Philosophy 405: Knowledge, Truth and Mathematics
Spring 2008
M, W: 1-2:15pm

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A Proof that 2+2=4

We presume the language of first-order logic with identity.
Note two properties of identity, which I will use without explicitly mentioning in the proof:

T: $(\forall x)(\forall y)(\forall z)[(x=y \bullet y=z) \supset x=z]$
Id: $(\forall x)x=x$

We will need a predicate 'N', for the property of being a number, the addition symbol, +, which stands for a function from numbers to numbers, and a successor function, s (all standard in axiomatizations of number theory), with the following governing axioms. (The functions and their compositions are governed by axioms of any standard set theory, which I presume implicitly.)

Z: N_0
S: $(\forall x)(Nx \supset Nsx)$
R: $(\forall x)(\forall y)(x+y = y+x)$
A: $(\forall x)(\forall y)(x+sy = s(x+y))$
IE: $(x)(x+0=x)$

Note that for convenience, I will write the constant '0' as it is standardly written, rather than as a lower-case letter, as is typical in first-order logic. I will write the successor symbol as 'S' when it precedes numerals, such as the other numbers which I introduce as follows:

$1 =_{df} S0$
$2 =_{df} S1$
$3 =_{df} S2$
$4 =_{df} S3$

The proof:

1.	2+2 = 2+2	by Id
2.	= 2 + S1	by definition of '2'
3.	= S(2 + 1)	by A
4.	= S(2 + S0)	by definition of '3'
5.	= SS(2 + 0)	by A
6.	= SS2	by IE
7.	= S3	by definition of '3'
8.	= 4	by definition of '4'

QED