Philosophy 405: Knowledge, Truth and Mathematics
Spring 2008
Mondays and Wednesdays: 1-2:15pm
Library 209

Hamilton College
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Syllabus

Course Description and Overview:

This course is a survey of the philosophical questions which arise from considering historical and contemporary approaches to explaining our knowledge of mathematics. Do we have a priori knowledge of necessary truths? Is our knowledge of mathematics empirical? Do we really have mathematical knowledge at all? The readings divide into three periods. The first part of the course surveys historical positions through the mid-nineteenth century. The second part of the course focuses on the fruitful period between Frege and Gödel. The last part covers contemporary approaches.

Mathematics has a long and prominent place in philosophy. Plato's students were implored to excel in mathematics; a sign over the door to his Academy said, "Let no one enter who is ignorant of geometry." Some prominent philosophers in the early modern period were mathematicians, including Descartes, who helped to found analytic geometry, and Leibniz, who developed the calculus. In the late nineteenth and early twentieth centuries, philosophers including Frege and Russell made advances in the foundations of mathematics proper. In recent years, many philosophers have made contributions to set theory and mathematical logic, independently of their philosophical work.

In the other direction, mathematicians from Euclid forward have contributed to philosophy. Cantor's work on transfinite numbers transformed the philosopher's concept of infinity, which had played a central role in philosophical debate about God and the origins of the universe for millennia. Other philosophical topics like necessity and contingency have received mathematical treatment which has changed the way philosophers argue about these concepts. Indeed some mathematicians, like Hilbert, Gödel, von Neumann, and Tarski, are central philosophical figures.

Even philosophers who have not contributed to mathematics often think about mathematics, and have made mathematical insights central to their work. Berkeley tried to debunk the calculus on philosophical grounds. Kant's transcendental idealism begins with the question of what the structure of our reasoning must be in order to yield mathematical certainty. Wittgenstein's *Remarks on the Foundations of Mathematics* contain core elements of his philosophical positions.

Still, even philosophers who spend time with mathematics deny that the relationship of mathematics to philosophy is particularly close. Wittgenstein wrote that philosophy, "Leaves mathematics as it is, and no mathematical discovery can advance it." (*Philosophical Investigations*, §124) Kripke implored that, "There is no mathematical substitute for philosophy."

In this course, in addition to examining the philosophical questions which arise from considerations of our knowledge of mathematics, we will try to see what makes mathematics so interesting to philosophers, and also what contributions mathematics can make to philosophy.

Texts:

Course packet of primary readings, most available on reserve. See the course schedule and course bibliography for more details.
James Robert Brown, *Philosophy of Mathematics: An Introduction to the World of Proofs and Pictures*, New York: Routledge, 2000.
Stewart Shapiro, *Thinking About Mathematics: The Philosophy of Mathematics*, New York: Oxford, 2000.

On-Line Resources:

The website for this course is:
www.thatmarcusfamily.org/philosophy/Phil_Math/Course_Home.htm

Limited material will be available on the Blackboard course pages. The Blackboard page will include a link to the course website. The course website includes an html syllabus, course schedule, course bibliography, class notes, assignments, other readings and handouts, and links to websites specifically selected for this course.

Assignments and Grading

Readings are to be completed before the class indicated on the course schedule, to be distributed in class, and available on the course website. Most class discussions will be initiated by short (2-4 page) papers written by the members of the seminar. Each student will be expected to complete three seminar papers during the term. Students will also be expected to complete a longer paper, in two drafts (preliminary and final). You have a choice of either a participation and preparation grade, or a final exam grade. By the penultimate week of classes, I will be able to tell you what your participation and preparation grade will be. At that time, you may choose to take a final exam. Assignments will be weighted as follows:

1. All the primary readings listed below, including seminar papers.
2. Seminar papers (45%; 15% each)
3. Term paper (30%)
4. Participation and preparation / final exam (25%)

The preliminary draft of your term paper is due on April 14. If you choose to expand one of your first two seminar papers, the draft must show evidence of further research. Failure to hand in a draft, or handing in an insufficient draft, will reduce your final paper grade by two steps (e.g. from B+ to B-).

The Hamilton College Honor Code will be enforced.

Seminar Papers and Class Discussion:

Classes will generally run as discussions of one or two seminar papers. Seminar papers should assimilate the assigned readings and summarize the main arguments. They must demonstrate attempts to grapple with the primary sources. You should also consider the secondary readings. Critical discussion is encouraged, and need not be fully developed. You are instigating class discussion, focusing our thoughts on the central theses, and raising questions. It is good practice to end a seminar paper with a few questions you believe will be useful for the class to discuss.

Each seminar paper is due at noon the day before the class in which it will be discussed (i.e. Sunday or Tuesday). This deadline is necessary for all participants in the seminar to be able to read the paper and prepare comments and questions for class. Classes will begin with an opportunity to present your paper, at which time you may discuss any particular difficulties in the material, or topics that you were unable to cover in the paper.

Students may be allowed to work on one seminar paper, either their second or third, in pairs, with a consequent increase in expectations of length and breadth.

Both the Writing Center and the Oral Communications Center have an astoundingly wonderful set of resources to help you write and speak more effectively.

Topics and Readings:

I have divided the readings for this course into two groups. The primary readings are available on reserve. Alternatively, you can find full citations for each reading in the course bibliography. The secondary readings are almost all from the Shapiro and Brown books; the rest are on reserve. A detailed schedule of readings will be distributed, and will be available on the course website.

I. Introduction
- What is mathematics? What is philosophy of mathematics?
  - Readings:
    - Brown, Chapter 1
    - Shapiro, pp 21-39

II. Historical Readings
- A. Pythagoras and the Pythagoreans
  - Primary readings:
    - Kline, "The Creation of Classical Greek Mathematics"
    - Kline, "The Greek Rationalization of Nature"
- B. Plato's Platonism
  - Primary readings:
    - Selections from Plato on Mathematics
  - Secondary readings:
    - Shapiro, pp 49-63
    - Brown, Chapter 2
- C. Aristotle
  - Primary reading:
    - Aristotle, "Books XIII and XIV"
  - Secondary reading:
    - Shapiro, pp 63-71
- D. Modern Rationalism
  - Primary readings:
    - Descartes, "Third Meditation"
    - Descartes, "Fifth Meditation"
    - Leibniz, "Meditations on Knowledge, Truth, and Ideas"
    - Locke, *Essay*, Bk 1, Ch. 1
    - Leibniz, Selections from *New Essays*
  - Secondary readings:
    - Kline, "The Mathematization of Science"
    - Kline, "The Creation of the Calculus"
- E. Modern Empiricism
  - Primary readings:
    - Selections from Berkeley's *Principles*
    - Selections from Hume on Mathematics
- F. The Synthetic A Priori
  - Primary readings:
    - Selections from Kant's *Critique*
    - Kant, *Prolegomena*, §§1-2

Secondary reading:
- Shapiro, pp 76-91

G. Radical Empiricism
- Primary readings:
  - Mill, *System of Logic,* Book II, §§V and VI
  - Frege, from *The Foundations of Arithmetic*, I
- Secondary reading:
  - Shapiro, pp 91-102

H. Cantor's Paradise
- Primary readings:
  - Tiles, "Cantor's Transfinite Paradise"
- Secondary reading:
  - Dauben, "Cantor's Philosophy of the Infinite"
  - Tiles, "Numbering the Continuum"

III. The Early 20th Century

A. Logicism
- Primary readings:
  - Frege, from *The Foundations of Arithmetic*, II
  - Russell, "Letter to Frege"
  - Frege, "Letter to Russell"
- Secondary reading:
  - Russell, "On Our Knowledge of General Principles"
  - Russell, "How *A Priori* Knowledge is Possible"
  - Shapiro, pp 107-115

B. Formalism and Incompleteness
- Primary readings:
  - Hilbert, "On the Infinite"
  - Johann (John) von Neumann, "The Formalist Foundations of Mathematics"
  - Smullyan, "The General Idea Behind Gödel's Proof"
- Secondary readings:
  - Brown, pp 62-71
  - Shapiro, pp 140-168

C. Intuitionism
- Primary readings:
  - Heyting, "Disputation"
  - Brouwer, "Intuitionism and Formalism"
  - Brouwer, "Consciousness, Philosophy, and Mathematics"
- Secondary readings:
  - Brown, Chapter 8
  - Shapiro, pp 172-189

D. Carnap
- Primary readings:
  - Carnap, "Empiricism, Semantics and Ontology"
- Secondary reading:
  - Shapiro, pp 124-133

- E. Wittgenstein's Conventionalism
  - Primary readings:
    - Wittgenstein, Selections from *Remarks on the Foundations of Mathematics*
    - Ayer, "The A Priori"
  - Secondary readings:
    - Brown, Chapter 9
- F. Gödel Platonism
  - Primary reading:
    - Gödel, "What is Cantor's Continuum Problem? (1964)"
  - Secondary reading:
    - Shapiro, pp 201-212
    - Feferman et al., "Introductory Note..."
    - Gödel, "What is Cantor's Continuum Problem? (1947)"

IV. Contemporary Views

- A. The Problem
  - Primary Readings
    - Benacerraf, "Mathematical Truth"
    - Field, "Knowledge of Mathematical Entities"
  - Secondary Reading
    - Shapiro, pp 29-39
- B. Quineans
  - Primary readings:
    - Quine, "Existence and Quantification"
    - Quine, "On What There Is"
    - Marcus, "Quine's Indispensability Argument"
    - Marcus, "Problems with Quine's Indispensability Argument"
  - Secondary reading:
    - Shapiro, pp 212-220
    - Quine, "Two Dogmas of Empiricism"
    - Grice and Strawson, "In Defense of a Dogma"
- C. Structuralism
  - Primary readings:
    - Benacerraf, "What Numbers Could Not Be"
    - Shapiro, "Structure"
  - Secondary reading:
    - Shapiro, Chapter 10
- D. Fictionalism
  - Primary reading:
    - Field, "Introduction: Fictionalism, Epistemology, and Modality"
  - Secondary reading:
    - Shapiro, pp 226-237
- E. Contemporary Platonism
  - Primary readings:
    - Katz, "Conclusion: The Problems of Philosophy"
    - Katz, "The Epistemic Challenge to Realism"
    - Katz, "Toward a Realistic Rationalism"
    - Balaguer, "A New Platonist Epistemology"

F. Modalism

Primary readings:

Putnam, "Mathematics without Foundations"

Chihara, "The Constructibility Theory"

Secondary reading:

Shapiro, pp 237-243

G. Computer Proofs

Primary reading:

Thomas Tymoczko, "The Four-Color Theorem and its Philosophical Significance."

Secondary reading:

Brown, pp 154-158

V. Epitaph

Putnam, "Philosophy of Mathematics: Why Nothing Works"