize. Still, I don't think anyone thought that the picture proof failed to be convincing. It was just that we wanted more than a picture.

I wondered whether there really was any more to get. Wittgenstein's suggestion is that we overvalue the algebraic proof, since we do not know how to extend, or project, results to new cases. Wittgenstein's skepticism seems unfounded, prima facie, but we will look at his arguments through the term.

I should have asked the following question: doesn't the picture proof give you more than the algenraic one, in some ways, as well?

We will also return to the question of picture proofs through the course.
We will return to the Pythagoreans on Wednesday.

III. The Nature of Mathematics

Brown characterizes the "mathematical image"

1. Mathematical results are certain.
2. Mathematics is objective.
3. Proofs are essential.
4. Diagrams are psychologically useful, but prove nothing.
5. Diagrams can even be misleading.
6. Mathematics is wedded to classical logic.
7. Mathematics is independent of sense experience.
8. The history of mathematics is cumulative.
9. Computer proofs are merely long and complicated regular proofs.
10. Some mathematical problems are unsolvable in principle.

Note that Brown omits the term 'a priori', at #7.
What does 'a priori' mean?
The question of whether we have a priori knowledge is widely debated.
The debates over the a priori are subtle and complex.
But, the question of whether there is a priori knowledge seems easily answered in mathematics.
Brown's example is excellent: the square root of two.
We could never discover that the square root of two is irrational by experience.
Since the rationals are dense, we could not get to the result by measuring.
We can always find a rational which will fulfill our measurement needs.