Philosophy 405: Knowledge, Truth and Mathematics Fall 2010

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Class #6: September 15 Modern Rationalism II, Locke and Leibniz

I. Locke, Leibniz, and Innate Ideas

The portions of Locke's work relevant to the philosophy of mathematics contain both a positive project and a negative project.

The positive project is an explanation of how minds born as blank slates, or *tabulae rasa*, can formulate mathematical theories using merely sense experience and our psychological capacities for reflection. We will look at Locke's positive project, and the related ones of Berkeley and Hume, in our next class.

Locke's negative project is his attack on the innate ideas of the rationalists, specifically of Descartes. In the first book of Locke's *Essay*, 1690, he criticizes arguments for the existence of innate ideas, in order to clear the way for his positive, empiricist project.

Leibniz, who was twelve years younger than Locke, wrote an extended commentary on Locke's *Essay*, called *New Essays on Human Understanding*, in the years around 1700.

In the New Essays, Leibniz defends a theory of innate ideas from Locke's attack.

Our task for today is to evaluate the Locke-Leibniz debate about innate ideas.

Our interest in the debate over innate ideas comes from the fact that mathematical ideas are supposed to be among the primary candidates for innateness.

The questions how and whether we can justify our knowledge of mathematics are on the line.

On the surface, Leibniz and Locke present two very different views of mathematical knowledge. Leibniz claims that we care born with all of mathematical knowledge imprinted on our souls. Locke claims that we are born as blank slates, and derive our mathematical knowledge from sense experience.

Although the author of the *Essay* says hundreds of fine things which I applaud, our systems are very different. His is closer to Aristotle and mine to Plato... (Leibniz, *New Essays*, 47).

Leibniz's comparisons to Plato and Aristotle are evocative, but can be misleading if taken too literally. Leibniz does believe that mathematical theorems are both eternal and necessary, as Plato did. But, he rejects Plato's doctrine of recollection, takes a modern view of causation, and has little use for Plato's more general theory of forms.

Similarly, Locke, like Aristotle, presents an account of mathematical knowledge which can be broadly called abstractionist.

But, he rejects Aristotle's theories of causation and sensation.

While Locke and Leibniz have significant disagreements, I wonder if some of them are more semantic than contentful.

If we look at the actual arguments, rather than just their labels, we might find that the distinction between empiricism and rationalism, between the defenders of innate ideas and their opponents, is not so great.

II. The Argument from Universal Assent

Locke, like most philosophers of the modern period (Berkeley is one significant exception), defended the new science and its method of experimentation.

The new science posits a world of material objects, available to sense perception.

We think about material objects through the use of our imagination, our capacity to receive sense images. The rationalists derogated those beliefs that were based on sense perception.

For Descartes, these images are confused, and the only real properties are those we can understand by pure reason, through innate ideas.

An innate idea is one that is implanted in our minds, or souls, rather than learned from sense experience. We are born with innate ideas, according to their proponents, which is why everyone has them, and everyone agrees about them.

The ideas of mathematics are among the most important of the innate ideas. Locke argues that he can avoid positing innate ideas by accounting for all of human knowledge, including mathematical knowledge, on the basis of sense experience.

Men, barely by the use of their natural faculties, may attain to all the knowledge they have, without the help of any innate impressions, and may arrive at certainty without any such original notions or principles (Locke, *Essay*, §I.2.1).

Locke focuses on 'What is, is' and 'It is impossible for the same thing to be and not to be'.

If these "first principles" of knowledge and science are found not to be innate, no other speculative maxims can (I suppose) with better right pretend to be so (Locke, *Essay*, §I.2.28).

He begins his attack on innate ideas by criticizing a general principle of universal assent that Locke attributes to the defender of innate ideas:

UA: If everyone agrees that p, then p is innate.

Locke calls UA the great argument, but it is unlikely that any defender of innate ideas accepted it. Descartes and Leibniz present no such principle, for example.

Still, some defenders of innate ideas may have appealed to universal assent as evidence for innateness. Locke argues that UA is false by presenting some claims that engender widespread agreement while at the same time being tied to sense experience.

For example, he argues that the claim that green is not red is self-evident. But, experience of color is not innate.

I imagine everyone will easily grant that it would be impertinent to suppose the *ideas* of colors innate in a creature to whom God has given sight and a power to receive them by the eyes from external objects... (Locke, *Essay*, §I.2.1).

Locke claims that arguments which appeal to UA to establish the existence of innate ideas are both invalid and unsound.

An argument is invalid if the conclusion does not follow from the premises.

An argument is unsound if the premises are false.

Locke's argument that defenses of innate ideas which rely on UA are invalid is that there are better, alternative accounts of any such universal assent. Locke is relying on a principle of parsimony. We should prefer simpler explanations of any phenomenon.

If it were true in matter of fact, that there were certain truths wherein all mankind agreed, it would not prove them innate, if there can be any other way shown how men may come to that universal agreement, in the things they do consent in, which I presume may be done (Locke, *Essay*, §I.2.3).

Thus, Locke's claim that UA supports an invalid argument depends on his positive account of our knowledge of innate ideas, which we will examine in our next class.

To argue that the use of UA in an argument for the existence of innate ideas is unsound, Locke presents examples of people who do not know the most obvious, purportedly-innate principles, and of principles which should be innate but are not known.

I'll call these two arguments the arguments from transparency.

III. The Arguments from Transparency

Locke argues that UA, and thus the doctrine of innate ideas, is easily shown false by considering that we do not know some of the ideas which are alleged to be innate. For example, children do not know lots of them.

It is evident that all *children*...do not have the least apprehension or thought of them. And the lack of that is enough to destroy that universal assent which must be the necessary concomitant of all innate truths... (Locke, *Essay*, §I.2.5).

Since we are not born apprehending any ideas, Locke claims, the rationalist's claim leads to a contradiction.

The purportedly-innate ideas are known (since they are built-in) and also unknown (since they are not apprehended) at the same time.

We need not appeal to the limitations of children to establish the claim that we lack awareness of some propositions which are supposedly innate.

Consider Goldbach's conjecture, that every even number can be written as the sum of two odd primes. Even the best mathematicians do not know if Goldbach's conjecture is true.

Since it is supposed to be innate, Locke could argue, it is hard to see why we couldn't find a proof of it.

If the doctrine of innate ideas depended on all innate ideas being conscious, then such examples would surely be decisive.

As Descartes argued, consciousness is the primary mark of the mental.

Conscious awareness of our thoughts is sufficient for showing that they are mental, though the nature of mental states remains an open question.

Locke seems to think that consciousness is also a necessary condition for a thought.

That is, Locke presumes that there can be no unconscious thoughts, or thoughts of which we are not aware.

It [seems] to me near a contradiction to say that there are truths imprinted on the soul which it does not perceive or understand (Locke, *Essay*, §I.2.5).

Let's call this claim the argument from transparency: there can be no innate ideas since all thoughts must be transparently conscious and we are unaware of many ideas which are supposed to be innate. The argument from transparency does not destroy the doctrine of innate ideas, but it forces the defender of innate ideas to adopt a theory of unconscious mental states.

Until the late nineteenth century, the notion of unconscious thought was undeveloped.

Still, any defender of innate ideas, in the face of Locke's argument, seems forced to admit that we are unaware of many of our innate ideas.

Indeed, when we are just born we are unaware of all of them.

Leibniz, perhaps uniquely among the moderns and in contrast to Descartes, flirted with such a view.

Cannot - and should not - a substance like our soul have various properties and states which could not all be thought about straight away or all at once? (Leibniz, *New Essays*, 78).

Not only did a theory of unconscious mental states deflect the argument from transparency, but it fit neatly into Leibniz's broader metaphysical, epistemological, and semantic views.

For example, Leibniz's solution to the problem of free will requires a distinction between finite and infinite analysis.

According to the traditional problem of free will, an omniscient God already knows all the future states of the universe, including all of my future actions.

Thus, it seems that no action is contingent; everything that happens is already determined to happen, and happens of necessity.

Leibniz agrees that an infinite mind can, by merely analyzing the current state of the universe, know all the future states of the universe.

But, our finite minds can only analyze the current state of the universe to a small degree.

We do our best to analyze complex ideas into their most simple component parts.

As we have seen, the fundamental truths will be what Leibniz calls identities.

When we analyze a complex proposition, and decide on its truth value, we are prone to err.

The likelihood of error increases with our distance from one of these fundamental identities.

In the *New Essays*, Leibniz recalls an excellent geometer who insisted, falsely, that ellipses, as conic sections, are distinct from oblique sections of cylinders.

I mentioned him only to indicate how far wrong one can go in denying one idea of another, if the case is one where the ideas need to be explored in depth and this has not been done (Leibniz, *New Essays*, 408).

The apparent contingency of my future actions, the appearance of free will, is grounded in such ignorance of infinite analysis.

Still, there is an infinite analysis of the current state of the universe, which could be performed by an infinite mind.

That analysis would reveal all the contents of my mind, including my unconscious mental states. Unconscious mental states are the kinds of things that an omniscient God could know, and which would support the absolute metaphysical determinism of the universe.

Let's review.

Locke argued that the defender of innate ideas relies on an unsound argument from UA.

He claimed that the argument from UA is unsound because we are unaware of many of our supposedlyinnate ideas.

Locke's argument presumes that an innate idea must be transparent to our consciousness.

The defender of innate ideas, Leibniz in this case, can deflect Locke's criticism by rejecting

transparency, and adopting a more-plausible theory of mental states on which we can have innate ideas of which we are unaware.

In fact, we do not need a substantial theory of unconscious mental processes to see that Locke's transparency claim is odd.

Even memory seems to refute the claim.

We are only thinking of a small portion of our beliefs at any one time.

We sometimes have to work to recall our experiences, or the capital of Kentucky, or the value of seventeen squared.

Recollection seems to be precisely the kind of process that shows that our beliefs are not transparent.

IV. Reason

Given that every one doesn't know some of their innate ideas and some people don't know any of them, the defender of innate ideas might claim that such ideas require development.

The whole of arithmetic and of geometry should be regarded as innate, and contained within us in a potential way, so that we can find them within ourselves by attending carefully and methodically to what is already in our minds, without employing any truth learned through experience or through being handed on by other people (Leibniz, *New Essays*, 77).

The proponent of a mature theory of innate ideas, then, can claim that we should expect our innate ideas not to be immediately apparent.

We have to reason to them, or unfold them from within.

Leibniz says that they are in us potentially.

Here is a passage which recalls both Leibniz's earlier work in "Meditation on Knowledge, Truth and Ideas," on the primary truths, as well as foreshadowing Frege's nineteenth-century work on logic.

In a larger sense, which is a good one to use if one is to have notions which are more comprehensive and determinate, any truths which are derivable from primary innate knowledge may also be called innate, because the mind can draw them from its own depths, though often only with difficulty (Leibniz, *New Essays*, 78)

Locke takes recourse to reason or development on the part of the rationalist to be a concession, and no help in avoiding the accusation of contradiction.

To make reason discover those truths thus imprinted, is to say, that the use of reason discovers to a man what he knew before: and if men have those innate impressed truths originally, and before the use of reason, and yet are always ignorant of them till they come to the use of reason, it is in effect to say, that men know and know them not at the same time (Locke, *Essay*, §I.2.9).

Locke's claim that the defenders of innate ideas must claim that we are aware of all of our ideas is implausible, turning the rationalist into a straw person. As Leibniz argues, Locke is relying on a weak principle.

I cannot accept the proposition that *whatever is learned is not innate*. The truths about numbers are in us; but we still learn them... (Leibniz, *New Essays*, 85).

For Leibniz, we have both primary innate ideas, and derivative ones.

We have to learn them all, through the use of reason, and we will never learn some of them.

Even if the Lockean accepts that the transparency thesis is indefensible, she might remain uncomfortable with Leibniz's appeal to reason.

To think that there are innate ideas that are forever inaccessible to us seems, to the empiricist, to push the claim of innateness too far.

But, we are finding ways in which the rhetoric of innateness obscures some points of agreement between the empiricists and the rationalists.

Most importantly, Leibniz's appeal to our capacity for reasoning may be compatible, to some degree, with Locke's appeal to our psychological capacities for reflection.

The question of how compatible the two view are will have to be approached by examining the differences between innate maxims and capacities.

V. Maxims and Capacities

Locke considers a possible rationalist response to the problematic contradiction of both knowing and not knowing the supposedly-innate ideas.

The rationalist can distinguish between innate maxims and innate capacities.

The capacity, they say, is innate; the knowledge acquired. But then to what end such contest for certain innate maxims? (Locke, *Essay*, §I.2.5).

If only the capacity for acquiring innate ideas is built in, then the rationalist can avoid the problematic contradiction.

The rhetoric used by Locke and Leibniz may obscure the fundamental similarities in their work. Locke admits that we have psychological capacities to reflect on our ideas, to compare and contrast and recombine ideas as we will.

Leibniz's claim that we have dispositions to discover innate ideas may be not so different.

Leibniz welcomes the view that our capacities are innate.

The actual knowledge of [propositions of arithmetic and geometry] is not innate. What is innate is what might be called the potential knowledge of them... (Leibniz, *New Essays*, 86).

But, Leibniz does not thus believe that his views are compatible with Locke's.

I have also used the analogy of a veined block of marble, as opposed to an entirely homogeneous block of marble, or to a blank tablet - what the philosophers call a *tabula rasa*. For if the soul were like such a blank tablet then truths would be in us as the shape of Hercules is in a piece of marble when the marble is entirely neutral as to whether it assumes this shape or some other. However, if there were veins in the block which marked out the shape of Hercules rather than other shapes, then that block would be more determined to that shape and Hercules would be innate in it, in a way, even though labour would be required to expose the veins and to polish them into clarity, removing everything that prevents their being seen. This is how ideas and truths are innate in us - as inclinations, dispositions, tendencies, or natural potentialities, and not as actions... (Leibniz, *New Essays*, 52).

Leibniz, taking thought to be an action, is willing to give up any claim to innate thoughts. Locke thinks that by admitting that only our capacities are innate, the rationalist must cede the argument. If the rationalist is only committed to innate capacities, then everything we know, including empirical propositions, would be innate.

Consider the obviously empirical proposition that, say, my copy of Locke's *Essay* has a white cover. In order to learn that proposition, we have to have built-in capacities for perception of colors and shapes. All learning depends on innate capacities.

But reliance on innate capacities is no evidence that the content a belief is really innate. Thus, Locke believes that he has a reductio argument against the rationalist's view.

- LR LR1. The doctrine of innate ideas amounts to no more than the claim that we have an innate capacity for receiving ideas.
 - LR2. But an innate capacity is just a potential presupposed by all acquisition of ideas, indeed all acquisition of knowledge. In other words, if logic and mathematics are innate, then so are all the beliefs that we ordinarily take to be empirical.
 - LR3. It is not the case that all knowledge is innate.
 - LRC. So, no knowledge is innate.

One way for the rationalist to avoid Locke's reductio argument is to deny LR3, and accept that all knowledge is innate.

In fact, I think that Descartes, like Plato, is best interpreted as holding the claim that everything we know is innate.

His view is more similar to Plato than is ordinarily recognized.

Leibniz, too, is best interpreted as denying LR3.

But, denying LR3 is really implausible.

It would be prudent for Leibniz to be able to resist LR in a more palatable way.

To that end, Leibniz also denies LR2, arguing that there is a difference between our uses of innate capacities to acquire ideas using the senses and our uses of innate capacities to reveal the eternal truths.

The question of whether our innate capacity allows us to find what is inside us, or whether it allows us to learn merely from experience, separates the rationalist from the empiricist.

Descartes and Leibniz are committed not merely to having innate capacities, but to the claim that significant content is built into the soul.

It is not a bare faculty, consisting in a mere possibility of understanding those truths: it is rather a disposition, an aptitude, a preformation, which determines our soul and brings it about that they are derivable from it (Leibniz, *New Essays*, 80).

It is difficult to see where exactly Locke and Leibniz part ways. Leibniz provides an evocative metaphor, of the statue of Hercules in the marble. But, to quote Berkeley, philosophers should not use metaphors. We want a literal understanding of the difference.

Locke believes that we have substantial psychological capacities, as we will see.

Leibniz believes that our innate ideas are predispositions.

We are not going to be able to get Locke to admit innate ideas.

We will not get Leibniz to give them up.

But, it is worth remembering that these philosophers are not as far apart on this issue as they seem.

VI. The Temporal Order and the Order of Justification

In contrast, Locke and Leibniz do have a serious disagreement about philosophical method. Whether the rationalist is ceding the claim that what is innate are certain capacities, or demanding that innateness is more than a mere capacity, she must distinguish between the kinds of mental processes that are acceptable to the empiricist, like Locke's psychological capacities to reflect and remember, and those which go beyond the *tabula rasa*.

Locke and Leibniz can agree that our minds are born with the capacity to receive sense images. (Leibniz, who denies transeunt causation, does not in fact agree with this point, but his reasons for opposing it are not relevant here.)

Leibniz, in the most important exchange of the debate, argues that we acquire certain maxims, the innate ideas, in ways that sense experience coupled with Lockean capacities could not explain.

Nobody questions whether experience is necessary for us to have knowledge.

The question is whether experience is sufficient to account for what we know.

Descartes argued that the information that we get from the senses is just not good enough to support clear and distinct judgments about the physical world.

That is the point of the wax argument in Meditation Two.

Leibniz, foreshadowing Hume, argues that some ideas could not be acquired without positing innate ideas beyond sense experience and Locke's psychological capacities.

Although the senses are necessary for all our actual knowledge, they are not sufficient to provide it all, since they never give us anything but instances, that is particular or singular truths. But however many instances confirm a general truth, they do not suffice to establish its universal necessity; for it does not follow that what has happened will always happen in the same way (Leibniz, *New Essays*, 49).

Leibniz's argument evokes Chomsky's poverty-of-the-stimulus argument for linguistic nativism. Linguistic nativism is the claim that the most fundamental rules of language are innate in our minds. Chomsky argues that language is grown, like an appendage, rather than learned.

He argues that what he calls universal grammar (UG) is innate, since children learn too much grammar too quickly to be explained by exposure to (experience with) language.

The central argument for nativism is called a poverty of the evidence, or poverty of the stimulus, argument (POTS).

We can generate an indefinite set of sentences of a natural language like English.

In order to generate the indefinite set of sentences of a natural language, we combine lexical particles

according to certain rules of formation, called a generative grammar.

A generative grammar is a formal system that produces the infinite set, like a set of logical axioms from which we can derive all logical truths.

The POTS argument is that children learn too much grammar too quickly for us to account for their grammatical abilities on the basis of behavioral stimulus.

Chomsky considers sentences like the following two:

- 1. I wonder who the men expected to see them.
- 2. The men expected to see them.

1 and 2 each contain the same clause.

In 1, 'them' refers back to the men.

In 2, 'them' does not refer to the men, but to some other people.

If children were learning grammar behaviorally, they would make the reasonable inductive conclusion that 'them' has the same reference in each case.

But, children just do not make that kind of mistake.

Similarly, Chomsky argues that children learn without instruction that the structural similarities of 3 and 4 do not entail that they are transformable into 5 and 6, respectively.

- 3. John is easy to please.
- 4. John is eager to please.
- 5. It is easy to please John.
- 6. It is eager to please John.

Children will make the transformation from 3 to 5, but not from 4 to 6.

If they were learning grammar merely behavioristically, we would expect that they would form sentences like 6, sometimes, requiring instruction to eliminate that formation.

Such instruction is never necessary, leading us to believe that the grammatical rules are built into the brain, in some way, rather than learned.

The POTS argument also relies on the claim that children learn the lexicon (vocabulary) of their first language too quickly to be explained purely behaviorally.

While they learn the specific words behaviorally, these words must hook onto pre-existing concepts.

It is a very difficult matter to describe the meaning of a word, and such meanings have great intricacy and involve the most remarkable assumptions, even in the case of very simple concepts, such as what counts as a possible "thing." At peak periods of language acquisition, children are "learning" many words a day, meaning that they are in effect learning words on a single exposure. This can only mean that the concepts are already available, with all or much of their intricacy and structure predetermined, and the child's task is to assign labels to concepts, as might be done with very simple evidence (Chomsky, "Language and Problems of Knowledge," 689).

Thus, Chomsky concluded, our abilities to use language must be built into our brains.

Poverty of the evidence arguments, from Leibniz and Chomsky, are important because they undermine arguments, like those of Locke, which arise from the temporal order of learning.

A child knows not that three and four are equal to seven, till he comes to be able to count seven, and has got the name and idea of equality; and then, upon explaining those words, he presently assents to, or rather perceives the truth of that proposition (Locke, *Essay*, §I.2.16).

Locke is arguing that our knowledge of mathematical ideas, and logical maxims, is just like our knowledge of empirical claims.

We don't know that, say, a whale is a mammal until we have knowledge of what those terms mean. Once we learn the meanings of the terms, then we can see that 'whales are mammals' is true. Similarly, according to Locke, we learn that 3+4=7 when we learn the meanings of 3, 4, 7, +, and =. Locke says that empiricism accounts for the temporal difference in learning 3+4=7 and 18+19=37. We learn the terms of the latter sentence later.

Poverty of evidence arguments show that the temporal order of our belief acquisition is irrelevant. Indeed, reliance on the temporal order of learning to reflect the order of justification is a species of the logical error called the genetic fallacy.

It confuses the origin of one's ideas with their justification.

I may learn that 2+2=4 by counting apples, but the truth of that claim is independent of how I learned it. There is an empirical element in the learning of terms, whether mathematical terms or empirical ones, and of associating terms with ideas.

But, justification is independent of the temporal order.

We are aware of particular claims before we know general ones.

But, the general truths are more fundamental, in the order of justification, than the particular ones.

We seek to reduce our knowledge to knowledge of axioms, which are simple and most perspicuous.

Leibniz puts the point in terms of intellectual ideas, which are the innate ones.

Intellectual ideas, from which necessary truths arise, do not come from the senses...It is true that explicit knowledge of truths is subsequent (in temporal or natural order) to the explicit knowledge of ideas; as the nature of truths depends upon the nature of ideas, before either are explicitly formed, and truths involving ideas which come from the senses are themselves at least partly dependent on the senses. But the ideas that come from the senses are confused; and so too, at least in part, are the truths which depend on them; whereas intellectual ideas, and the truths depending on them, are distinct, and neither [the ideas nor the truths] originate in the senses; though it is true that without the senses we would never think of them (Leibniz, *New Essays*, 81).

Leibniz's rejection of the importance of temporal order in learning is probably the most important lesson we can take from the debate between Locke and Leibniz over innate ideas.

The question whether mathematical truths are innate can not be decided by observing how we learn them. We have to look at the way in which we justify them, and their character.

We have to see whether sense experience is sufficient for their justification, or whether we have to posit an innate capacity, or disposition, to learn them.

The rationalist accepts, in general, all the psychological capacities that the empiricist accepts.

Thus, from a purely methodological perspective, the burden of proof is on the rationalists to establish the existence of innate ideas.

Still, if the empiricist wants us to believe that there are no innate ideas, she must present a plausible positive account of our knowledge of mathematics.

That's Locke's task, and ours, for next class.